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SECOND EDITION

JOHNATHAN MUN

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Tools and Techniques for Valuing Strategic Investments and Decisions

Second Edition

JOHNATHAN MUN

WILEY

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Real Options Analysis

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Specially dedicated to my wife, Penny, the love and sunshine of my life, for these amazing first few years, and with great anticipation of many more to come. Without your encouragement, advice, and support, this second edition would never have taken off.

"If you will walk in my ways and keep my requirements,
then you will govern my house and have charge of my courts,
and I will give you a place among these standing here."

Zechariah 3:7 (NIV)

About the Author

Dr. Johnathan C. Mun is the founder and CEO of Real Options Valuation, Inc., a consulting, training, and software development firm specializing in real options, employee stock options, financial valuation, simulation, forecasting, and risk analysis, located in northern California. He is the creator of the Super Lattice Solver software, Risk Simulator software, and Employee Stock Options Valuation software at the firm. The Super Lattice Solver software showcased in this book supersedes the previous Real Options Analysis Toolkit software that he also developed. He has also authored numerous other books including *Real Options Analysis Course: Business Cases* (Wiley 2003), *Applied Risk Analysis: Moving Beyond Uncertainty* (Wiley 2003), *Valuing Employee Stock Options* (Wiley 2004), and *Modeling Risk: Monte Carlo Simulation, Real Options Analysis, Forecasting, and Optimization* (Wiley 2006). His books and software are being used around the world at top universities (including the Bern Institute in Germany, Chung-Ang University in South Korea, Georgetown University, ITESM in Mexico, Massachusetts Institute of Technology, New York University, Stockholm University in Sweden, University of the Andes in Chile, University of Chile, University of Pennsylvania Wharton School, University of York in the United Kingdom, and Edinburgh University in Scotland, among others).

Dr. Mun is also currently a finance and economics professor and has taught courses in financial management, investments, real options, economics, and statistics at the undergraduate and the graduate MBA levels. He is currently teaching and has taught at universities all over the world, from the University of Applied Sciences (Switzerland and Germany) and U.S. Naval Postgraduate School to Golden Gate University (California) and St. Mary's College (California), and has chaired many graduate research thesis committees. He was formerly the vice president of analytics at Decisioneering, Inc., where he headed up the development of real options and financial analytics software products, analytical consulting, training, and technical support, and where he was the creator of the Real Options Analysis Toolkit software, the older predecessor of the Super Lattice Software discussed in this book. Prior to joining Decisioneering, he was a consulting manager and financial economist in the Valuation Services and Global Financial Services practice of KPMG Consulting and a manager with the Economic Consulting

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Dr. Mun received his Ph.D. in finance and economics from Lehigh University, where his research and academic interests were in the areas of Investment Finance, Econometric Modeling, Financial Options, Corporate Finance, and Microeconomic Theory. He also has an M.B.A. in business administration, an M.S. in management science, and a B.S. in biology and physics. He is Certified in Financial Risk Management (FRM), Certified in Financial Consulting (CFC), and is Certified in Risk Analysis (CRA). He is a member of the American Mensa, Phi Beta Kappa Honor Society, and Golden Key Honor Society as well as several other professional organizations, including the Eastern and Southern Finance Associations, American Economic Association, and Global Association of Risk Professionals. Finally, he has written many academic articles published in the *Journal of the Advances in Quantitative Accounting and Finance*, the *Global Finance Journal*, the *International Financial Review*, the *Journal of Financial Analysis*, the *Journal of Applied Financial Economics*, the *Journal of International Financial Markets, Institutions and Money*, the *Financial Engineering News*, and the *Journal of the Society of Petroleum Engineers*.

Preface

Real Options Analysis, Second Edition, provides a novel view of evaluating capital investment strategies by taking into consideration the strategic decision-making process. The book provides a qualitative and quantitative description of real options, the methods used in solving real options, why and when they are used, and the applicability of these methods in decision making. In addition, multiple business cases and real-life applications are discussed. This includes presenting and framing the real options problems, as well as introducing a stepwise quantitative process developed by the author for solving these problems using the different methodologies inherent in real options. Included are technical presentations of models and approaches used as well as their theoretical and mathematical justifications.

The book is divided into three parts: the qualitative discussions of real options; the quantitative analysis and mathematical concepts; and practical software and business case applications. The first part looks at the qualitative nature of real options, providing actual qualitative business cases and scenarios of real options in the industry, as well as high-level explanations of how real options provide the much-needed insights in decision making. The second part of the book looks at the step-by-step quantitative analysis, complete with worked-out examples and mathematical formulae. The third part illustrates the use of the Real Options Valuation's Super Lattice Solver software and Risk Simulator software, both developed by the author and included in the enclosed CD-ROM (standard 30-day trial with extended academic license). In this section, more detailed quantitative business cases are solved using the software.

This second edition provides many updates including:

- A trial version and introduction to the Super Lattice Solver software that supersedes the author's older Real Options Analysis Toolkit software (all bugs and computational errors have been fixed and verified).
- A trial version and introduction to the Risk Simulator software for running Monte Carlo simulation, forecasting, and optimization also created by the author.

- Extended examples and step-by-step computations of American, Bermudan, European, and customized options (including abandon, barrier, chooser, contraction, expansion, and other options).
- More extensive coverage of advanced and exotic real and financial options (multiple-phased sequential compound options, complex sequential compound options, barrier options, trinomial mean-reverting options, quadranomial jump-diffusion options, pentanomial dual-asset rainbow options, multiple-asset with multiple-phased options, engineering your own exotic options, and so forth).
- Extended real options cases with step-by-step worked-out solutions using the Super Lattice Solver software.
- Several brand new case studies on applying real options in the industry (manufacturing, pharmaceutical, biotechnology, real estate, Department of Defense, and others).
- An extended discussion on volatility estimates, risk, and uncertainty.

This book is targeted at both the uninitiated professional as well as those well-versed in real options applications. It is also applicable for use as a second-year M.B.A. level textbook or introductory Ph.D. reference book.

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Chapter Summaries

CHAPTER 1: A NEW PARADIGM?

Introduction

This chapter looks at the issues of new decision-making challenges and provides an introduction to real options analysis as the solution to these new challenges. The chapter briefly defines real options analysis and its many forms, when it is used, who has used it in the past, and why it is used. Examples provided come from multiple industries, including oil and gas exploration and production, pharmaceutical research and development, e-commerce valuation, IT infrastructure investment justification, prioritization of venture capital investments, mergers and acquisitions, research and development, Internet start-up valuation, structuring of venture capital contracts, timing of investments, parallel portfolio development, profitability profiling, and so forth. The chapter also profiles the types of options, defines real options analysis, and introduces several sample business cases of how real options are used as well as quotations of what the experts are saying. Finally, actual business cases from industry are provided in the appendixes. These appendixes are contributed by major corporations detailing the applications of real options in their respective companies.

A Paradigm Shift

The new economy provides a challenge for the corporate decision-maker. Corporate valuation may no longer depend on traditional fundamentals but rather on future expectations. Investment strategies with high risks and uncertainty or irreversible corporate decisions coupled with managerial flexibility provide the best candidates for real options. In this chapter, the reader will find that real options analysis is indeed a new way of thinking rather than simply the application of advanced analytical procedures.

Sample Business Cases Where Traditional Approaches Break Down

These sections introduce the issues, concerns, and problems of traditional methods, issues that are addressed using a real options framework. The sections also introduce several business cases requiring the use of real options analysis. These cases include IT investments in a new operating system, prioritizing e-commerce strategies, pharmaceutical research and development, oil and gas exploration, manufacturing contractual decisions, valuation of different venture capital opportunities, capital structuring and valuation of an Internet start-up firm, and selecting capital investment projects within the context of a portfolio. In each of these cases, the reader delves into the minds of people closest to the analysis and decision-making process, and examines their thinking and analytical approach.

The Real Options Solution and Issues to Consider

These two sections detail the use of real options in terms of thinking strategically, identifying strategic optionalities, valuing and prioritizing strategies, optimizing and timing strategies, as well as the overall management of strategies. In addition, they describe where real options value comes from and why in certain cases the true value of a project may be less than its option value.

Industry Leaders Embracing Real Options

This section details actual corporate cases and Fortune 500 firms embracing this new valuation concept. Firms highlighted include General Motors, HP-Compaq, Boeing, and AT&T. Included are consulting success stories of how these firms have looked at business decisions through the lens of real options. More industry cases are provided in the appendixes.

What the Experts Are Saying

This section details what the experts are saying in terms of the uses of real options, including quotations from the *Wall Street Journal, Business Week, Harvard Business Review, CFO,* and others. The upshot is that firms are fast embracing this new hot valuation approach, which has the potential of being the next new business breakthrough. It would seem apparent from the brief excerpts that real options analysis is not simply a financial fad but the methodology is here to stay for the long-term.

fad = novidade; moda; capricho; mania

CHAPTER 2: TRADITIONAL VALUATION APPROACHES

Introduction

This chapter introduces the pitfalls of using only traditional discounted cash flow analysis and how a real options process framework captures the strategic valuation a traditional approach cannot. A brief overview of traditional analyses includes the income approach, the market approach, and the cost approach. In addition, the chapter focuses on the issues and concerns regarding the discounted cash flow analysis. The chapter concludes with two appendixes discussing the details of financial statement analysis and the calculation of an appropriate discount rate.

The Traditional Views

Traditional analysis includes the income, cost, and market approaches, which involve using forecast profit and loss statements, comparable multiples, ratio analysis, common sizing, and so forth. The traditional approaches view risk and return on investment in a very static view. However, not all uncertainty is risk, and not all risk is bad. Real options view capital investments in terms of a dynamic approach and view upside risk as an ally that can be capitalized on.

Practical Issues Using Traditional Valuation Methodologies

This section highlights the pitfalls of the three fundamental approaches: income approach, cost approach, and market approach. These pitfalls include the incorrect use of discount rates, risk-free rates, terminal value calculations, and others.

CHAPTER 3: REAL OPTIONS ANALYSIS

Introduction

This chapter introduces the fundamental concepts of real options through several simple examples showing why an options framework provides much better insights than traditional valuation approaches do. In order to compare the results from different approaches, a simplified example is presented, starting with traditional analyses. The example continues with the application of Monte Carlo simulation and ends with the use of real options analysis.

The Fundamental Essence of Real Options

This section starts with the example of how an analyst would perform a financial analysis for the purpose of project selection. It then shows the virtues of using simulation to capture uncertainties rather than using simple single-point estimates. The analysis is complicated further by using active and passive waiting strategies. Finally, this section demonstrates how real options can be applied to more accurately assess a project's value by better defining the variables underlying a project and its potential value creation.

The Basics of Real Options, and a Simplified Example of Real Options in Action

A simple example illustrates the power of real options through the execution of an option to wait. The option to defer the execution of a second-phase clinical trial until receiving updated news of market demand adds value to a pharmaceutical research and development division's project in general. The example uses a simple discounted cash flow model to make the case.

Advanced Approaches to Real Options, and Why Are Real Options Important?

These two sections show the importance of looking at decision-making processes as a series of dynamic options and describe the types of generic options that exist in corporate investment strategies. In addition, several advanced real options techniques are introduced. Some of these techniques—for example, the use of binomial lattices, Monte Carlo simulation, partial-differential equations, and closed-form exotic options analysis—are also discussed briefly.

Comparing Traditional Approaches with Real Options

A protracted example is provided on a sample business case. The example starts from a simple static discounted cash flow analysis and proceeds with sensitivity analysis. Then an additional layer of sophistication is introduced, with the application of Monte Carlo simulation. Finally, real options analysis is applied to the problem. The results are then compared, starting with a static discounted cash flow approach, to the simulation results, as well as to the real options results.

CHAPTER 4: THE REAL OPTIONS PROCESS

Introduction and Critical Steps in Performing Real Options Analysis

This chapter introduces the eight phases in a real options process framework as developed by the author. The first phase starts with the qualification of projects through management screening, which weeds out the projects that management wishes to evaluate. The second phase starts with the construction of a traditional discounted cash flow model under the base case condition. Next, Monte Carlo simulation is applied, and the results are in turn inserted directly into the real options analysis. This phase covers the identification of strategic options that exist for a particular project under review. Based on the type of problem framed, the relevant real options models are chosen and executed. Depending on the number of projects as well as management-set constraints, portfolio optimization is performed. The efficient allocation of resources is the outcome of this analysis. The next phase involves creating reports and explaining to management the analytical results. This step is critical in that an analytical process is only as good as its expositional ease. Finally, the last phase involves updating the analysis over time. Real options analysis adds tremendous value to projects with uncertainty, but when uncertainty becomes resolved through the passage of time, old assumptions and forecasts have now become historical facts. Therefore, existing models must be updated to reflect new facts and data. This continual improvement and monitoring is vital in making clear, precise, and definitive decisions over time.

CHAPTER 5: REAL OPTIONS, FINANCIAL OPTIONS, MONTE CARLO SIMULATION, AND OPTIMIZATION

Introduction

This chapter explains the differences between financial options and real options by first describing the fundamentals of financial options theory. The chapter then goes into the importance of Monte Carlo simulation for financial analysis and ends with the application of portfolio optimization and the efficient allocation of resources.

Real Options versus Financial Options

This section details the basics of financial options and how they relate to real options. For instance, the underlying asset in most real options analysis is nontradable—that is, there usually exists no liquid market for the asset or project in question. Nonetheless, there exist many similarities between the two, and the underlying analytics of financial options may be applicable, with a few exceptions and modifications.

Monte Carlo Simulation

How are simulation techniques important in real options analysis? This discussion explains how certain key variables are obtained through the use of Monte Carlo simulation. An example depicts the error of means and why simulation should be used when uncertainty abounds. Further examples show the different strategies that would have been executed otherwise without the use of real options.

CHAPTER 6: BEHIND THE SCENES

This chapter introduces the reader to some common types of real options analytics. The two main methods introduced are closed-form differential equations and binomial lattices through the use of risk-neutral probabilities. The advantages and disadvantages of each are discussed in detail. In addition, the theoretical underpinnings surrounding the binomial equations are demystified here, leading the reader through a set of simplified discussions on how certain binomial equations are derived.

Real Options: Behind the Scenes

This section introduces the reader to the use of binomial models and closed-form solutions, which are the two mainstream approaches, used in solving real options problems. The section also discusses the advantages and disadvantages of using each approach, while demonstrating that the results from both methods approach each other at the limit.

Binomial Lattices

The binomial lattice is introduced here, complete with the application of risk-neutral probabilities, time-steps, and jump sizes.

The Look and Feel of Uncertainty, and a Firm's Real Options Provide Value in the Face of Uncertainty

The idea of uncertainty in cash flow predictions is presented in these two sections. With the use of Monte Carlo simulation, these uncertainties can be easily captured and quantified. However, if there are strategic options in these projects, there may be value in these uncertainties, which Monte Carlo simulation alone cannot capture. The upside and downside options can be better quantified using real options analysis.

Binomial Lattices as a Discrete Simulation of Uncertainty, Risk versus Uncertainty, Hard Options versus Soft Options, and Granularity Leads to Precision

The cone of uncertainty is explained through the idea of increasing uncertainty over time. This cone of uncertainty can be captured using stochastic simulation methods, such as the use of Brownian Motions. Then a discussion contrasting risk and uncertainty is provided and the linkage among risk, uncertainty, volatility, probability, and discount rate is further explored. The section continues with the discussion of how a binomial lattice approximates the simulation of stochastic processes. Indeed, the binomial lattice is a discrete simulation and, at the limit, approaches the results generated using continuous stochastic process simulation techniques, which can be solved using closed-form approaches.

An Intuitive Look at Binomial Equations, and Frolicking in a Risk-Neutral World

These sections look at the binomial equations and how they can be explained intuitively, without the need for difficult and high-level mathematics. The equations include the use of up and down jump-steps as well as the use of risk-neutral probabilities.

CHAPTER 7: REAL OPTIONS MODELS

This chapter looks at the different types of strategic real options, providing a step-by-step methodology in solving these options. The options covered include the options to abandon, expand, contract, and choose. In addition, compound options, changing strike options, changing volatility options, and sequential compound options are discussed. These basic option types provide the building blocks in analyzing more complex real options as discussed in the following chapters, including building more sophisticated real options models such as those included in the CD-ROM.

These different real options sections walk the reader through calculating by hand the various real options models. These models include using the binomial lattices and closed-form approaches. Examples of options calculated include the option to expand, contract, barrier, salvage, switch, and so on. There are also several technical appendixes on the derivation of the appropriate volatility estimate, a discussion of the Black-Scholes model, the use of path-dependent valuation using market-replicating portfolios, an example static binomial model, sensitivity models, reality checks, and trinomial lattices.

CHAPTER 8: ADDITIONAL ISSUES IN REAL OPTIONS

The additional issues in real options are discussed here, including exit and abandonment options, timing options, compound options, and the use of stochastic optimization. A discussion of the inappropriate use of decision trees is also included. Three technical appendixes follow the chapter, providing insights into different stochastic processes, differential equations, and a barrage of exotic options models.

The options models start from a simple European Black-Scholes model and extend to Black-Scholes with dividend outflows, chooser options, complex options, compound options, floating strike options, fixed strike options, forward start options, jump-diffusion options, spread options, discrete time switch options, and two correlated asset options. The approaches for estimating American-type options are also discussed.

CHAPTER 9: INTRODUCTION TO THE REAL OPTIONS VALUATION'S SUPER LATTICE SOLVER SOFTWARE AND RISK SIMULATOR SOFTWARE

This chapter introduces the readers to the author's Super Lattice Solver (SLS) and Risk Simulator software, trial versions of which are included in the CD-ROM.

The SLS software comprises several different modules. The Single Asset SLS is used for solving simple to complex and customized American, Bermudan, and European financial and real options with one underlying asset. The types of options solved include among others, the abandonment, American, barrier, Bermudan, chooser, contraction, deferment, European, expansion, and plain-vanilla options. The Multiple Asset SLS is used for solving options with multiple underlying assets and/or multiple-phased options. The types of options solved include multistaged sequential compound options, complex custom sequential options, multiple asset simultaneous compound options, options with multiple underlying assets, and switching options. The Multinomial SLS is used to solve mean-reverting options using trinomial lattices, jump-diffusion options using quadranomial lattices, and dual-asset rainbow options using pentanomial lattices. Excel-based SLS functions are also shown, where real options can be solved in existing Excel models (this allows

Monte Carlo simulation and optimization to be run on the results), as well as sample audit sheets generated by the SLS software.

The author's own Risk Simulator software is also introduced. This software is used to perform Monte Carlo simulation, time-series forecasting, and stochastic optimization within the Excel spreadsheet environment. Step-by-step getting started illustrations are presented in this chapter. It is also used for running regular simulations, nonparametric simulations, multivariate regressions, nonlinear extrapolations, stochastic processes, time-series analysis, sensitivity analysis, tornado and spider charts, bootstrapping, hypothesis testing, and many other methodologies.

CHAPTER 10: REAL OPTIONS VALUATION APPLICATION CASES

In this chapter, American, Bermudan, European, and Customized options are introduced and solved using the author's Super Lattice Solver software. The types of options introduced and solved include:

- American, European, Bermudan, and Customized Abandonment Options
- American, European, Bermudan, and Customized Contraction Options
- American, European, Bermudan, and Customized Expansion Options
- Contraction, Expansion, and Abandonment Options
- American, European, Bermudan, and Customized Call and Put Options
- Exotic Chooser Options
- Multiphased Complex Sequential Compound Options
- Multiphased Simultaneous Compound Options
- Mean-Reverting Options
- Jump-Diffusion Options
- Dual-Asset Rainbow Options
- Barrier Options (Upper, Lower, and Double-Barrier Options)
- Employee Stock Options (with Suboptimal Exercise Behavior Multiples, Forfeitures, Vesting, and Blackout Periods)

Other topics discussed include optimal timing and optimal trigger values in real options: path dependent, path independent, mutually exclusive, nonmutually exclusive, and complex nested options, as well as dominant and dominated options. Additional student exercises are included in this chapter.

CHAPTER 11: REAL OPTIONS CASE STUDIES

This chapter provides many solved case studies in various industries using real options and financial options. The cases are solved by illustrating the use of real options framing exercises. The cases show how the real or financial options are first framed in strategy trees and then solved using the Super Lattice Solver software. The cases introduced in this chapter include:

- High-tech manufacturing: Build or buy decision with real options.
- Financial options: Convertible warrants with a vesting period and put protection.
- Pharmaceutical development: Value of perfect information and trigger values.
- Oil and gas: Farm outs, options to defer, and value of information.
- Valuing employee stock options under 2004 FAS 123.
- Integrated risk modeling: Applying simulation, forcasting, and optimization on real options.
- Biopharmaceutical industry: Valuing strategic manufacturing flexibility.
- Real estate: Alternative use and development.
- United States Navy: Strategic flexibility in mission control centers.

CHAPTER 12: RESULTS INTERPRETATION AND PRESENTATION

This chapter walks the reader through the results and sample reports that should be generated by a real options analyst. The chapter includes information to help the reader in interpreting the results and being able to bring the results from the analyst's desktop to the desktop of the CEO.

How do you broach the subject of real options to management? What are the links between traditional approaches versus more advanced analytical approaches? Will management "bet the farm" based on a single number generated through a fancy mathematical model the analyst can't even interpret? This chapter provides a step-by-step methodology in presenting and explaining to management a highly complicated set of analyses through the eyes of an analyst. Complete with graphical displays, charts, tables, and process flows, this chapter provides a veritable cookbook of sorts, for the exposition of the results from a real options analysis.

The results interpretation and presentation proceed through 13 steps. The steps include comparing real options analysis with traditional financial analysis, comparing their similarities, and highlighting their differences. Next, the presentation shows where traditional analyses end and where the new analytics begin, through a simple-to-understand structured evaluation process. Then the results summary is presented, where different projects with different sized investments and returns are compared. This comparison is made on the basis of returns as well as risk structures. The final prognosis is presented as an impact to the bottom line for the company as a consequence of selecting different projects. A critical success factor analysis is also presented, together with its corresponding sensitivity analyses. A Monte Carlo simulation analysis is then presented as a means of identifying and measuring risks inherent in the analysis. Finally, the assumptions and results stemming from a real options analysis are discussed, as are its corresponding risk analyses.

PART One

Theory

CHAPTER 1

A New Paradigm?

INTRODUCTION

What are real options, how are companies using real options, what types of options exist, why are real options important, who uses real options, where are real options most appropriately used, and what are the experts saying about real options? This chapter attempts to demystify the concepts of real options and starts by reviewing the basics of real options as a new paradigm shift in the way of thinking about and evaluating projects. The chapter then reviews several business cases in different industries and situations involving pharmaceutical, oil and gas, manufacturing, IT infrastructure, venture capital, Internet start-ups, and e-business initiatives. The chapter then concludes with some industry “war stories” on using real options as well as a summary of what the experts are saying in journal publications and the popular press.

A PARADIGM SHIFT

In the past, corporate investment decisions were cut-and-dried. Buy a new machine that is more efficient, make more products costing a certain amount, and if the benefits outweigh the costs, execute the investment. Hire a larger pool of sales associates, expand the current geographical area, and if the marginal increase in forecast sales revenues exceeds the additional salary and implementation costs, start hiring. Need a new manufacturing plant? Show that the construction costs can be recouped quickly and easily by the increase in revenues it will generate through new and improved products, and the initiative is approved.

However, real-life business conditions are a lot more complicated. Your firm decides to go with an e-commerce strategy, but multiple strategic paths exist. Which path do you choose? What are the options that you have? If you choose the wrong path, how do you get back on the right track? How do you value and prioritize the paths that exist? You are a venture capital firm with multiple business plans to consider. How do you value a start-up firm with no proven track record? How do you structure a mutually beneficial investment deal? What is the optimal timing to a second or third round of financing?

Real options are useful not only in valuing a firm through its strategic business options but also as a strategic business tool in capital investment decisions. For instance, should a firm invest millions in a new e-commerce initiative? How does a firm choose among several seemingly cashless, costly, and unprofitable information technology infrastructure projects? Should a firm indulge its billions in a risky research and development initiative? The consequences of a wrong decision can be disastrous or even terminal for certain firms. In a traditional discounted cash flow (DCF) model, these questions cannot be answered with any certainty. In fact, some of the answers generated through the use of the traditional discounted cash flow model are flawed because the model assumes a static, one-time decision-making process while the real options approach takes into consideration the strategic managerial options certain projects create under uncertainty and management's flexibility in exercising or abandoning these options at different points in time, when the level of uncertainty has decreased or has become known over time.

The real options approach incorporates a learning model such that management makes better and more informed strategic decisions when some levels of uncertainty are resolved through the passage of time. The discounted cash flow analysis assumes a static investment decision, and assumes that strategic decisions are made initially with no recourse to choose other pathways or options in the future. To create a good analogy of real options, visualize it as a strategic road map of long and winding roads with multiple perilous turns and forks along the way. Imagine the intrinsic and extrinsic value of having such a strategic road map or global positioning system when navigating through unfamiliar territory, as well as having road signs at every turn to guide you in making the best and most informed driving decisions. This is the essence of real options.

Business conditions are fraught with uncertainty and risks. These uncertainties hold with them valuable information. When uncertainty becomes resolved through the passage of time, actions, and events, managers can make the appropriate midcourse corrections through a change in business decisions and strategies. Real options incorporate this learning model, akin to having a strategic road map, while traditional analyses that neglect this managerial flexibility will grossly undervalue certain projects and strategies.

The answer to evaluating such projects lies in real options analysis, which can be used in a variety of settings, including pharmaceutical drug development, oil and gas exploration and production, manufacturing, e-business, start-up valuation, venture capital investment, IT infrastructure, research and development, mergers and acquisitions, e-commerce and e-business, intellectual capital development, technology development, facility expansion, business project prioritization, enterprise-wide risk management, business unit capital budgeting, licenses, contracts, intangible asset valuation, and the like. The following section illustrates some business cases and how real options can assist in identifying and capturing additional strategic value for a firm.

EXPANSION AND COMPOUND OPTIONS: THE CASE OF THE OPERATING SYSTEM

You are the Chief Technology Officer of a large multinational corporation, and you know that your firm's operating systems are antiquated and require an upgrade, say to the new Microsoft Windows XP or Server 2003 series. You arrange a meeting with the CEO, letting him in on the situation. The CEO quips back immediately, saying that he'll support your initiative if you can prove to him that the monetary benefits outweigh the costs of implementation—a simple and logical request. You immediately arrange for a demonstration of the new operating system, and the highly technical experts from Microsoft provide you and your boss a marvelous presentation of the system's capabilities and value-added enhancements that took in excess of a few billion dollars and several years to develop. The system even fixes itself in times of dire circumstances and is overall more reliable and stable than its predecessors. You get more excited by the minute and have made up your mind to get the much-needed product upgrade. There is still one hurdle, the financial hurdle, to prove not only that the new system provides a better operating environment but also that the plan of action is financially sound. Granted, the more efficient and sophisticated system will make your boss's secretary a much happier person and hence more productive. Then again, so will an extra week's worth of vacation and a bigger bonus check, both of which are a lot cheaper and easier to implement. The new system will not help your sales force sell more products and generate higher revenues because the firm looks state-of-the-art only if a customer questions what version of Windows operating system you are using—hardly an issue that will arise during a sales call. Then again, when has using the latest software ever assisted in closing a deal, especially when you are a contract global-freight and logistics solutions provider?

You lose sleep over the next few days pondering the issue, and you finally decide to assemble a task force made up of some of your top IT personnel. The six of you sit in a room considering the same issues and trying to brainstorm a few really good arguments. You link up the value-added propositions provided in the Microsoft technician's presentation and come up with a series of potential cost reduction drivers. Principally, the self-preservation and self-fixing functionality will mean less technical assistance and help-desk calls, freeing up resources and perhaps leading to the need for fewer IT people on staff. Your mind races through some quick figures, you feel your heart pounding faster, and you see a light at the end of the tunnel. Finally you will have your long-awaited operating system, and all your headaches will go away. Wait—not only does it reduce the help-desk time, but also it increases efficiency because employees will no longer have to call or hold for technical assistance.

Your team spends the next few days scouring through mountains of data on help-desk calls and issues—thank God for good record-keeping and relational databases. Looking for issues that could potentially become obsolete with the new system, you find that at least 20 percent of your help-desk calls could be eliminated by having the new system in place because it is more stable, is capable of self-fixing these critical issues, can troubleshoot internal hardware conflicts, and so forth. Besides, doesn't employee morale count? Satisfied with your analysis, you approach the CEO and show him your findings.

Impressed with your charts and analytical rigor in such a short time frame, he asks several quick questions and points out several key issues. The cost reduction in technical assistance is irrelevant because you need these people to install and configure the new system. The start-up cost and learning curve might be steep, and employees may initially have a tough time adjusting to the new operating environment—help-desk calls may actually increase in the near future, albeit slowing down in time. But the firm's mission has always been to cultivate its employees and not to fire them needlessly. Besides, there are five people on staff at the help desk, and a 20 percent reduction means one less full-time employee out of 5,000 in the entire firm—hardly a cost reduction strategy! As for the boss's secretary's productivity, you noticed two first-class air tickets to Maui on his desk, and you're pretty sure one of them is for her. Your mind races with alternate possibilities—including taking a trip to Hawaii with a high-powered digital-zoom camera but deciding against it on your way out. He notices your wandering eyes and tries to change the subject. You still have not sufficiently persuaded your boss on getting the new operating system, and you are up a tree and out on a limb. Thoughts of going shopping for a camera haunt you for the rest of the day.

Sound familiar? Firms wrestle with similar decisions daily, and vendors wrestling with how to make their products more marketable have to first address this financial and strategic issue. Imagine you're the sales director for

Microsoft, or any software and hardware vendor for that matter. How do you close a sale like this?

Performing a series of simple traditional analyses using a discounted cash flow methodology or economic justification based on traditional analyses will fail miserably, as we have seen above. The quantifiable financial benefits do not exceed the high implementation costs. How do you justify and correctly value such seemingly cashless and cash-flow draining projects? The answer lies in real options. Instead of being myopic and focusing on current savings, the implementation of large-scale servers or operating systems will generate future strategic options for the firm. That is, having the servers and system in place provides you a springboard to a second-, third-, or fourth-phase IT implementation. That is, having a powerful connected system gives you the technical feasibility to pursue online collaboration, global data access, videoconferencing, digital signatures, encryption security, remote installations, document recovery, and the like, which would be impossible to do without it.

> ***An expansion option provides management the right and ability to expand into different markets, products, and strategies or to expand its current operations under the right conditions. A chooser option implies that management has the flexibility to choose among several strategies, including the option to expand, abandon, switch, contract, and so forth. A sequential compound option means that the execution and value of future strategic options depend on previous options in sequence of execution.***

Hence, the value of upgrading to a new system provides the firm an *expansion option,* which is the right and ability, but not the obligation, to invest and pursue some of these value-added technologies. Some of these technologies such as security enhancements and global data access can be highly valuable to your global freight company's supply chain management. You may further delineate certain features into groups of options to execute at the same time—that is, create a series of *sequential compound options* where the success of one group of initiatives depends on the success of another in sequence, similar to a stage-gate investment process.

Notice that using an extrapolation of the traditional analytic approaches would be inappropriate here because all these implementation possibilities are simply options that a senior manager has, and not guaranteed execution by any means. When you view the whole strategic picture, value is created and identified where there wasn't any before, thereby making you able to clearly justify financially your plans for the upgrade. You would be well on your way to getting your new operating system installed.

EXPANSION OPTIONS: THE CASE OF THE E-BUSINESS INITIATIVE

The e-business boom has been upon us for a few years now, and finally the investment bank you work for has decided to join the Internet age. You get a decree from the powers that be to come up with a solid e-commerce initiative. The CEO calls you into his office and spends an hour expounding on the wisdom of bringing the firm closer to the electronic Web. After hours of meetings, you are tasked with performing a feasibility analysis, choosing the right strategy, and valuing the wisdom of going e-commerce. Well, it sounds simple enough, or so you think.

The next two weeks are spent with boardroom meetings, conference calls with e-commerce consulting firms, and bottles of Alka-Seltzer. Being a newly endowed expert on the e-business strategies after spending two weeks in Tahiti on a supposedly world-renowned e-commerce crash course, you realize you really still know nothing. One thing is for certain: the Internet has revolutionized the way businesses are run. The traditional Sun Tzu business environment of "know thy enemy and know thyself and in a hundred battles you will be victorious" hadn't met the Internet. The competitive playing field has been leveled, and your immediate competitors are no longer the biggest threat. The biggest threat is globalization, when new competitors halfway around the world crawl out of the woodwork and take half of your market share just because they have a fancy Web site capable of attracting, diverting, and retaining Web traffic, and capable of taking orders around the world, and you don't. Perhaps the CEO's right; it's a do-or-die scenario. When a 12-year-old girl can transform her parents' fledgling trinket store into an overnight success by going to the Internet, technology seems to be the biggest foe of all. You either ride the technological wave or are swept under.

Convinced of the necessity of e-commerce and the strong desire to keep your job, you come up with a strategic game plan. You look at the e-commerce options you have and try to ascertain the correct path to traverse, knowing very well that if you pick the wrong one, it may be ultimately disastrous, for you and your firm, in that particular order. In between updating your curriculum vitae, you decide to spend some time pondering the issues. You realize that there are a large number of options in going e-commerce, and you have decided on several potential pathways to consider as they are most appropriate to the firm's core business.

Do we simply create a static Web site with nice graphics, text explaining what we do, and perhaps a nice little map showing where we are located and the hours of availability, and get fired? Do we perhaps go a little further and provide traditional banking services on the Web? Perhaps a way for our customers to access their accounts, pay bills, trade stocks, apply for loans, and perhaps get some free stock advice or free giveaways and pop-up ads to divert traffic on the Web? Perhaps we can take it to the extreme and use state-of-the-art technology to enable items like digital television access, live continuous streaming technology, equity trading on personal digital assistants and cellular phones, interaction with and direct access to floor specialists and traders on the New York Stock Exchange for the larger clients, and all the while using servers in Enron-like offshore tax havens. The potentials are endless.

You suddenly feel queasy, and the inkling of impending doom. What about competition? Ameritrade and a dozen other online trading firms currently exist. Most major banks are already on the Web, and they provide the same services. What makes us so special? Then again, if we do not follow the other players, we may be left out in the cold. Perhaps there are some ways to differentiate our services. Perhaps some sort of geographical expansion; after all, the Internet is global, so why shouldn't we be? What about market penetration effects and strategies, country risk analysis, legislative and regulatory risks? What if the strategy is unsuccessful? What will happen then? Competitive effects are unpredictable. The threats of new entrants and low barriers to entry may elicit even more competitors than you currently have. Is the firm ready to play in the big leagues and fight with the virtual offshore banking services? Globalization—what an ugly word it is right about now. What about new technology: Do we keep spending every time something new comes out? What about market share, market penetration, positioning, and being first to market with a new and exciting product? What about future growth opportunities, e-traffic management, and portal security? The lists go on and on. Perhaps you should take a middle ground, striking an alliance with established investment banking firms with the applicable IT infrastructure already in place. Why build when you can buy? You reach for your Alka-Seltzer and realize you need something a lot stronger.

How do you prioritize these potential strategies, perform a financial and strategic feasibility analysis, and make the right decision? Will the firm survive if we go down the wrong path? If we find out we are on the wrong path, can we navigate our way back to the right one? What options can we create to enable this? Which of these strategies is optimal? Upon identifying what these strategies are, including all their downstream *expansion options,* you can then value each of these strategic pathways. The identification, valuation, prioritization, and selection of strategic projects are where real options analysis can provide great insights and value. Each project initiative should not be viewed in its current state. Instead, all downstream opportunities should be viewed and considered as well. Otherwise, wrong decisions may be made because only projects with immediate value will be chosen, while projects that carry with them great future potential are abandoned simply because management is setting its sights on the short term.

EXPANSION AND SEQUENTIAL OPTIONS: THE CASE OF THE PHARMACEUTICAL R&D

Being the chief chemist of a small pharmaceutical firm that is thinking of developing a certain drug useful in gene therapy, you have the responsibility to determine the right biochemical compounds to create. Understanding very well that the future of the firm rests on pursuing and developing the right portfolio of drugs, you take your evaluation task rather seriously. Currently, the firm's management is uncertain whether to proceed with developing a group of compounds and is also uncertain regarding the drug development's financial feasibility. From historical data and personal experience, you understand that development "home runs" are few and far between. As a matter of fact, you realize that less than 5 percent of all compounds developed are superstars. However, if the right compounds are chosen, the firm will own several valuable patents and bolster its chances of receiving future rounds of funding. Armed with that future expectation, you evaluate each potential compound with care and patience.

For example, one of the compounds you are currently evaluating is called Creatosine. Management knows that Creatosine, when fully developed, can be taken orally, but has the potential to be directly injected into the bloodstream, which increases its effectiveness. As there is great uncertainty in the development of Creatosine, management decides to develop the oral version for now and wait for a period of several years before deciding on investing additional funds to develop the injectable version. Thus, management has created an *expansion option*—that is, the option but not the obligation to expand Creatosine into an injectable version at any time between now and several years. The firm thus creates no value in developing the injection version after that time period. By incorporating real options strategy, your firm has mitigated its risks in developing the drug into both an oral and injectable form at initiation. By waiting, scientific and market risks become resolved through the passage of time, and your firm can then decide whether to pursue the second injectable phase. This risk-hedging phenomenon is common in financial options and is applicable here for real options.

However, there are other drug compounds to analyze as well. You go through the list with a fine-tooth comb and realize that you must evaluate each drug by not only its biochemical efficacies, but also by its financial feasibility. Given the firm's current capital structure, you would need to not only value, prioritize, and select the right compounds, but also find the optimal portfolio mix of compounds, subject to budget, timing, and risk constraints. On top of that, you would have to value your firm as a whole in terms of a portfolio of strategic options. The firm's value lies not only in its forecast revenues less its costs subject to time valuation of money but also in all the current research and development initiatives under way, where a single home run will double or triple the firm's valuation. These so-called future *growth options,* which are essentially growth opportunities that the firm has, are highly valuable. These *growth options* are simply *expansion options* because your firm owns the right infrastructure, resources, and technology to pursue these future opportunities but not the obligation to do so unless both internal research and external market conditions are amenable.

Another approach you decide to use is to create a strategic development road map, knowing that every drug under development has to go through multiple phases. At each phase, depending on the research results, management can decide to continue its development to the next phase or abandon it assuming it doesn't meet certain prespecified criteria. That is, management has the *option to choose* whether a certain compound will continue to the next stage. Certain drugs in the initial phases go through a *sequential compound option,* where the success of the third phase, for example, depends on the success of the second phase, which in turn depends on the success of the first phase in the stage-gate drug development cycle. Valuing such sequences of options using a traditional approach of taking expected values with respect to the probabilities of success is highly dubious and incorrect. The valuation will be incorrect at best and highly misleading at worst, driving management to select the wrong mix of compounds at the wrong time.

EXPANSION AND SWITCHING OPTIONS: THE CASE OF THE OIL AND GAS EXPLORATION AND PRODUCTION

The oil and gas industry is fraught with strategic options problems because oil and gas exploration and production involves significant amounts of risk and uncertainty. For example, when drilling for oil, the reservoir properties, fluidic properties, trap size and geometry, porosity, seal containment, oil and gas in place, expulsion force, losses due to migration, development costs, and so forth are all unknowns. How then is a reservoir engineer going to recommend to management the value of a particular drill site? Let's explore some of the more frequent real options problems encountered in this industry.

Being a fresh M.B.A. graduate from a top finance program, you are hired by a second-tier independent oil and gas firm, and your first task is to value several primary and secondary reservoir recovery wells. You are called into your boss's office, and she requests you to do an independent financial analysis on a few production wells. You were given a stack of technical engineering documents to review. After spending a fortnight scouring through several books on the fundamentals of the oil and gas industry, you finally have some basic understanding of the intricacies of what a secondary recovery well is. Needing desperately to impress your superiors, you decide to investigate a little further into some new analytics for solving these types of recovery-well problems.

Based on your incomplete understanding of the problem, you begin to explore all the possibilities and come to the conclusion that the best analytics to use may be the application of a Monte Carlo simulation and real options analysis. Instead of simply coming up with the value of the project, you decide to also identify where value can be added to the projects by incorporating strategic real optionality.

Suppose that the problem you are analyzing is a primary drilling site that has its own natural energy source, complete with its gas cap on one side and a water drive on the other. These energy sources maintain a high upward pressure on the oil reservoir to increase the ease of drilling and, therefore, the site's productivity. However, knowing that the level of energy may not be sustainable for a long time and its efficacy is unknown currently, you recognize that one of the strategies is to create an *expansion option* to drill a secondary recovery well near the primary site. Instead of drilling, you can use this well to inject water or gas into the ground, thereby increasing the upward pressure and keeping the reservoir productive. Building this secondary well is an option and not an obligation for the next few years.

The first recommendation seems to make sense given that the geological structure and reservoir size are difficult to estimate. Yet these are not the only important considerations. The price of oil in the market is also something that fluctuates dramatically and should be considered. Assuming that the price of oil is a major factor in management's decisions, your second recommendation includes separating the project into two stages. The first stage is to drill multiple wells in the primary reservoir, which will eventually maximize on its productivity. At that time a second phase can be implemented through smaller satellite reservoirs in the surrounding areas that are available for drilling but are separated from the primary reservoir by geological faults. This second stage is also an *expansion option* on the first; when the price of oil increases, the firm is then able to set up new rigs over the satellite reservoirs, drill, and complete these wells. Then, using the latest technology in subsurface robotics, the secondary wells can be tied back into the primary platform, thereby increasing and expanding the productivity of the primary well by some expansion factor. Obviously, although this is a strategic option that the firm has, the firm does not have the obligation to drill secondary wells unless the market price of oil is favorable enough. Using some basic intuition, you plug some numbers into your models and create the optimal oil price levels such that secondary drillings are profitable. However, given your brief conversation with your boss and your highly uncertain career future, you decide to dig into the strategy a little more.

Perhaps the company already has several producing wells at the reservoir. If that is so, the analysis should be tweaked such that instead of being an *expansion option* by drilling more wells, the firm can retrofit these existing wells in strategic locations from producers into injectors, creating a *switching option*. Instead of drilling more wells, the company can use the existing wells to inject gas or water into the surrounding geological areas in the hopes that this will increase the energy source, forcing the oil to surface at a higher rate. Obviously, these secondary production wells should be switched into injectors when the recovery rate of the secondary wells is relatively low and the marginal benefits of the added productivity on primary wells far outstrip the retrofit costs. In addition, some of the deep-sea drilling platforms that are to be built in the near future can be made into *expansion options,* where slightly larger platforms are built at some additional cost (premium paid to create this option), such that if oil prices are optimally high, the flexible capacity inherent in this larger platform can be executed to boost production.

Finally, depending on the situation involved, you can also create a *sequential compound option* for the reservoir. That is, the firm can segregate its activities into different phases. Specifically, we can delineate the strategic option into four phases. Phases I to III are exploration wells, and Phase IV is a development well.

Phase I: Start by performing seismic surveys to get information on the structures of subsurface reservoirs (the costs incurred include shooting the survey, processing data, mapping, etc.).

Phase II: If autoclines and large structures are found, drill an exploration well; if not, then abandon now.

Phase III: If the exploration well succeeds industrially or commercially (evaluated on factors such as cost, water depth, oil price, rock, reservoir, and fluid properties), drill more delineation or "step out" wells to define the reservoir.

Phase IV: If the reservoir is productive enough, commit more money for full development (platform building, setting platform, drilling development wells).

A switching option provides the right and ability but not the obligation to switch among different sets of business operating conditions, including different technologies, markets, or products.

ABANDONMENT OPTIONS: THE CASE OF THE MANUFACTURER

You work for a midsized hardware manufacturing firm located in the heartland of America. Having recently attended a corporate finance seminar on real options, you set out to determine whether you can put some of your newfound knowledge to good use within the company. Currently, your firm purchases powerful laser-guided robotic fabrication tools that run into tens and even hundreds of millions of dollars each. These tools have to be specially ordered more than a year in advance, due to their unique and advanced specifications. They break down easily, and if any one of the three machines that your firm owns breaks down, it may be disastrous because part of the manufacturing division may have to be shut down temporarily for a period exceeding a year. So, is it always desirable to have at least one fabrication tool under order at all times, just as a precaution? A major problem arises when the newly ordered tool arrives, but the three remaining ones are fully functional and require no replacement. The firm has simply lost millions of dollars. In retrospect, certainly having a backup machine sitting idle that costs millions of dollars is not optimal. However, millions can also be lost if indeed a tool breaks down and a replacement is a year away. The question is, what do you do, and how can real options be used in this case, both as a strategic decision-making tool and as a valuation model?

Using traditional analysis, you come to a dead end, as the tool's breakdown has never been consistent and the ordered parts never arrive on schedule. Turning to real options, you decide to create a strategic option with the vendor. Instead of having to wait more than a year before a new machine arrives, while during that time not knowing when your existing machines will break down, you decide to create a mutually agreeable contract. Your firm decides to put up a certain amount of money and to enter into a contractual agreement whereby the vendor will put you on its preferred list. This cuts down delivery time from one year to two months. If your firm does not require the equipment, you will have to pay a penalty exit fee equivalent to a certain percentage of the machine's dollar value amount, within a specified period, on a ratcheted scale, with different exit penalties at different exit periods. In essence, you have created an *abandonment option* whereby your firm has the right not to purchase the equipment should circumstances force your hand, but hedging yourself to obtain the machine at a moment's notice should there be a need. The price of the option's premium is the contractual price paid for such an arrangement. The savings come in the form of not having to close down part of your plant and losing revenues. By incorporating real options insights into the problem, the firm saves millions and ends up with the optimal decision.

EXPANSION AND BARRIER OPTIONS: THE CASE OF THE LOST VENTURE CAPITALIST

You work in a venture capital firm and are in charge of the selection of strategic business plans and performing financial analysis on their respective feasibility and operational viability. The firm gets more than a thousand business plans a year, and your boss does not have the time to go through each of them in detail and relies on you to sniff out the ones with the maximum potential in the least amount of time. Besides, the winning plans do not wait for money. They often have money chasing after them. Having been in the field of venture capital funding for 10 years and having survived the bursting of the dot-com bubble, your judgment is highly valued in the firm, and you are more often than not comfortable with the decisions made. However, with the changing economic and competitive landscape, even seemingly bad ideas may turn into the next IPO success story. Given the opportunity of significant investment returns, the money lost on bad ideas is a necessary evil in not losing out on the next eBay or Yahoo! just because the CEO is not a brilliant business plan author. Your qualitative judgment may still be valid, but the question is what next? What do you do after you've selected your top 100 candidates? How do you efficiently allocate the firm's capital to minimize risk and maximize return? Picking the right firms the wrong way only gets you so far, especially when banking on start-ups hoping for new technological breakthroughs. A diversified portfolio of firms is always prudent, but a diversified portfolio of the right firms is much better. Prioritizing, ranking, and coming up with a solid financing structure for funding start-ups is tricky business, especially when traditional valuation methodologies do not work.

The new economy provides many challenges for the corporate decision maker. Market equity value of a firm now depends on expectations and anticipation of future opportunities in novel technologies rather than on a traditional bricks-and-mortar environment. This shift in the underlying fundamentals from tangible goods to technological innovation has created an issue in valuing the firm. Even the face of the intangibles created by technological innovation has changed. In most cases, a significant portion of a firm's value or its strategic investment options is derived from the firm's intangibles. Intangibles generally refer to elements in a business that augment the revenue-generating process but do not themselves have a physical or monetary appearance while still holding significant value to the firm. Intangibles may range from more traditional items like intellectual property, property rights, patents, branding, and trademarks to a new generation of so-called e-intangibles created in the new economy.

Examples of this new generation of e-intangibles include items like marketing intangibles, process and product technologies, trade dress, customer loyalty, branding, proprietary software, speed, search engine efficiency, online data catalogs, server efficiency, traffic control and diversion, streaming technology, content, experience, collaborative filtering, universal-resource-locator-naming conventions, hubs, Web page hits, imprints, blogs, and community relationships. New entries in the e-commerce economy over the past few years include the financial sector (bank wires, online bill payments, online investing), health care sector (cross-border medical teaching), publication and retail auctions (e-pocket books, Web magazines, Web papers, eBay, Web-Van, Auto-Web). The new trend seems to continue, and new start-ups emerge in scores by the minute to include sophisticated and complex structures like online cross-border banking services, virtual offshore banks, cross-border medical diagnostic imaging, and online-server game playing. However, other less sophisticated e-business strategies have also been booming of late, including service-based Web sites, which provide a supposedly value-added service at no charge to consumers, such as online greeting cards and online e-invitations. Lower barriers to entry and significant threat of new entrants and substitution effects characterize these strategies.

Even using fairly well-known models like the discounted cash flow analysis is insufficient to value these types of firms. For instance, as a potential venture capitalist, how do you go about identifying the intangibles and intellectual property created when traditional financial theory is insufficient to justify or warrant such outrageous price-to-earnings multiples? Trying to get on the bandwagon in initial public offerings with large capital gains is always a good investment strategy, but randomly investing in start-ups with little to no fundamental justification of potential future profitability is a whole other issue. Perhaps there is a fundamental shift in the way the economy works today or is expected to work in the future as compared to the last decade. Whether there is indeed an irrational exuberance in the economy, or whether there is perhaps a shift in the fundamentals, we need a newer, more accurate, and sophisticated method of quantifying the value of such intangibles.

How do you identify, value, select, prioritize, justify, optimize, time, and manage large corporate investment decisions with high levels of uncertainty such that when a decision is made, the investment becomes irreversible? How do you value and select among several start-up firms to determine whether they are ideal venture candidates, and how do you create an optimal financing structure? These types of cashless return investments provide no immediate increase in revenues, and the savings are only marginal compared to their costs. How do you justify such outrageous market equity prices?

A barrier option means that the execution and value of a strategic option depend on either breaching or not breaching an artificial barrier.

There must be a better way to value these investment opportunities. Having read press releases by Motley Fool on Credit Suisse First Boston, and how the firm used real options to value stocks of different companies, you begin looking into the possibilities of applying real options yourself. The start-up firm has significant value even when its cash flow situation is hardly something to be desired because the firm has strategic *growth options*. That is, a particular start-up may have some technology that may seem untested today, but it has the *option to expand* into the marketplace quickly and effortlessly should the technology prove to be highly desirable in the near future. Obviously the firm has the right to also pursue other ancillary technologies but only if the market conditions are conducive. The venture firm can capitalize on this *option to expand* by hedging itself with multiple investments within a venture portfolio. The firm can also create strategic value through setting up contractual agreements with a *barrier option* (and *option to defer*) where for the promise of seed financing, the venture firm has the right of first refusal, but not the obligation, to invest in a second or third round should the start-up achieve certain management-set goals or barriers. The cost of this *barrier option* is seed financing, which is akin to the premium paid on a stock option. Should the option be in-the-money, the option will be executed through second- and third-round financing. By obtaining this strategic option, the venture firm has locked itself into a guaranteed favorable position should the start-up be highly successful, similar to the characteristics of a financial call option of unlimited upside potential. At the same time, the venture firm has hedged itself against missing the opportunity with limited downside proportional to the expenditure of a minimal amount of seed financing.

When venture capital firms value a group of companies, they should consider all the potential upsides available to these companies. These strategic options may very well prove valuable. A venture firm can also hedge itself through the use of barrier-type or deferment options. The venture firm should then go through a process of portfolio optimization analysis to decide what proportion of its funds should be disseminated to each of the chosen firms. This portfolio optimization analysis will maximize returns and minimize the risks borne by the venture firm on a portfolio level subject to budget or other constraints.

COMPOUND EXPANSION OPTIONS: THE CASE OF THE INTERNET START-UP

In contrast, one can look at the start-up entrepreneur. How do you obtain venture funding, and how do you position the firm such that it is more attractive to the potential investor? Your core competency is in developing software or Web-enabled vehicles on the Internet, not financial valuation. How do you then structure the financing agreements such that your firm will be more attractive yet at the same time the agreements are not detrimental to your operations, strategic plans, or worse, your personal equity stake? What are your projected revenues and costs? How do you project these values when you haven't even started your business yet? Are you undervaluing your firm and its potential such that an unscrupulous venture firm will capitalize on your lack of sophistication and take a larger piece of the pie for itself? What are your strategic alternatives when you are up and running, and how do you know it's optimal for you to proceed with the next phase of your business plan?

All these questions can be answered and valued through a real options framework. Knowing what strategic options your firm has is significant because this value-added insight not only provides the firm an overall strategic road map but also increases its value. The real option that may exist in this case is something akin to a *compound expansion option*. For example, the firm can expand its product and service offerings by branching out into ancillary technologies or different applications, or expanding into different vertical markets. However, these expansions will most certainly occur in stages, and the progression from one stage to another depends heavily on the success of the previous stages.

THE REAL OPTIONS SOLUTION

Simply defined, real options is a systematic approach and integrated solution using financial theory, economic analysis, management science, decision sciences, statistics, and econometric modeling in applying options theory in valuing real physical assets, as opposed to financial assets, in a dynamic and uncertain business environment where business decisions are flexible in the context of strategic capital investment decision making, valuing investment opportunities, and project capital expenditures. Real options are crucial in:

- Identifying different corporate investment decision pathways or projects that management can navigate given the highly uncertain business conditions;
- Valuing each strategic decision pathway and what it represents in terms of financial viability and feasibility;
- Prioritizing these pathways or projects based on a series of qualitative and quantitative metrics;
- Optimizing the value of your strategic investment decisions by evaluating different decision paths under certain conditions or using a different sequence of pathways to lead to the optimal strategy;

> ***Real options are useful for identifying, understanding, valuing, prioritizing, selecting, timing, optimizing, and managing strategic business and capital allocation decisions.***

- Timing the effective execution of your investments and finding the optimal trigger values and cost or revenue drivers; and
- Managing existing or developing new optionalities and strategic decision pathways for future opportunities.

ISSUES TO CONSIDER

Strategic options do have significant intrinsic value, but this value is only realized when management decides to execute the strategies. Real options theory assumes that management is logical and competent and that it acts in the best interests of the company and its shareholders through the maximization of wealth and minimization of risk of losses. For example, suppose a firm owns the rights to a piece of land that fluctuates dramatically in price. An analyst calculates the volatility of prices and recommends that management retain ownership for a specified time period, where within this period there is a good chance that the price of real estate will triple. Therefore, management owns a call option, an *option to wait* and defer sale for a particular time period. The value of the real estate is therefore higher than the value that is based on today's sale price. The difference is simply this option to wait. However, the value of the real estate will not command the higher value if prices do triple but management decides not to execute the option to sell. In that case, the price of real estate goes back to its original levels after the specified period and then management finally relinquishes its rights.

> ***Strategic optionality value can only be obtained if the option is executed; otherwise, all the options in the world are worthless.***

Was the analyst right or wrong? What was the true value of the piece of land? Should it have been valued at its explicit value on a deterministic basis where you know what the price of land is right now and, therefore, this is its value; or should it include some type of optionality where there is a good probability that the price of land could triple in value and, hence, the piece of land is truly worth more than it is now and should therefore be valued accordingly?

The latter is the real options view. The additional strategic optionality value can only be obtained if the option is executed; otherwise, all the options in the world are worthless. This idea of explicit versus implicit value becomes highly significant when management's compensation is tied directly to the actual performance of particular projects or strategies.

To further illustrate this point, suppose the price of the land in the market is currently $10 million. Further, suppose that the market is highly liquid and volatile, and that the firm can easily sell it off at a moment's notice within the next five years, the same amount of time the firm owns the rights to the land. If there is a 50 percent chance the price will increase to $15 million and a 50 percent chance it will decrease to $5 million within this time period, is the property worth an expected value of $10 million? If prices rise to $15 million, management should be competent and rational enough to execute the option and sell that piece of land immediately to capture the additional $5 million premium. However, if management acts inappropriately or decides to hold off selling in the hopes that prices will rise even further, the property value may eventually drop back down to $5 million. Now, how much is this property really worth? What if there happens to be an *abandonment option*? Suppose there is a perfect counterparty to this transaction who decides to enter into a contractual agreement whereby for a contractual fee, the counterparty agrees to purchase the property for $10 million within the next five years, regardless of the market price and executable at the whim of the firm that owns the property. Effectively, a safety net has been created whereby the minimum floor value of the property has been set at $10 million (less the fee paid). That is, there is a limited downside but an unlimited upside, as the firm can always sell the property at market price if it exceeds the floor value. Hence, this strategic abandonment option has increased the value of the property significantly and hedged its downside risks. Logically, with this abandonment option in place, the value of the land with the option is definitely worth more than $10 million after having such a safety net or downside insurance. The question is how much this insurance is worth and only real options analysis can answer this.

INDUSTRY LEADERS EMBRACING REAL OPTIONS

Industries using real options as a tool for strategic decision making started with oil and gas as well as mining companies, and later expanded into utilities, biotechnology, pharmaceuticals, and now into telecommunications, high-tech, and across all industries. Following are some examples of how real options have been or should be used in different industries.

Automobile and Manufacturing Industry In automobile manufacturing, General Motors (GM) applies real options to create *switching options* in producing its new series of autos. This is essentially the option to use a cheaper resource over a given period of time. GM holds excess raw materials and has multiple global vendors for similar materials with excess contractual obligations above what it projects as necessary. The excess contractual cost is outweighed by the significant savings of switching vendors when a certain raw material becomes too expensive in a particular region of the world. By spending the additional money in contracting with vendors as well as meeting their minimum purchase requirements, GM has essentially paid the premium on purchasing a *switching option*. This is important especially when the price of raw materials fluctuates significantly in different regions around the world. Having an option here provides the holder a hedging vehicle against pricing risks.

Computer Industry In the computer industry, HP-Compaq used to forecast sales of printers in foreign countries months in advance. It then configured, assembled, and shipped the highly specific printers to these countries. However, given that demand changes rapidly and forecast figures are seldom correct, the preconfigured printers usually suffer a higher inventory holding cost or the cost of technological obsolescence. HP-Compaq can create an *option to wait* and defer making any decisions too early through building assembly plants in these foreign countries. Parts can then be shipped and assembled in specific configurations when demand is known, possibly weeks in advance rather than months in advance. These parts can be shipped anywhere in the world and assembled in any configuration necessary, while excess parts are interchangeable across different countries. The premium paid on this option is building the assembly plants, and the upside potential is the savings from not making wrong demand forecasts.

Airline Industry In the airline industry, Boeing spends billions of dollars and several years to decide if a certain aircraft model should even be built. Should the wrong model be tested in this elaborate strategy, Boeing's competitors may gain a competitive advantage relatively quickly. Because so many technical, engineering, market, and financial uncertainties are involved in the decision-making process, Boeing can conceivably create an *option to choose* through parallel development of multiple plane designs simultaneously, knowing very well the increased cost of developing multiple designs simultaneously with the sole purpose of eliminating all but one in the near future. The added cost is the premium paid on the option. However, Boeing will be able to decide which models to abandon or continue when these uncertainties and risks become known over time. Eventually, all the models will be eliminated save one. This way, the company can hedge itself against making the wrong initial decision and benefit from the knowledge gained through multiple parallel development initiatives.

Oil and Gas Industry In the oil and gas industry, companies spend millions of dollars to refurbish their refineries and add new technology to create an *option to switch* their mix of outputs among heating oil, diesel, and other petrochemicals as a final product, using real options as a means of making capital and investment decisions. This option allows the refinery to switch its final output to one that is more profitable based on prevailing market prices, to capture the demand and price cyclicality in the market.

Telecommunications Industry In the telecommunications industry, in the past, companies like Sprint and AT&T installed more fiber-optic cable and other telecommunications infrastructure than other companies in order to create a *growth option* in the future by providing a secure and extensive network, and to create a high barrier to entry, providing a first-to-market advantage. Imagine having to justify to the board of directors the need to spend billions of dollars on infrastructure that will not be used for years to come. Without the use of real options, this would have been impossible to justify.

Utilities Industry In the utilities industry, firms have created an *option to execute* and an option to switch by installing cheap-to-build, inefficient energy generator peaker plants only to be used when electricity prices are high and to shut down when prices are low. The price of electricity tends to remain constant until it hits a certain capacity utilization trigger level, when prices shoot up significantly. Although this occurs infrequently, the possibility still exists, and by having a cheap standby plant, the firm has created the option to turn on the switch whenever it becomes necessary, to capture this upside price fluctuation.

Real Estate Industry In the real estate arena, leaving land undeveloped creates an option to develop at a later date at a more lucrative profit level. However, what is the optimal wait time and the optimal trigger price to maximize returns? In theory, one can wait for an infinite amount of time, and real options provide the solution for the optimal timing and price-trigger value.

Pharmaceutical Research and Development Industry In pharmaceutical research and development initiatives, real options can be used to justify the large investments in what seems to be cashless and unprofitable under the discounted cash flow method but actually creates *compound expansion options* in the future. Under the myopic lenses of a traditional discounted cash flow analysis, the high initial investment of, say, a billion dollars in research and development may return a highly uncertain projected few million dollars over the next few years. Management will conclude under a net-present-value analysis that the project is not financially feasible. However, a cursory look at the industry indicates that research and development is performed everywhere. Hence, management must see an intrinsic strategic value in research and development. How is this intrinsic strategic value quantified? A real options approach would optimally time and spread the billion-dollar initial investment into a multiple-stage investment structure. At each stage, management has an *option to wait* and see what happens as well as the *option to abandon* or the *option to expand* into the subsequent stages. The ability to defer cost and proceed only if situations are permissible creates value for the investment.

High-Tech and e-Business Industry In e-business strategies, real options can be used to prioritize different e-commerce initiatives and to justify those large initial investments that have an uncertain future. Real options can be used in e-commerce to create incremental investment stages, *options to abandon,* and other future growth options, compared to a large one-time investment (invest a little now, wait and see before investing more).

Mergers and Acquisition In valuing a firm for acquisition, you should not only consider the revenues and cash flows generated from the firm's operations but also the strategic options that come with the firm. For instance, if the acquired firm does not operate up to expectations, an *abandonment option* can be executed where it can be sold for its intellectual property and other tangible assets. If the firm is highly successful, it can be spun off into other industries and verticals or new products and services can be eventually developed through the execution of an *expansion option*. In fact, in mergers and acquisition, several strategic options exist. For instance, a firm acquires other entities to enlarge its existing portfolio of products or geographic location, to obtain new technology (*expansion option*), or to divide the acquisition into many smaller pieces and sell them off as in the case of a corporate raider (*abandonment option*); or it merges to form a larger organization due to certain synergies and immediately lays off many of its employees (*contraction option*). If the seller does not value its real options, it may be leaving money on the negotiation table. If the buyer does not value these strategic options, it is undervaluing a potentially highly lucrative acquisition target.

All these cases where the high cost of implementation with no apparent payback in the near future seems foolish and incomprehensible in the traditional discounted cash flow sense are fully justified in the real options sense when taking into account the strategic options the practice creates for the future, the uncertainty of the future operating environment, and management's flexibility in making the right choices at the appropriate time.

WHAT THE EXPERTS ARE SAYING

The trend in the market is quickly approaching the acceptance of real options, as can be seen from the following sample publication excerpts.[1]

According to a *Harvard Business Review* article (December 2004):

> *Companies that rely solely on discounted cash flow (DCF) analysis underestimate the value of their projects and may fail to invest enough in uncertain but highly promising opportunities. Far from being a replacement for DCF analysis, real options are an essential complement, and a project's total value should encompass both. DCF captures a base estimate of value; real options take into account the potential for big gains.*

According to another *Harvard Business Review* article (March 2004):

> *The complexity of real options can be eased through the use of a binomial valuation model. Many of the problems with real options analysis stem from the use of the Black-Scholes-Merton model, which isn't suited to real options. Binomial models, by contrast, are simpler mathematically, and you can tinker with binomial model until it closely reflects the project you wish to value.*

According to an article in *Bloomberg Wealth Manager* (November 2001):

> *Real options provide a powerful way of thinking and I can't think of any analytical framework that has been of more use to me in the past 15 years that I've been in this business.*

According to a *Wall Street Journal* article (February 2000):

> *Investors who, after its IPO in 1997, valued only Amazon.com's prospects as a book business would have concluded that the stock was significantly overpriced and missed the subsequent extraordinary price appreciation. Though assessing the value of real options is challenging, without doing it an investor has no basis for deciding whether the current stock price incorporates a reasonable premium for real options or whether the shares are simply overvalued.*

CFO Europe (July/August 1999) cites the importance of real options in that:

> *[A] lot of companies have been brainwashed into doing their valuations on a one-scenario discounted cash flow basis . . . and sometimes our recommendations are not what intuition would suggest, and that's where the real surprises come from—and with real options, you can tell exactly where they came from.*

According to a *Business Week* article (June 1999):

> *The real options revolution in decision making is the next big thing to sell to clients and has the potential to be the next major business breakthrough. Doing this analysis has provided a lot of intuition you didn't have in the past . . . and as it takes hold, it's clear that a new generation of business analysts will be schooled in options thinking. Silicon Valley is fast embracing the concepts of real options analytics, in its tradition of fail fast so that other options may be sought after.*

In *Products Financiers* (April 1999):

> *Real options are a new and advanced technique that handles uncertainty much better than traditional evaluation methods. Since many managers feel that uncertainty is the most serious issue they have to face, there is no doubt that this method will have a bright future as any industry faces uncertainty in its investment strategies.*

A *Harvard Business Review* article (September/October 1998) hits home:

> *Unfortunately, the financial tool most widely relied on to estimate the value of a strategy is the discounted cash flow which assumes that we will follow a predetermined plan regardless of how events unfold. A better approach to valuation would incorporate both the uncertainty inherent in business and the active decision making required for a strategy to succeed. It would help executives to think strategically on their feet by capturing the value of doing just that—of managing actively rather than passively and real options can deliver that extra insight.*

This book provides a novel approach in applying real options to answering these issues and more. In particular, a real options framework is presented. It takes into account managerial flexibility in adapting to ever-changing strategic, corporate, economic, and financial environments over time as well as the fact that in the real business world, opportunities and uncertainty exist and are dynamic in nature. This book provides a real options process framework to identify, justify, time, prioritize, value, and manage corporate investment strategies under uncertainty in the context of applying real options.

The recommendations, strategies, and methodologies outlined in this book are not meant to replace traditional discounted cash flow analysis but to complement it when the situation and the need arise. The entire analysis could be done, or parts of it could be adapted to a more traditional approach. In essence, the process methodology outlined starts with traditional analyses and continues with value- and insight-adding analytics, including Monte Carlo simulation, real option analysis, and portfolio optimization. The real options approach outlined is not the only viable alternative nor will it provide a set of infallible results. However, if utilized correctly with the traditional approaches, it may lead to a set of more robust, accurate, insightful, and plausible results. The insights generated through real options analytics provide significant value in understanding a project's true strategic value.

CRITICISMS, CAVEATS, AND MISUNDERSTANDINGS IN REAL OPTIONS

Before embarking on a real options analysis, analysts should be aware of several caveats. First, the following five requirements need to be satisfied before a real options analysis can be run:

- ***A financial model must exist.*** Real options analysis requires the use of an existing discounted cash flow model, as real options build on the existing tried-and-true approaches of current financial modeling techniques. If a model does not exist, it means that strategic decisions have already been made and no financial justifications are required, and hence, there is no need for financial modeling or real options analysis.
- ***Uncertainties must exist.*** Otherwise the option value is worthless. If everything is known for certain in advance, then a discounted cash flow model is sufficient. In fact, when volatility (a measure of risk and uncertainty) is zero, everything is certain, the real options value is zero, and the total strategic value of the project or asset reverts to the net present value in a discounted cash flow model.
- ***Uncertainties must affect decisions when the firm is actively managing the project and these uncertainties must affect the results of the financial model.*** These uncertainties will then become risks, and real options can be used to hedge the downside risk and take advantage of the upside uncertainties.
- ***Management must have strategic flexibility or options to make midcourse corrections when actively managing the projects.*** Otherwise, do not apply real options analysis when there are no options or management flexibility to value.
- ***Management must be smart enough and credible enough to execute the options when it becomes optimal to do so.*** Otherwise, all the options in the world are useless unless they are executed appropriately, at the right time, and under the right conditions.

There are also several criticisms against real options analysis. It is vital that the analyst understands what they are, and what the appropriate responses are, prior to applying real options.

- ***Real options analysis is merely an academic exercise and is not practical in actual business applications.*** Nothing is further from the truth. Although it was true in the past that real options analysis was merely academic, however, many corporations have begun to embrace and apply real options analysis. Also, its concepts are very pragmatic and with the use of the Real Options Valuation's Super Lattice Solver software, even very difficult problems can be easily solved, as will become evident later in this book. This book and software have helped bring the theoretical a lot closer to practice. Firms are using it and universities are teaching it. It is only a matter of time before real options analysis becomes part of normal financial analysis.
- ***Real options analysis is just another way to bump up and incorrectly increase the value of a project to get it justified.*** Again, nothing is further from the truth. If a project has significant strategic options but the analyst does not value them appropriately, he or she is leaving money on the table. In fact, the analyst will be incorrectly undervaluing the project or asset. Also, one of the requirements foregoing states that one should never run real options analysis unless strategic options and flexibility exist. If they do not exist, then the option value is zero, but if they do exist, neglecting their valuation will grossly and significantly underestimate the project or asset's value.
- ***Real options analysis ends up choosing the highest risk projects as the higher the volatility, the higher the option value.*** This criticism is also incorrect. The option value is zero if no options exist. However, if a project is highly risky and has high volatility, then real options analysis becomes more important. That is, if a project is strategic but is risky, then you better incorporate, create, integrate, or obtain strategic real options to reduce and hedge the downside risk and take advantage of the upside uncertainties. Therefore, this argument is actually heading in the wrong direction. It is not that real options will overinflate a project's value, but for risky projects, you should create or obtain real options to reduce the risk and increase the upside, thereby increasing the total strategic value of the project. Also, although an option value is always greater than or equal to zero (as will be seen in later chapters), sometimes the cost to obtain certain options may exceed its benefit, making the entire strategic value of the option negative, although the option value itself is always zero or positive.

So, it is incorrect to say that real options will always increase the value of a project or only risky projects are selected. People who make these criticisms do not truly understand how real options work. However, having said that, real options analysis is just another financial analysis tool, and the old axiom of "garbage in garbage out" still holds. But if care and due diligence are exercised, the analytical process and results can provide highly valuable insights. In fact, this author believes that 50 percent (rounded, of course) of the challenge and value of real options analysis is simply *thinking about it*. Understanding that you have options, or obtaining options to hedge the risks and take advantage of the upside, and to think in terms of strategic options, is half the battle. Another 25 percent of the value comes from actually running the analysis and obtaining the results. The final 25 percent of the value comes from being able to explain it to management, to your clients, and to yourself, such that the results become *actionable intelligence* and not merely another set of numbers.

SUMMARY

Real options analysis simply defined is the application of financial options, decision sciences, corporate finance, and statistics to evaluate real or physical assets as opposed to financial assets. Industry analysts, experts, and academics all agree that real options provide significant insights to project evaluation that traditional types of analysis like the discounted cash flow approach cannot provide. Sometimes the simple task of thinking and framing the problem within a real options context is highly valuable. The simple types of real options discussed include expansion, abandonment, contraction, chooser, compound, barrier, growth, switching, and sequential compound options.

CHAPTER 1 QUESTIONS

1. What are some of the characteristics of a project or a firm that is best suited for a real options analysis?
2. Define the following:
   a. Compound option
   b. Barrier option
   c. Expansion option
3. If management is not credible in acting appropriately through profit-maximizing behavior, are strategic real options still worth anything?

APPENDIX 1A

The Timken Company on Real Options in R&D and Manufacturing

The following is contributed by Kenneth P. English, Director of R&D Emerging Technology, The Timken Company, Canton, Ohio. The Timken Company is a public company traded on the NYSE, and is a leading international manufacturer of highly engineered bearings, alloy and specialty steels and components, as well as related products and services. With operations in 24 countries, the company employs about 18,700 associates worldwide.

The Timken Company's journey toward real options analysis began in 1996 when the corporation made the decision to focus on profitably growing the business by 10 percent per year. We started with the creation of a gate-type process to identify and evaluate project opportunities that would generate the necessary profits for our growth requirements. During the numerous gate meetings, the process actually highlighted gaps in our process more than the anticipated growth project opportunities we had expected. The first group of gaps identified during the process was the lack of expertise in project management and market research; the second was poorly defined and documented product and corporate strategies; and, finally, financial evaluation capabilities. The gaps identified in project management, market research, and strategy were addressed over the following years by recruiting various consulting firms to assist with those disciplines. The financial evaluation gap was initially addressed with the assistance of our internal financial department by applying the same financial modeling tools used when the corporation built new physical plants. These models focused on NPV, payback, and project terminal value. Project terminal value caused considerable controversy with the reviewers.

As these parallel consulting efforts continued for months/years, the corporation became more adroit at the terminology of product development. As the refinements and understanding of these other areas evolved, it was realized that the financial model used on the gate templates was not adequate for the dynamic uncertain environment of product development. At this time, Monte Carlo simulation was being used in benchmarked growth industries to determine the range of risk for projects. Our first response was to acquire books on the subject of Monte Carlo simulation.

The financial department was familiar with the model but was not prepared to assist with implementation of it in the product development environment. After some time and frustration, the Risk Simulator software product for Monte Carlo simulation at a company named Real Options Valuation was discovered. The timing was excellent, since the corporation was reviewing a high-profile project that contained hidden ranges of risk. The simulation product was immediately purchased and inserted into our gate templates to address the issue of risk. Within weeks, some of our corporate leadership was looking at risk with a much different perspective. Previously, we identified risk and noted it, then proceeded on a product development path without sensitivity to the dynamic ramifications of the risk. The Monte Carlo simulation put focus on the importance of the corporation's gaps in detailed market research and the absence of aligned product and corporate strategy for Horizon II projects. The software made the complex and time-consuming financial formulas into a quick, user-friendly tool to assist with the difficult task of defining the range of risk and promoting timely decision making. It was painfully obvious that the real object of successful product development was to enable speedy decisions to either fund or kill projects and not the joy of being comfortable with seeing the old favorite projects and connected potential acquisitions lingering on with several lives.

Two and a half years into the quest for profitable growth, we identified the next barrier to our success. That barrier was the absence of a project portfolio process. The major issue with any initially installed gate-oriented process in a previous incremental corporate culture structure is that the gatekeepers only have the opportunity to evaluate the presented projects against other projects presented during that particular gate meeting. This situation exerts pressure to find a tool/process that will allow the gatekeepers to prioritize all the product and project efforts of the corporation to give maximum return on investment. The concept of projects in a portfolio becomes very important to the corporate allocation of funds. Portfolio management was a very foreign concept to us because our corporate orientation to projects was based on NPV and payback and not mitigation of risk, maximizing efforts, and cost of capital. We responded to the corporate learning piece of the puzzle by creating a manual portfolio simulation exercise to sensitize our executives and gatekeepers to how they looked at projects and their synergies. It also broadened their view of the significant impact that strategic fit, selection, and timing has with respect to financial success.

With the success of the portfolio simulation, we were then sensitized to the issue of the corporate benefit of cultivating a mindset of timing projects *(timing options)* in a way that could maximize the impact to our growth requirements. The writings regarding real options began appearing in the business literature, magazines, and seminars, but the application was initially geared toward the practice of financial options. Again, we were put in a position of educating ourselves (the change agents) and subsequently the corporate culture to a different way of thinking. We searched the available real options course selection taught at the university level. The universities were interested in real options but did not have coursework in place to conduct educational sessions.

The Timken Company established the R&D Emerging Technology Department in June of 2002. The focus of the department is to scan the world for dispersed technologies that are not part of the present corporate portfolio. These technologies contain varying degrees of risk, which require an even higher level of evaluation techniques to take advantage of numerous options.

Publicity from Dr. Johnathan Mun about the upcoming real options software and the lectures and workshops on real options appeared to be the best vehicle to take us to the next level of portfolio decision making. We contacted Dr. Mun to give a real options lecture and workshop to bring our financial department and executives up to speed. The time spent was very useful, and the culture is starting to communicate in real option terms. We at The Timken Company are anticipating that the new software for real options from Real Options Valuation, Inc., will get us closer to the target of achieving more confident corporate project decisions, resulting in assisting us in our goal of sustained profitability and growth.

APPENDIX 1B

Schlumberger on Real Options in Oil and Gas

The following is contributed by William Bailey, Ph.D., Senior Research Engineer at Schlumberger—Doll Research, Ridgefield, Connecticut. The company is involved in global technology services, with corporate offices in New York, Paris, and The Hague. Schlumberger has more than 80,000 employees, representing 140 nationalities, working in nearly 100 countries. The company consists of two business segments: Schlumberger Oilfield Services, which includes Schlumberger Network Solutions, and Schlumberger-Sema. Schlumberger Oilfield Services supplies products, services, and technical solutions to the oil and gas exploration and production (E&P) industry, with Schlumberger Network Solutions providing information technology (IT) connectivity and security solutions to both the E&P industry and a range of other markets. Schlumberger-Sema provides IT consulting, systems integration, managed services, and related products to the oil and gas, telecommunications, energy and utilities, finance, transport, and public-sector markets.

Long gone are those heady days in the petroleum industry when a pith-helmeted geologist could point to an uninspiring rock outcrop, declare confidently "drill here," and then find an oil field the size of a small country. Over the past 30 years, however, the situation for the oil industry has become very different indeed. As we search to replenish our ever-decreasing hydrocarbon supplies, oil explorationists now find themselves looking in some of the most inaccessible parts of the globe and in some of the deepest and most inhospitable seas. What could have been achieved in the past with a relatively small investment is now only attainable at a considerably greater cost. In other words, developing an oil and/or gas field nowadays is subject to considerably larger investments in time, money, and technology. Furthermore, such large investments are almost always based on imperfect, scant, and uncertain information. It is no accident, therefore, that when teaching the concepts of risk analysis, many authors cite the oil industry as a classic case in point. This is not by accident for few other industries exhibit such a range of uncertainty and possible downside exposure (in technical, financial, environmental, and human terms). Indeed this industry is almost ubiquitous when demonstrating risk analysis concepts.

Consequently real options have a natural place in the oil industry management decision-making process. The process and discipline in such an analysis captures the presence of uncertainty, limited information, and the existence of different—but valid—development scenarios. The fact that petroleum industry management are faced with multimillion (sometimes billion) dollar decisions is nothing new. Such people are used to making critical decisions on a mixture of limited information, experience, and best judgment. What is new is that we now have a coherent tool and framework that explicitly considers uncertainty and available choices in a timely and effective manner.

This short appendix is intended to provide just a brief glimpse into the types of applications real options have been used for in the petroleum industry. To guide the reader unfamiliar with the finer points of the oil and gas industry, it may be prudent to outline the basic process in an "average" petroleum development. In so doing, the reasons why the oil industry is deemed such a prime example for use of real options (and risk analysis in general) will become clear.

In the 1959 film of Jules Verne's 1864 novel *Journey to the Center of the Earth,* James Mason and others found themselves sailing on a dark sea in a mighty cavern many miles down in the earth's crust. This was, of course, just science fiction, not science fact. Unfortunately it is still a common misconception that oil is found in such caverns forming black lakes deep beneath our feet. While such images may be romantic and wishful, reality is far more intricate. For the most part, oil (and gas) is found in the microscopic pore spaces present between individual grains making up the rock. For example, hydrocarbon-bearing sandstone may have porosity levels (the percentage of pore space in the rock) of about 15 percent. This means that if all the pore space in the rock is full of oil, then 15 percent of the total rock volume contains oil. Of course, things are not as simple as that because water and other minerals serve to reduce the available pore volume.[1] As oil and gas are liquid, they will flow. Unless the rock itself provides some form of seal (or trap) to contain these fluids, over time they will simply seep to the surface and be lost. (Azerbaijan has some good examples of such seepage with whole hillsides being awash with flame from seeping gas for as long as recorded history.) So not only do we need a rock that contains oil (or gas) but also the oil (or gas) must be trapped somehow, ready for exploitation. For a readable and well-informed summary of petroleum geology, refer to Selley (1998).[2]

Extraction of oil (and/or gas) from a virgin field is undertaken in typically four stages: exploration and appraisal; development; production; and abandonment. This is a gross simplification, of course, for within each phase there are a multitude of technical, commercial, and operational considerations. Keeping one eye on real options, in their crudest form these phases can be briefly described as follows:

- ***Exploration and Appraisal.*** Seismic data is obtained and a picture of the subsurface is then revealed. Coupled with geological knowledge, experience, and observations, it is then possible to generate a more detailed depiction of a possible hydrocarbon-bearing zone. Seismic data cannot tell what fluids are present in the rock, so an exploratory well needs to be drilled, and from this, one is then able to better establish the nature, size, and type of an oil and gas field.

  Exploration and Appraisal Phase—Where Real Options Come In. The decision maker has numerous options available to him/her, which may include:
  - Extent of investment needed in acquiring seismic data. For example should one invest in 3D seismic studies that provide greater resolution but are significantly more expensive? Should 4D (time-dependent) seismic data be considered? While advanced seismic data (and interpretation) certainly provides improved representation of the subsurface environment, one needs to assess whether it is worthwhile investing in this information. Will it reduce uncertainty concerning the size and nature of the reservoir sufficiently to pay off the investment?
  - Given inherent uncertainty about the reserves, if possible, how much should the company share in the risk (extent of contract partnership)?
  - How many exploration wells are appropriate to properly delineate the field? One, two, five, or more?
- ***Development.*** Once sufficient data has been obtained (from seismic or exploratory wells) to make an educated judgment on the size of the prize, we enter into the development phase. Here we decide on the most commercially viable way for exploiting this new resource by engineering the number (and type) of producing wells, process facilities, and transportation. We must also establish if, at all, any pressure support is necessary.[3]

  Development Phase—Where Real Options Come In. This phase is where decision makers face possibly the greatest number of valid alternatives. Valid development options include:
  - How many wells should be drilled? Where should they be located? In what order should they be drilled?
  - Should producers be complex (deviated/horizontal) wells located at the platform, or should they be simple but tied-back to a subsea manifold?

- How many platforms or rigs will be needed? If offshore, should they be floating or permanent?
- What potential future intervention should be accommodated? Intervention refers to an ability to reenter a well to perform either routine maintenance or perform major changes—referred to as a work-over.
- How many injectors (if any at all) should be drilled? Where should they be located?
- How large should the processing facility[4] be? If small, then capital expenditure will be reduced but may ultimately limit throughput (the amount of hydrocarbons sent to market thereby restricting cash flow). If the process facility is made too large, then it may be costly and also operationally inefficient.
- Are there adjacent fields waiting to be developed? If so, should the process facility be shared? Is this a valid and reasonable future possibility in anticipation of uncertain future throughput?
- Should a new pipeline be laid? If so, where would it be best to land it, or is it possible to tie it into an existing pipeline elsewhere with available capacity? Should other transportation methods be considered (e.g., FPSO, or floating production and storage operation[5])?

The number of different engineering permutations available at this stage means that management may be faced with several viable alternatives, which are contingent on the assumptions on which they were developed. Real options enable uncertainty to be explicitly quantified at this stage.

- ***Production.*** Depending on the size of the reserve (and how prolific the wells are) the engineer must manage this resource as carefully as any other valuable asset. Reservoir management (the manner and strategy in which we produce from a field) has become increasingly important over the past few years. Older, less technically advanced, production methods were inefficient, often leaving 75 percent or more of the oil in the ground—oil that cannot be easily extracted afterward, if at all. Increasing the efficiency of our production from our reservoirs is now a crucial part of any engineering effort (unfortunately, nature prevents us from extracting 100 percent of the oil; there will always be some left behind).

Production Phase—Where Real Options Come In. Valid production options include:

- Are there any areas of the field that are unswept[6] and can be exploited by drilling more wells?
- Should we farm out (divest) some, or all, of the asset to other companies?
- Should we consider further seismic data acquisition?
- Should we consider taking existing production wells and converting them into injection wells to improve the overall field performance?

  - What options does one have to extend the life of the field?
  - Should we consider reentering certain wells and performing various actions to improve their performance (e.g., reperforating some or all of the well, shutting off poorly producing zones, drilling a smaller branch well [known as a *sidetrack*] to access unswept reserves, etc.)? What information needs to be collected to be able to make these operational decisions? How is such information best obtained? At what cost and at what operational risk? (Reentering a well may be a hazardous and potentially damaging act.)

  Once again, there are many opportunities during the production phase to make decisions that are still subject to considerable uncertainty. Even though the field may be mature and much experience has been accumulated, the operator is still faced with many management options that can impact ultimate reservoir performance and economic viability.

  ***Decommissioning (also known as* Abandonment).** Once reserves have been depleted, the infrastructure can either be left to decay or—increasingly—it must be dismantled in an environmentally and economically efficient manner. This is especially true for the North Sea and offshore United States.

- ***Decommissioning Phase—Where Real Options Come In.*** Valid production options include:
  - What will the ultimate abandonment cost be, and what is the likelihood that this will remain true at the end of the life of the field?
  - Should the full cost of abandonment be included in the initial development strategy, or is there a way to hedge some or all of this cost?
  - What contingency should be built in to account for changes in legislation?
  - At what threshold does abandonment cost make the project unprofitable, and how would this impact our initial development strategy?

This brief (and admittedly incomplete) list of bullet points at least demonstrates why the oil industry is ideally suited for a real options-type analysis because the companies exhibit all the necessary ingredients:

- Large capital investments.
- Uncertain revenue streams.
- Often long lead times to achieve these uncertain cash flows.
- Uncertainty in the amount of potential production (reservoir size and quality).
- Numerous technical alternatives at all stages of development.
- Political risk and market exposure (external influences outside the control of the operating company).

Final Word: Early examples of options-based analysis are found in the oil and gas industry.[7] The impact and wholesale adoption so far has been limited. Why this is the case in the oil industry raises important issues that should be kept in mind when considering adopting real options as a practice in any company.

Real options are technically demanding, with a definite learning curve in the oil industry, and have three main hurdle classifications:[8]

- ***Marketing Problem.*** Selling real options to management, appreciating the utility and benefit, understanding their capabilities and strengths (as well as weaknesses), and ultimately communicating these ideas (companies usually have a few volunteer champions/early adopters, but they often remain isolated unless there is suitable communication of these concepts, particularly in a nontechnical capacity, which may be easier said than done).
- ***Analysis Problem.*** Problem framing and correct technical analysis (not too difficult to resolve if suitably trained technical people are available—and have read this book).
- ***Impact Problem.*** Not really the interpretation of results but rather acting on them, implementing them, monitoring and benchmarking them, then communicating them (a recurring theme), and finally managing the whole process.

These issues should be kept in mind when communicating the concepts and results of a real options analysis.

APPENDIX 1C

Intellectual Property Economics on Real Options in Patent and Intangible Valuation

The following is contributed by A. Tracy Gomes, President and CEO of Intellectual Property Economics, LLC, located in Dallas, Texas. Gomes's firm specializes in the valuation of intellectual property and intangibles, for the purposes of corporate financial planning and tax transactions.

Real options analysis is designed to explicitly incorporate and analyze risk and uncertainty associated with real assets. Intellectual property (IP), whether defined in its strictest, most narrow legal sense—patents, trademarks, trade secrets, and copyrights—or more broadly to encompass all intellectual/intangible assets created from human conceptual endeavor, is the poster child of uncertainty, and exemplifies the great challenge and promise that is real options analysis.

In this information- and knowledge-based age that is the postmodern economy, IP is the most fundamental and valuable asset in business today. From 1978 to 1998, the composition of market value of the S&P 500 has been transformed from 80 percent physical assets, 20 percent intangible assets, to 20 percent physical assets, 80 percent intangible assets.[1] Since 1990, the annual revenue realized from just the licensing of patented technology has grown from less than $10 billion to nearly $120 billion (not counting the direct administrative and maintenance costs, which are likely less than one-half of one percent; that is $120 billion in net, bottom-line profit).

But this is just the IP that is visible, that the marketplace can actually see and has already put a value on. The goal and application for real options analysis lies in the vast uncovered trove of IP that is unseen and hidden, and like a giant iceberg lies just below the surface. For younger, emerging companies, this is likely to be IP that is in process—research and development projects in varying stages of development. For older companies, IP value is likely to be found not only in those efforts still in the pipeline but perhaps even more so in those efforts long ago completed and placed on the shelf.

Kevin Rivette, in his seminal book, *Rembrandts in the Attic,* recounts the embarrassing legacy of Xerox, which discarded such "worthless" ideas as the PC, laser printing, the Ethernet, and graphical user interface (GUI), only to see them transformed from trash to cash by someone else. Leading industry companies have gotten religious and are fast about combing through their patent portfolios. Procter & Gamble, after a three-year internal audit, estimates that it is utilizing only about 10 percent of its 25,000-patent portfolio. Dupont has allocated each of its 29,000 patents to one of 15 business units. And IBM has literally thrown open its vaults, declaring each and every patent, each technology and process, even trade secrets as potentially "up for sale."

Recognizing that something is of potential value, and knowing what the value of that something is, are two different things. Information and knowledge are the guideposts of strategic business decision making and the glue of economic transactions. When information is incomplete or unknown, business decisions tend to be delayed and markets fail to clear. Stereotypical examples in the case of IP are the individual sole inventors who think their ideas are worth millions and the giant multinational corporations who are only willing to pay pennies. Unfortunately, the reality of today's IP business transactions is all too often characterized by divergent bid/offer sheets, lengthy negotiations, and tortured contractual terms,[2] leading to excessively high and wasteful transaction costs. Perhaps even more disheartening are the thousands of IP deals in which buyer and seller don't even get a chance to meet—IP left orphaned on countless Internet exchanges, or projects abandoned or put back on the shelf because they are thought to be too costly or their markets too remote or too shallow.

It is here that real options analysis holds so much promise, to be applied to those IP assets and projects that were thought to be too vague, too unknown, and too iffy. Not that it can predict the future success or failure of IP development or the creation of some still hypothetical market, or that it can turn perennial duds into potential deals. Real options analysis is not magic, nor does it make risk or uncertainty vanish and go away. What it does do is attempt to make risk and uncertainty explicit through rational statistical means. In this way, uncertainty is bounded and risk quantified such that information becomes more clear and tangible, and the knowledge base expanded, thereby aiding decision making.

Unlike financial assets, there are no existing liquid markets for intangible "real" assets. Real options analysis seeks to change that by providing a means to demystify the risk and uncertainty surrounding IP and supply potential buyers and sellers with objective, quantifiable information to shortcut uncertainty, clarify risk, and clear the path to shorter, smoother, and less costly IP deal making. Two examples provide a case in point.

A small automotive engineering start-up identifies a cutting-edge technology being developed by a private research institute. They approach the institute, seeking the acquisition or license of the technology. Given that the technology is a few years from commercialization, and the expectant market, which is being driven by governmental regulation (and resisted by manufacturers), is several more years into the future, instead of jumping into negotiations, the two sides agree to an outside independent economic analysis.

Due to cost considerations, simplified real options analysis was performed modeling future auto demand and holding government regulation constant. The real options valuation, though nearly twice as high as the conventional DCF analysis, gave both parties a clearer view of uncertainty and amount of risk facing the technology. After the two-month analysis, the parties entered into negotiations and within two months completed discussions, and drafted and signed an agreement.

A second case involves a medium-sized contract research organization with a proprietary portfolio of nearly 400 patents, processes, trade secrets, and disclosures spread over an area of half a dozen different fields of technology. Seeking to extract value from its IP assets, and develop an additional revenue stream, the firm selected a sampling of assets (in varying stages of development) from several of its portfolio segments and contracted for a risk assessment—the beginning stages of an options analysis prior to modeling. The assessment identified several key parameters, including various risks (technical, competition, and regulatory) as well as timing issues, both technical and market. And, while not a complete options analysis, the assessment did provide management with valuable, tangible information with which to assess and prioritize the sampling of assets and develop a template to evaluate all its IP on a go-forward basis.

Uncertainty and risk are nowhere more real and tangible than in the case of intellectual property. Understandably, this uncertainty makes firms hesitant in decision making regarding IP and virtually hamstrung in IP deal making. The role of real options analysis in IP is to identify and quantify uncertainty, to illuminate risk, and thereby to increase confidence and realize the full value of the IP.

APPENDIX 1D

Gemplus on Real Options in High-Tech R&D

The following is contributed by Jim Schreckengast, Sr. Vice President of Gemplus International SA, the world's leading provider of smart cards and solutions for the telecommunications, financial, government, and IT industries. Gemplus is one of the most innovative companies of its kind, carrying a significant investment in research and development (R&D) aimed at driving forward the state-of-the-art in secure, ultrathin computing platforms, wireless security, identity, privacy, content protection, and trusted architectures.

Gemplus is a high-tech company. Such companies assign great importance to R&D, because high-tech companies often derive their primary competitive advantage through technology, and R&D plays a pivotal role in determining the technology position of these companies. Effective management of R&D is difficult and involves significant uncertainty. Moreover, company resources are limited, so it is critical for management to invest R&D resources wisely, considering the many types of value that can be produced by these resources. Gemplus recognizes the complexity of managing R&D efforts in rapidly changing and competitive environments and has used real options analysis to improve the effectiveness of R&D investment decisions.

One of the most significant challenges in R&D is the management of innovation. Management of this process is difficult, because successful innovation usually involves the discovery and generation of knowledge, while exploiting existing knowledge and capabilities in an attempt to generate value through new products and services, to differentiate existing offerings, to lower costs, and to disrupt the competitive landscape. Each successful innovation may be used as a building block for further R&D efforts, enabling the firm to create a sustainable competitive advantage through a cohesive R&D program that blends and builds on previous results. For example, a firm pursuing low-power, wireless communications technologies for tiny wearable computers might discover that their latest approach to reducing power requirements has the capability to generate fine-grained location and speed information as a side effect of communication. This location information could enable the firm to begin generating a new family of location-based services, while enhancing routing and switching in dynamic wireless networks. Further, the company might recognize that expansion of investment in fine-grained location technologies could position the firm favorably to compete in context-rich service delivery, if the market for these services materializes in two years. In addition, the R&D director might conclude that investment in this technology will improve the firm's ability to manage bandwidth and resource utilization, if the results of current research in peer-to-peer network architectures prove promising.

The chain of loosely connected innovations in the previous example is surprisingly representative of the events that unfold in practice within a high-tech industry. While prediction of a specific sequence of innovations is usually infeasible, successful companies often develop innovation systems that recognize the potential for these chains and develop R&D systems that can stimulate their creation while retaining the flexibility to capitalize on the most promising among them over time.

The complexities of analyzing technical uncertainty, market uncertainty, and competitive movements in a rapidly changing industry often drive management to either shorten the time horizon of R&D projects to the extent that each project has a very predictable (and often unremarkable) outcome, or to assemble R&D projects as a collection of desperate "bets" in the hopes of finding one that "wins" for the company. The former approach tends to restrict flexibility, because project managers will focus energy and resources on short-term tasks that are directly linked to the limited scope of each project. The latter approach dilutes R&D resources across many unrelated projects and overlooks potential synergies between the outcomes of these efforts. Furthermore, by viewing the R&D portfolio as a collection of "bets," management may fail to recognize the many opportunities that usually exist to control, refine, and combine the intermediate results of these projects in a way that enhances the total value of R&D.

Traditional valuation techniques for R&D (e.g., decision trees and NPV) may exacerbate the fundamental problems associated with investment analysis and portfolio management, because these techniques rely solely on information available at the time of the analysis and cannot accurately value flexibility over time. The limitations of these techniques often go unrecognized by decision makers, resulting in suboptimal R&D investment decisions.

Gemplus uses an R&D management approach that recognizes three key realities for its industry:

1. Uncertainties are resolved on a continuous basis as R&D is conducted, competitive conditions change, and market expectations evolve.

2. There is a significant lag that exists between the time a company begins to invest in a technology and the time when the company can wield that technology effectively to generate new products and services.
3. The most valuable R&D investments are those that simultaneously build on existing, distinctive competencies while generating capabilities that enhance the firm's flexibility in light of existing uncertainties.

These realities compel Gemplus to manage R&D on a continuous basis and to invest while significant uncertainties exist, valuing the flexibility created by R&D investments. Each R&D project carries a primary purpose but may also carry a number of secondary objectives that relate to the value of real options associated with expected capabilities delivered by the project. Gemplus manages a portfolio of R&D innovation efforts in the context of a technology road map that makes the most significant real options apparent, and relates the strategic direction of the company to the flexibility and competitive advantage sought by its R&D efforts.

Once the most significant real options have been identified, each R&D project is valued in the context of this road map. Research proposals are evaluated based on the value of the information generated by the work, together with the relevant capabilities that may be generated and the flexibility this affords the company. Gemplus has seen up to 70 percent of the value of a research proposal arise from the real options generated by the research. Development projects are typically valued for their primary purpose and for real options arising from R&D management flexibility (i.e., expansion and contraction in the course of portfolio management), technology switching capabilities (e.g., when it is unclear which technology will emerge as a dominant design), and real options created in the context of the technology road map (e.g., multipurpose technologies). Although a development project usually has a much smaller percentage of its value attributed to real options, the difference can be significant enough to alter R&D investment decisions that would have otherwise favored a less flexible or less synergistic effort.

R&D efforts also result in the generation of intellectual property. Patents are of particular interest, because they can affect the firm's ability to protect products and services derived from the patented technology. Further, patents may be licensed, sold, or used to erect barriers (i.e., entry, switching, substitution, and forward or backward integration), as well as to counter infringement claims by third parties. Thus, patents may carry significant value for the firm, and this value reflects the real options associated with the invention now and in the future. Gemplus believes that correct valuation of intellectual property, and patents in particular, leads to improved intellectual property strategies and more effective research prioritization. Thus, Gemplus has changed its intellectual property strategy and valuation process to explicitly incorporate the value of real options created by R&D patents.

Acquiring technology from outside the firm is often an integral element of a good technology strategy. Thus, R&D managers must determine how to value technologies accurately. Like direct R&D investments, technology acquisitions may carry a significant value associated with real options. Acquisition valuation should include comparables, the value for direct exploitation of the technology, and the value of real options associated with the technology. Such valuation should also consider real options forgone by the current technology owner and the game theoretic aspects of bidding competitively for the technology with others. Gemplus considers real options analysis to be a critical ingredient to accurate valuation of technology acquisitions and has augmented its process to include this analysis.

It should be noted that recognition of the value associated with real options in R&D must be combined with a process for acting on the decisions associated with these real options. If real options are not effectively linked with the ongoing R&D management process, it may be difficult to realize the values projected by the real options analysis. For instance, Gemplus found that the value of R&D management options was highly dependent on the life cycle and review that was applied to projects and programs. For example, a hardware development project often follows a traditional "waterfall" life cycle with natural checkpoints at the conclusion of investigation, specification, design, and implementation. These checkpoints present an opportunity to take advantage of what has been learned over time and to alter the course of the project. The project could be expanded or reduced, changed to incorporate a new capability from a recently completed research project, or perhaps altered in a more fundamental way. Software development projects, however, may follow a more iterative life cycle, with less time between cycles and fewer natural checkpoints. These differences should be considered carefully when identifying management options associated with a project.

Of course, all of the activity associated with real options analysis in R&D is aimed at more accurately valuing technological choices, so that the best decisions are made for the firm. The experience at Gemplus thus far suggests that these efforts are worthwhile. Real options analysis is a powerful financial tool that meshes nicely with the complexities of managing a collection of projects and research activities that inherently carry significant uncertainty, but also represent great potential value for the firm.

APPENDIX 1E

Sprint on Real Options in Telecommunications

The following is contributed by Marty Nevshemal (FMDP, Global Markets Division) and Mark Akason (FMDP, Local Telecommunications Division) of Sprint. Sprint is a global communications company serving 23 million business and residential customers in more than 70 countries. With more than 80,000 employees worldwide and $23 billion in annual revenues, the Westwood, Kansas-based company is represented on the New York Stock Exchange by the FON group and the PCS group. On the wireline side, the Sprint FON Group (NYSE: FON) comprises Sprint's Global Markets Group and the Local Telecommunications Division, as well as product distribution and directory publishing businesses. On the wireless side, the Sprint PCS Group (NYSE: PCS) consists of Sprint's wireless PCS operations. Sprint is widely recognized for developing, engineering, and deploying state-of-the-art technologies in the telecommunications industry, including the nation's first nationwide all-digital, fiber-optic network. The Global Markets Group provides a broad suite of communications services to business and residential customers. These services include domestic long-distance and international voice service; data service like Internet, frame relay access and transport, Web hosting, and managed security; and broadband.

In the twentieth century, telecommunications has become ubiquitous in developed countries. In 1999, total telecommunications revenues in the United States were in excess of $260 billion and had grown in excess of 10 percent per year for the prior four years.[1] By December 2000, there were more than 100 million mobile wireless subscribers in the United States.[2] Even more staggering is the capital intensity necessary to drive this revenue and provide this service. In 2000, the largest telecom company, AT&T, required assets of $234 billion to drive revenue of $56 billion, a ratio of greater than 4:1.[3] Not only is simple growth in population and locations driving the industry, but also new technologies and applications such as wireless and the

Internet are fueling that growth. Given the capital intensity and sheer size of the investments, to be successful in the telecom industry it is critical that companies make decisions that properly value and assess the new technologies and applications. This is where real options can play a role.

The goal of any company is to make the right decision regarding its investments. One of the goals of any investment decision is to optimize value while preserving flexibility. Often though, optimum value and flexibility are at odds. An example of this dichotomy is that one can choose a strategy of leading the industry by investing in and implementing new, unproven technologies that will hopefully become the platform(s) for future profitable products and services. Or one can choose a wait-and-see strategy, holding back on investments until the technology standard is recognized industry-wide. Both strategies have obvious advantages and disadvantages. The first strategy opens a telecom company up to the risk of investing in a technology that may not become the industry standard, may be a dead end (remember BETA tapes?), or may not meet all the desired specifications. Furthermore, the magnitude of the "cutting edge" technology bet, if it does not work, could adversely impact the financial viability of the firm.

Therefore, it is critically important for a telecommunications company such as Sprint to ensure that their decision-making process includes a structured method that recognizes both the benefits and pitfalls of a particular technology investment as soon as information about that technology becomes available. This method should quickly obtain information in a usable form to decision makers so that they can take appropriate action. Finally, this structured method must ensure that timely decision points be identified, where actions can be taken to either improve the development results or obtain the option to redeploy resources to better opportunities.

One of the ways that Sprint believes that this strategic flexibility for technology investments can be systematically implemented throughout the organization is through the adoption of real options analysis. The very nature of the analysis forces managers to think about the growth and flexibility options that may be available in any technology investment decision. Real options analysis has a process for valuing these options, and it identifies decision points along the way.

Systemic to the telecom industry is the requirement of management to make critically important strategic decisions regarding the implementation and adaptation of various telecom technologies that will have significant impact on the value of their firm over the long term. Overall, these technologies are extremely capital intensive, especially in the start-up phase, and take an extended period to develop and implement, and have an extended payoff period.

Here are a couple of examples of capital-intensive telecom technology bets that a telecom company has to make:

- Selection of wireless technology (e.g., TDMA—Time Division Multiple Access, CDMA—Code Division Multiple Access, or GSM—Global System for Mobile Communications).
- Third Generation (3G) build-out and timing of a commercial rollout.
- 3G wireless technology applications.
- Location and construction of Metropolitan Area Networks (MANs), Central Offices (COs), and Points-of-Presence (POPs).
- Capacity of its backbone fiber network.
- Technology of the backbone network (ATM—Asynchronous Transfer Mode versus IP—Internet Protocol).

Generally speaking, there are three basic outcomes to any technology decision using a strategy to lead. Each outcome has distinct effects on the company, both operationally and financially:

- The right technology choice generally leads to success in its many forms: sustainable competitive advantage in pricing/cost structure, first-to-market benefits, greater market share, recognition as a superior brand, operational efficiencies, superior financial results, and industry recognition.
- The wrong technology choice without strategic options to redirect the assets or redeploy resources could lead to a sustained competitive disadvantage and/or a technology dead end from which it takes considerable financial and operational resources to recover.
- However, the wrong initial choice can also lead to success eventually—if viable strategic and tactical options are acted on in a timely manner. At a minimum, these options can help avoid financial distress and/or reduce its duration and/or the extent of a competitive disadvantage.

It is important to implement valuation techniques that improve the analysis of business opportunities, but perhaps more important, telecom managers should strive to implement a structured thought/analysis process that builds operational flexibility into every business case.

This is where real options analysis has shown to have definite benefits. More specifically, the thought process that forces management to look for and demand strategic flexibility is critical. Furthermore, similar to the value of Monte Carlo simulations that educate management to better understand the input variables as opposed to concentrating on a final output NPV (net present value), real options analysis also forces management to better understand these input variables. However, it goes a couple of steps further by valuing strategic flexibility and identifying trigger points where the direction of the business plan may be amended. The challenge is how to implement the mechanics of real options analysis.

In the telecom industry there are typically no natural trigger points where hard-stop reviews are required as there are in the pharmaceutical industry or in the oil and gas industry. Within the pharmaceutical industry, for example, there are natural gates/decision points, such as FDA reviews, that act as trigger points where the strategic direction of the product/project can be and typically must be revisited.

For technology companies like Sprint, these trigger points are not implicit. Instead, they need to be actively defined by management and built into a structured analysis. These trigger points can be based on fixed time line reviews (monthly/quarterly/yearly) or can occur when a technology reaches a natural review stage such as the completion of product design, product development, market analysis, or pricing. Other milestones include when financial and operational thresholds are realized (project overspent/competing technology introduced/growth targets exceeded).

When implementing new technology, historical benchmark data regarding the chance that a particular event will occur is not available. For example, there is no historical precedent to show the percentage chance that CDMA rather than GSM technology will be the preferred wireless technology in the United States over the long run. Yet, the adoption of one technology over the other may have serious financial ramifications for the various wireless carriers. Therefore, management, in many cases, will base the value of the option on their subjective analysis of the situation. With real options, the final outcome of management's analysis is determined through thorough analysis and critical thinking, and the result has considerable value.

Similar cases are present throughout the telecom industry and may result in considerable subjective leeway that allows for wide swings in the value of any particular option. This is not to say that this dilutes the value of real options analysis. On the contrary, just having the structured thought process that recognizes that there is value in strategic flexibility and in trying to put a value on this flexibility is important unto itself.

In summary, applying the key principles of real options analysis is important and valuable; and overall, real options analysis complements traditional analysis tools and in many cases is an improvement over them.

The following examples are telecom-specific areas where various types of real options can be used to determine the financial viability of the project.

- ***Wireless Minutes of Use (MOU) and Replacement of Wireline MOU.*** In today's competitive wireless landscape, most, if not all, of the nationwide wireless carriers are offering long-distance plans as part of their wireless package. As wireless penetration increases, this drives MOU to the long-distance carriers and may change the economics of the wireline build-out for some carriers. Furthermore, because wireless subscribers have long distance bundled into their monthly recurring charge (MRC), many are replacing their wireline phones with wireless phones for long distance. Therefore, real options analysis can be done to place a value on both wireless and wireline carriers.

- ***Valuing New Technologies Using Sequential Compound and Redeploy Resources Options.*** 3G can be seen as both a sequential compound and redeploy resources option. Some wireless carriers in the United States, such as Sprint, will be able to implement 3G by deploying software upgrades throughout the existing network, while others will need to build out new networks. Companies that find 3G a *sequential compound option* can upgrade their network to 3G capable with very little (if any) incremental investment over what the wireless company would normally invest to build-out capacity. In addition, the subscribers of these companies will be able to utilize existing phones that are not 3G capable for voice services.
  - Wireless companies that cannot upgrade sequentially to 3G must redeploy resources from the existing wireless network. These companies must spend billions of dollars acquiring the spectrum to enable the build-out of 3G networks. In Europe alone, an estimated $100 billion was spent acquiring the spectrum to allow 3G. These companies must redeploy these significant resources to build out their 3G-capable network, while maintaining their existing networks. In addition, the customers of these companies must purchase new handsets because their existing phones will not function on the new 3G-capable networks.
  - The options that wireless carriers face today can be traced back to an option that faced the companies years ago. As stated earlier, a real option existed between CDMA, TDMA, and GSM for wireless technologies that the industry is just now getting better visibility on. The decisions made then have consequences today and in the future for the viability of these companies' 3G offerings.
- ***Leveraging Local Assets Using New Market Penetration and Change Technology Options.*** Incumbent local exchange carriers (ILEC) are in a unique competitive position and therefore can use a new market penetration option analysis when valuing expenditures on their existing local infrastructure. This can be used to offer long-distance service with minimal infrastructure upgrade to allow the ILECs to enter new markets as well as to offer new technologies like high speed data, video, and the Internet to existing local customers.
  - The change of technology option is one that is facing or will be facing all major ILECs. The existing circuit-switched network is not efficient enough to handle the increasing amount of traffic from both data and voice. Carriers built their networks to handle peak voice traffic during the business day, but in actuality the peak times of the

network now occur during the evening because of Internet and data use. The change to a packet-based data network is an option that local carriers are facing today. The change to packet technology will create a much more efficient network that will handle both the increasing voice and data traffic. This change technology option to a packet-based network also enables more options in the future by opening the possibility of creating new markets and products that cannot exist on the old circuit-based network.

- ***Infrastructure Build-Out (Expand versus Contract Options).*** The expand/contract option is gaining more and more validity in the current telecom environment. Network build-out is a capital-intensive requirement that has forced many carriers to leverage their balance sheets with a large amount of debt. However, demand for telecommunications services has not kept up with supply, resulting in excess fiber capacity. Depending on the location and availability of excess capacity, it may be cheaper for a telco to lease existing capacity from another telco than to build out its own network. In addition, due to the strained finances of some carriers, this capacity may be acquired at prices that offer a considerable discount to any unilateral build-out scenario.

CHAPTER 2

Traditional Valuation Approaches

INTRODUCTION

This chapter begins with an introduction to traditional analysis, namely, the discounted cash flow model. It showcases some of the limitations and shortcomings through several examples. Specifically, traditional approaches underestimate the value of a project by ignoring the value of its flexibility. Some of these limitations are addressed in greater detail, and potential approaches to correct these shortcomings are also addressed. Further improvements in the areas of more advanced analytics are discussed, including the potential use of Monte Carlo simulation, real options analysis, and portfolio resource optimization.

THE TRADITIONAL VIEWS

Value is defined as the single time-value discounted number that is representative of all future net profitability. In contrast, the market price of an asset may or may not be identical to its value. (Assets, projects, and strategies are used interchangeably.) For instance, when an asset is sold at a significant bargain, its price may be somewhat lower than its value, and one would surmise that the purchaser has obtained a significant amount of value. The idea of valuation in creating a fair market value is to determine the price that closely resembles the true value of an asset. This true value comes from the physical aspects of the asset as well as the nonphysical, intrinsic, or intangible aspect of the asset. Both aspects have the capabilities of generating extrinsic monetary or intrinsic strategic value. Traditionally, there are three mainstream approaches to valuation, namely, the market approach, the income approach, and the cost approach.

Market Approach The market approach looks at comparable assets in the marketplace and their corresponding prices and assumes that market forces will tend to move the market price to an equilibrium level. It is further assumed that the market price is also the fair market value, after adjusting for transaction costs and risk differentials. Sometimes a market-, industry-, or firm-specific adjustment is warranted, to bring the comparables closer to the operating structure of the firm whose asset is being valued. These approaches could include common-sizing the comparable firms, performing quantitative screening using criteria that closely resemble the firm's industry, operations, size, revenues, functions, profitability levels, operational efficiency, competition, market, and risks.

Income Approach The income approach looks at the future potential profit or free-cash-flow-generating potential of the asset and attempts to quantify, forecast, and discount these net free cash flows to a present value. The cost of implementation, acquisition, and development of the asset is then deducted from this present value of cash flows to generate a net present value (NPV). Often, the cash flow stream is discounted at a firm-specified hurdle rate, at the weighted average cost of capital, or at a risk-adjusted discount rate based on the perceived project-specific risk, historical firm risk, or overall business risk.

> ***The three approaches to valuation are market approach, income approach, and cost approach.***

Cost Approach The cost approach looks at the cost a firm would incur if it were to replace or reproduce the asset's future profitability potential, including the cost of its strategic intangibles if the asset were to be created from the ground up. Although the financial theories underlying this approach are sound in the more traditional deterministic view, they cannot reasonably be used in isolation when analyzing the true strategic flexibility value of a firm, project, or asset.

Other Approaches Other approaches used in valuation, more appropriately applied to the valuation of intangibles, rely on quantifying the economic viability and economic gains the asset brings to the firm. There are several well-known methodologies to intangible-asset valuation, particularly in valuing trademarks and brand names. These methodologies apply the combination of the market, income, and cost approaches just described.

The first method compares pricing strategies and assumes that by having some dominant market position by virtue of a strong trademark or brand recognition—for instance, Coca-Cola—the firm can charge a premium price for its product. Hence, if we can find market comparables producing similar products, in similar markets, performing similar functions, facing similar market uncertainties and risks, the price differential would then pertain exclusively to the brand name. These comparables are generally adjusted to account for the different conditions under which the firms operate. This price premium per unit is then multiplied by the projected quantity of sales, and the outcome after performing a discounted cash flow analysis will be the residual profits allocated to the intangible. A similar argument can be set forth in using operating profit margin in lieu of price per unit. Operating profit before taxes is used instead of net profit after taxes because it avoids the problems of comparables having different capital structure policies or carry-forward net operating losses and other tax-shield implications.

Another method uses a common-size analysis of the profit and loss statements between the firm holding the asset and market comparables. This takes into account any advantage from economies of scale and economies of scope. The idea here is to convert the income statement items as a percentage of sales, and balance sheet items as a percentage of total assets. In addition, in order to increase comparability, the ratio of operating profit to sales of the comparable firm is then multiplied by the asset-holding firm's projected revenue structure, thereby eliminating the potential problem of having to account for differences in economies of scale and scope. This approach uses a percentage of sales, return on investment, or return on asset ratio as the common-size variable.

PRACTICAL ISSUES USING TRADITIONAL VALUATION METHODOLOGIES

The traditional valuation methodology relying on a discounted cash flow series does not get at some of the intrinsic attributes of the asset or investment opportunity. Traditional methods assume that the investment is an all-or-nothing strategy and do not account for managerial flexibility that exists such that management can alter the course of an investment over time when certain aspects of the project's uncertainty become known. One of the value-added components of using real options is that it takes into account management's ability to create, execute, and abandon strategic and flexible options.

There are several potential problem areas in using a traditional discounted cash flow calculation on strategic optionalities. These problems include undervaluing an asset that currently produces little or no cash flow, the non-constant nature of the weighted average cost of capital discount rate through time, the estimation of an asset's economic life, forecast errors in creating the future cash flows, and insufficient tests for plausibility of the final results.

Real options when applied using an options theoretical framework can mitigate some of these problematic areas. Otherwise, financial profit level metrics, such as NPV or internal rate of return (IRR), will be skewed and not provide a comprehensive view of the entire investment value. However, the discounted cash flow model does have its merits:

Discounted Cash Flow Advantages

- Clear, consistent decision criteria for all projects.
- Same results regardless of risk preferences of investors.
- Quantitative, decent level of precision, and economically rational.
- Not as vulnerable to accounting conventions (depreciation, inventory valuation, and so forth).
- Factors in the time value of money and risk structures.
- Relatively simple, widely taught, and widely accepted.
- Simple to explain to management: "If benefits outweigh the costs, do it!"

In reality, an analyst should be aware of several issues prior to using discounted cash flow models, as shown in Table 2.1. The most important aspects include the business reality that risks and uncertainty abound when decisions have to be made and that management has the strategic flexibility to make and change decisions as these uncertainties become known over time. In such a stochastic world, using deterministic models like the discounted cash flow may potentially grossly underestimate the value of a particular project. A deterministic discounted cash flow model assumes at the outset that all future outcomes are fixed. If this is the case, then the discounted cash flow model is correctly specified as there would be no fluctuations in business conditions that would change the value of a particular project. In essence, there would be no value in flexibility. However, the actual business environment is highly fluid, and if management has the flexibility to make appropriate changes when conditions differ, then there is indeed value in flexibility, a value that will be grossly underestimated using a discounted cash flow model.

Figure 2.1 shows a simple example of applying discounted cash flow analysis. Assume that there is a project that costs \$1,000 to implement at Year 0 that will bring in the following projected positive cash flows in the subsequent five years: \$500, \$600, \$700, \$800, and \$900. These projected values are simply subjective best-guess forecasts on the part of the analyst. As can be seen in Figure 2.1, the time line shows all the pertinent cash flows and their respective discounted present values. Assuming that the analyst decides that the project should be discounted at a 20 percent risk-adjusted discount rate using a weighted average cost of capital (WACC), we calculate the NPV to be \$985.92 and a corresponding IRR of 54.97 percent.[1] Furthermore, the analyst assumes that the project will have an infinite economic life

TABLE 2.1 Disadvantages of DCF: Assumptions versus Realities

DCF Assumptions	Realities
Decisions are made now, and cash flow streams are fixed for the future.	Uncertainty and variability in future outcomes. Not all decisions are made today, as some may be deferred to the future, when uncertainty becomes resolved.
Projects are "mini firms," and they are interchangeable with whole firms.	With the inclusion of network effects, diversification, interdependencies, and synergy, firms are portfolios of projects and their resulting cash flows. Sometimes projects cannot be evaluated as stand-alone cash flows.
Once launched, all projects are passively managed.	Projects are usually actively managed through project life cycle, including checkpoints, decision options, budget constraints, and so forth.
Future free cash flow streams are all highly predictable and deterministic.	It may be difficult to estimate future cash flows as they are usually stochastic and risky in nature.
Project discount rate used is the opportunity cost of capital, which is proportional to nondiversifiable risk.	There are multiple sources of business risks with different characteristics, and some are diversifiable across projects or time.
All risks are completely accounted for by the discount rate.	Firm and project risk can change during the course of a project.
All factors that could affect the outcome of the project and value to the investors are reflected in the DCF model through the NPV or IRR.	Because of project complexity and so-called externalities, it may be difficult or impossible to quantify all factors in terms of incremental cash flows. Distributed, unplanned outcomes (e.g., strategic vision and entrepreneurial activity) can be significant and strategically important.
Unknown, intangible, or immeasurable factors are valued at zero.	Many of the important benefits are intangible assets or qualitative strategic positions.

and assumes a long-term growth rate of cash flows of 5 percent. Using the Gordon constant growth model, the analyst calculates the terminal value of the project's cash flow at Year 5 to be $6,300. Discounting this figure for five years at the risk-adjusted discount rate and adding it to the original NPV yields a total NPV with terminal value of $3,517.75.

The calculations can all be seen in Figure 2.1, where we further define *w* as the weights, *d* for debt, *ce* for common equity, and *ps* for preferred stocks, *FCF* as the free cash flows, *tax* as the corporate tax rate, *g* as the long-term growth rate of cash flows, and *rf* as the risk-free rate.

Even with a simplistic discounted cash flow model like this, we can see the many shortcomings of using a discounted cash flow model that are worthy of mention. Figure 2.2 lists some of the more noteworthy issues. For instance, the NPV is calculated as the present value of future net free cash flows (benefits) less the present value of implementation costs (investment costs). However, in many instances, analysts tend to discount both benefits and investment costs at a single identical market risk-adjusted discount rate, usually the WACC. This, of course, is flawed.

The benefits should be discounted at a market risk-adjusted discount rate like the WACC, but the investment cost should be discounted at a reinvestment rate similar to the risk-free rate. Cash flows that have market risks should be discounted at the market risk-adjusted rate, while cash flows that have private risks should be discounted at the risk-free rate because the market will only compensate the firm for taking on the market risks but not private risks. It is usually assumed that the benefits are subject to market risks (because benefit free cash flows depend on market demand, market prices, and other exogenous market factors), while investment costs depend on internal private risks (such as the firm's ability to complete build-

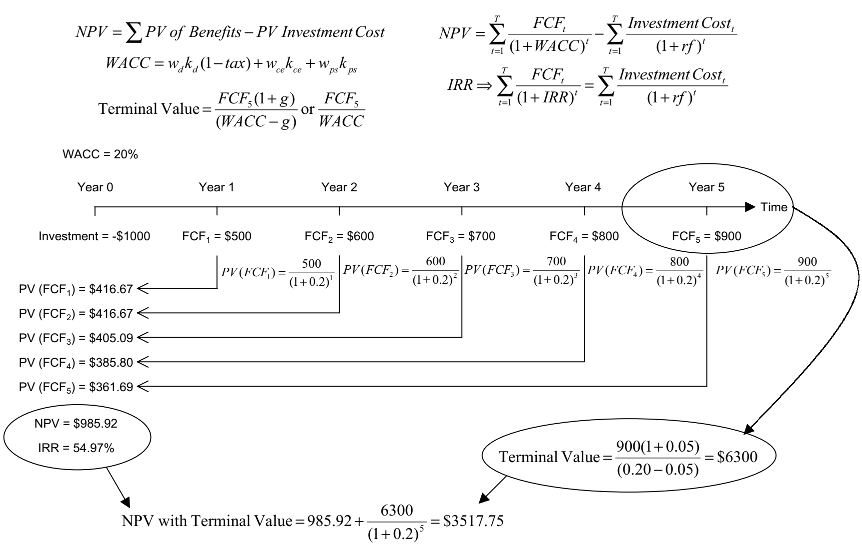


FIGURE 2.1 Applying Discounted Cash Flow Analysis

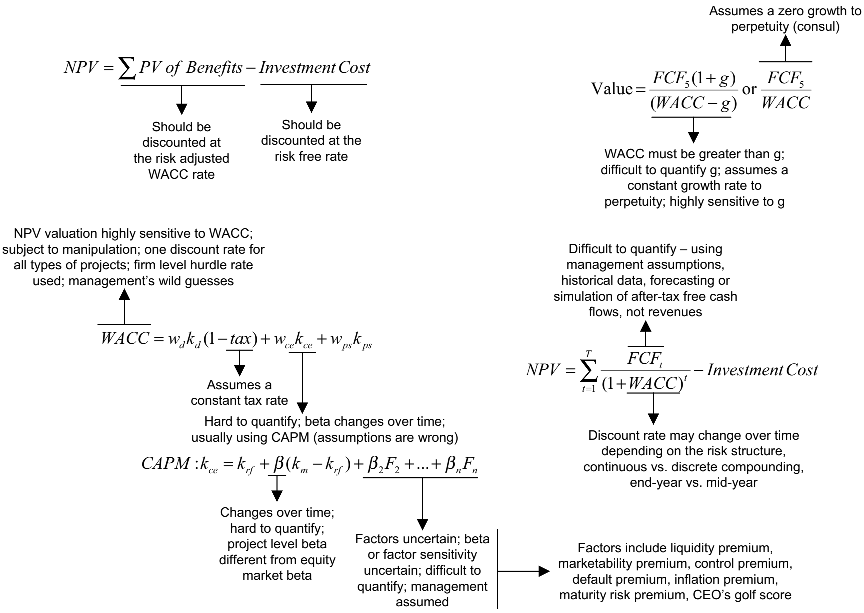


FIGURE 2.2 Shortcomings of Discounted Cash Flow Analysis

ing a project in a timely fashion or the costs and inefficiencies incurred beyond what is projected). On occasion, these implementation costs may also be discounted at a rate slightly higher than a risk-free rate, such as a money-market rate or at the opportunity cost of being able to invest the sum in another project yielding a particular interest rate. Suffice it to say that benefits and investment costs should be discounted at different rates if they are subject to different risks. Otherwise, discounting the costs at a much higher market risk-adjusted rate will reduce the costs significantly, making the project look as though it were more valuable than it actually is.

Variables with market risks should be discounted at a market risk-adjusted rate, which is higher than the risk-free rate, which is used to discount variables with private risks.

The discount rate that is used is usually calculated from a WACC, capital asset-pricing model (CAPM), multifactor asset-pricing theory (MAPT), or arbitrage pricing theory (APT), set by management as a requirement for the firm, or as a hurdle rate for specific projects.[2] In most circumstances, if we were to perform a simple discounted cash flow model, the most sensitive variable is usually the discount rate. The discount rate is also the most difficult variable to correctly quantify. Hence, this leaves the discount rate open to potential abuse and subjective manipulation. A target NPV value can be obtained by simply massaging the discount rate to a suitable level. In addition, certain input assumptions required to calculate the discount rate are also subject to questions. For instance, in the WACC, the input for cost of common equity is usually derived using some form of the CAPM. In the CAPM, the infamous beta (β) is extremely difficult to calculate. In financial assets, we can obtain beta through a calculation of the covariance between a firm's stock prices and the market portfolio, divided by the variance of the market portfolio. Beta is then a sensitivity factor measuring the co-movements of a firm's equity prices with respect to the market. The problem is that equity prices change every few minutes! Depending on the time frame used for the calculation, beta may fluctuate wildly. In addition, for nontraded physical assets, we cannot reasonably calculate beta this way. Using a firm's tradable financial assets' beta as a proxy for the beta on a single nontraded and nonmarketable project within a firm that has many other projects is ill advised. Chapter 6 introduces a new method of obtaining discount rates through the use of internal comparables, Monte Carlo simulation, and real options volatility estimates. This approach, discussed in the Risk versus Uncertainty section of Chapter 6, provides a more robust discount rate estimate than the CAPM with external market comparables.

There are risk and return diversification effects among projects as well as investor psychology and overreaction in the market that are not accounted for. There are also other more robust asset-pricing models that can be used to estimate a project's discount rate, but they require great care. For instance, the APT models are built on the CAPM and have additional risk factors that may drive the value of the discount rate. These risk factors include maturity risk, default risk, inflation risk, country risk, size risk, nonmarketable risk, control risk, minority shareholder risk, and others. Even the firm's CEO's golf score can be a risk hazard (e.g., rash decisions may be made after a bad game or bad projects may be approved after a hole in one, believing in a lucky streak). The issue arises when one has to decide which risks to include and which not to include. This is definitely a difficult task, to say the least.[3]

The methods to find a relevant discount rate include using a WACC, CAPM, APT, MAPT, comparability analysis, management assumptions, and a firm- or project-specific hurdle rate.

One other widely used method is that of comparability analysis. By gathering publicly available data on the trading of financial assets by stripped-down entities with similar functions, markets, risks, and geographical location, analysts can then estimate the beta (a measure of systematic risk) or even a relevant discount rate from these comparable firms. For instance, an analyst who is trying to gather information on a research and development effort for a particular type of drug can conceivably gather market data on pharmaceutical firms performing only research and development on similar drugs, existing in the same market, and having the same risks. The median or average beta value can then be used as a market proxy for the project currently under evaluation. Obviously, there is no silver bullet, but if an analyst were diligent enough, he or she could obtain estimates from these different sources and create a better estimate. Monte Carlo simulation is most preferred in situations like these.[4] The analyst can define the relevant simulation inputs using the range obtained from the comparable firms and simulate the discounted cash flow model to obtain the range of relevant variables (typically the NPV or IRR).

Now that you have the relevant discount rate, the free cash flow stream should then be discounted appropriately. Herein lies another problem: forecasting the relevant free cash flows and deciding if they should be discounted on a continuous basis or a discrete basis, versus using end-of-year or mid-year conventions. Free cash flows should be net of taxes, with the relevant noncash expenses added back.[5] Because free cash flows are generally calculated starting with revenues and proceeding through direct cost of goods sold, operating expenses, depreciation expenses, interest payments, taxes, and so forth, there is certainly room for mistakes to compound over time.

Forecasting cash flows several years into the future is oftentimes very difficult and may require the use of fancy econometric regression modeling techniques, time-series analysis, management hunches, and experience. A recommended method is not to create single-point estimates of cash flows at certain time periods but to use Monte Carlo simulation and assess the relevant probabilities of cash flow events. In addition, because cash flows in the distant future are certainly riskier than in the near future, the relevant discount rate should also change to reflect this. Instead of using a single discount rate for all future cash flow events, the discount rate should incorporate the changing risk structure of cash flows over time. This can be done by either weighing the cash flow streams' probabilistic risks (standard deviations of forecast distributions) or using a stepwise technique of adding the maturity risk premium inherent in U.S. Treasury securities at different maturity periods. This bootstrapping approach allows the analyst to incorporate what the market experts predict the future market risk structure looks like. That is, discount the cash flows twice: once for time value of money, and once for risk.

This way, changes in risk structure and risk-free rate can be adjusted accordingly over time.

Finally, the issue of terminal value is of major concern for anyone using a discounted cash flow model. Several methods of calculating terminal values exist, such as the Gordon constant growth model (GGM), zero growth perpetuity consul, and the supernormal growth models. The GGM is the most widely used, where at the end of a series of forecast cash flows, the GGM assumes that cash flow growth will be constant through perpetuity. The GGM is calculated as the free cash flow at the end of the forecast period multiplied by a relative growth rate, divided by the discount rate less the long-term growth rate. Shown in Figure 2.2, we see that the GGM breaks down when the long-term growth rate exceeds the discount rate. This growth rate is also assumed to be fixed, and the entire terminal value is highly sensitive to this growth rate assumption. In the end, the value calculated is highly suspect because a small difference in growth rates will mean a significant fluctuation in value. Perhaps a better method is to assume some type of growth curve in the free cash flow series. These growth curves can be obtained through some basic time-series analysis as well as using more advanced assumptions in stochastic modeling. Nonetheless, we see that even a well-known, generally accepted and applied discounted cash flow model has significant analytical restrictions and problems. These problems are rather significant and can compound over time, creating misleading results. Great care should be taken when performing such analyses. Later chapters introduce the concepts of Monte Carlo simulation, real options, and portfolio optimization. These new analytical methods address some of the issues discussed above. However, it should be stressed that these new analytics do not provide the silver bullet for valuation and decision making. They provide value-added insights, and the magnitude of insights and value obtained from these new methods depend solely on the type and characteristic of the project under evaluation.

The applicability of traditional analysis versus the new analytics across a time horizon is depicted in Figure 2.3. During the shorter time period, holding everything else constant, the ability for the analyst to predict the near future is greater than when the period extends beyond the historical and forecast periods. This is because the longer the horizon, the harder it is to fully predict all the unknowns, and hence, management can create value by being able to successfully initiate and execute strategic options.

The traditional and new analytics can also be viewed as a matrix of approaches as seen in Figure 2.4, where the analytics are segregated by its analytical perspective and type. With regard to perspective, the analytical approach can be either a top-down or a bottom-up approach. A top-down approach implies a higher focus on macro variables than on micro variables. The level of granularity from the macro to micro levels include starting from the global perspective, and working through market or economic conditions,

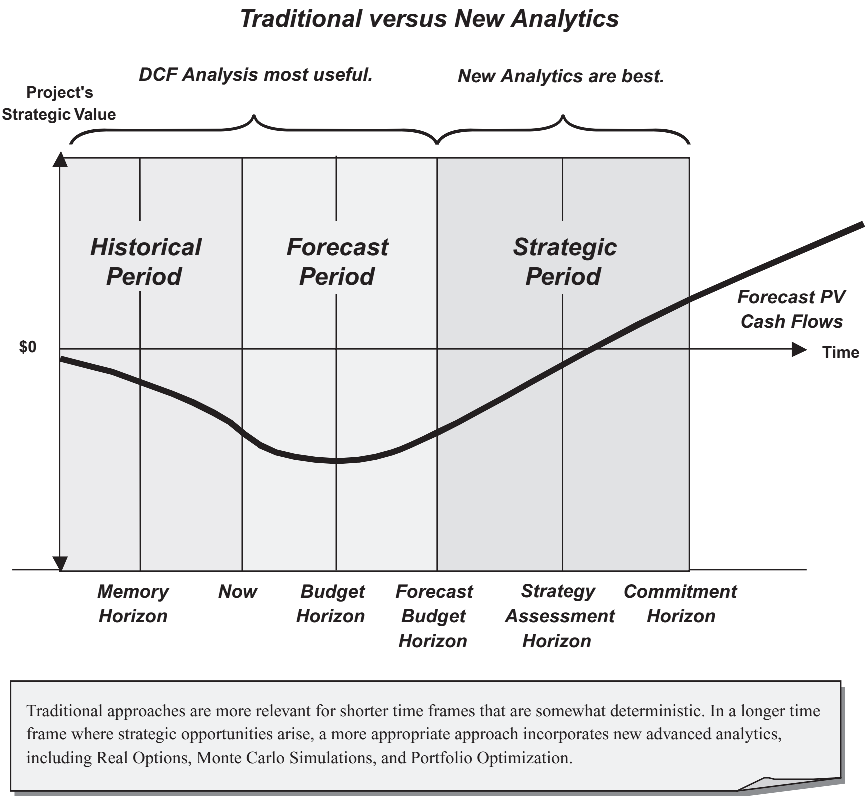


FIGURE 2.3 Using the Appropriate Analysis

impact on a specific industry, and more specifically, the firm's competitive options. At the firm level, the analyst may be concerned with a single project and the portfolio of projects from a risk management perspective. At the project level, detail focus will be on the variables impacting the value of the project.

SUMMARY

Traditional analyses like the discounted cash flow are fraught with problems. They underestimate the flexibility value of a project and assume that all outcomes are static and all decisions made are irrevocable. In reality, business decisions are made in a highly fluid environment where uncertainties abound and management is always vigilant in making changes in decisions when the

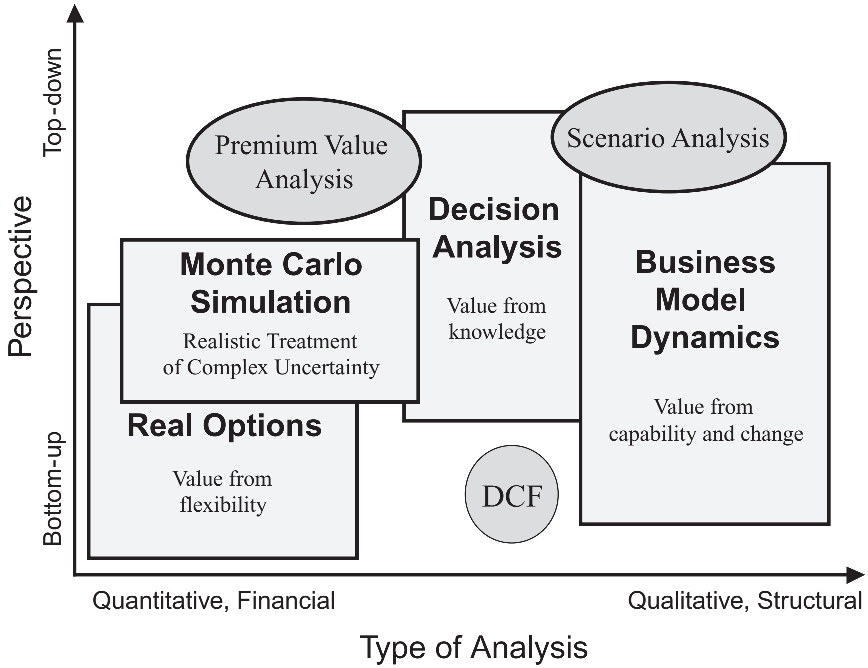


FIGURE 2.4 An Analytical Perspective

circumstances require a change. To value such decisions in a deterministic view may potentially grossly underestimate the true intrinsic value of a project. New sets of rules and methodology are required in light of these new managerial flexibilities. It should be emphasized that real options analysis builds upon traditional discounted cash flow analysis, providing value-added insights to decision making. In later chapters, it will be shown that discounted cash flow analysis is a special case of real options analysis when there is no uncertainty in the project.

CHAPTER 2 QUESTIONS

1. What are the three traditional approaches to valuation?
2. Why should benefits and costs be discounted at two separate discount rates?
3. Is the following statement true? Why or why not? “The value of a firm is simply the sum of all its individual projects.”
4. What are some of the assumptions in order for the CAPM to work?

5. Using the discrete and continuous discounting conventions explained in Appendix 2A, and assuming a 20 percent discount rate, calculate the net present value of the following cash flows:

Year	2002	2003	2004	2005	2006	2007
Revenues		\$100	\$200	\$300	\$400	\$500
Operating Expenses		10	20	30	40	50
Net Income		90	180	270	360	450
Investment Costs	(\$450)					
Free Cash Flow	(\$450)	90	180	270	360	450

APPENDIX 2A

Financial Statement Analysis

This appendix provides some basic financial statement analysis concepts used in applying real options. The focus is on calculating the free cash flows used under different scenarios, including making appropriate adjustments under levered and unlevered operating conditions. Although many versions of free cash flows exist, these calculations are examples of more generic free cash flows applicable under most circumstances. An adjustment for inflation and the calculation of terminal cash flows are also presented here. Finally, a market multiple approach that uses price-to-earnings ratios is also briefly discussed.

FREE CASH FLOW CALCULATIONS

Below is a list of some generic financial statement definitions used to generate free cash flows based on GAAP (generally accepted accounting principles):

- Gross Profits = Revenues − Cost of Goods Sold.
- Earnings Before Interest and Taxes = Gross Profits − Selling Expenses − General and Administrative Costs − Depreciation − Amortization.
- Earnings Before Taxes = Earnings Before Interest and Taxes − Interest.
- Net Income = Earnings Before Taxes − Taxes.
- Free Cash Flow to Equity = Net Income + Depreciation + Amortization − Capital Expenditures ± Change in Net Working Capital − Principal Repayments + New Debt Proceeds − Preferred Dividends – Interest (1 – Tax Rate).
- Free Cash Flow to the Firm = EBIT (1 – Tax Rate) + Depreciation + Amortization – Capital Expenditures ± Change in Net Working Capital = Free Cash Flow to Equity + Principal Repayment – New Debt Proceeds + Preferred Dividends + Interest (1 – Tax Rate).

FREE CASH FLOW TO A FIRM

An alternative version of the free cash flow for an unlevered firm can be defined as:

- Free Cash Flow = Earnings Before Interest and Taxes [1 − Effective Tax Rate] + Depreciation + Amortization − Capital Expenditures ± Change in Net Working Capital.

LEVERED FREE CASH FLOW

For a levered firm, the free cash flow becomes:

- Free Cash Flow = Net Income + α [Depreciation + Amortization] ± α [Change in Net Working Capital] − α [Capital Expenditures] − Principal Repayments + New Debt Proceeds − Preferred Debt Dividends

where

α	is the equity-to-total-capital ratio; and
$(1-\alpha)$	is the debt ratio.

INFLATION ADJUSTMENT

The following adjustments show an inflationary adjustment for free cash flows and discount rates from nominal to real conditions:

- Real $CF = \dfrac{\text{Nominal } CF}{(1+E[\pi])}$
- Real $\rho = \dfrac{1+\text{Nominal } \rho}{(1+E[\pi])} - 1$

where

CF	is the cash flow series;
π	is the inflation rate;
$E[\pi]$	is the expected inflation rate; and
ρ	is the discount rate.

TERMINAL VALUE

The following are commonly accepted ways of getting terminal free cash flows under zero growth, constant growth, and supernormal growth assumptions:

- Zero Growth Perpetuity:

$$\sum_{t=1}^{\infty} \frac{FCF_t}{[1 + WACC]^t} = \frac{FCF_T}{WACC}$$

- Constant Growth:

$$\sum_{t=1}^{\infty} \frac{FCF_{t-1}(1 + g_t)}{[1 + WACC]^t} = \frac{FCF_{T-1}(1 + g_T)}{WACC - g_T} = \frac{FCF_T}{WACC - g_T}$$

- Punctuated Supernormal Growth:

$$\sum_{t=1}^{N} \frac{FCF_t}{[1+WACC]^t} + \frac{\left[\frac{FCF_N(1 + g_N)}{[WACC - g_N]}\right]}{[1 + WACC]^N}$$

$$WACC = \omega_e k_e + \omega_d k_d (1 - \tau) + \omega_{pe} k_{pe}$$

where

FCF	is the free cash flow series;
$WACC$	is the weighted average cost of capital;
g	is the growth rate of free cash flows;
t	is the individual time periods;
T	is the terminal time at which a forecast is available;
N	is the time when a punctuated growth rate occurs;
ω	is the respective weights on each capital component;
k_e	is the cost of common equity;
k_d	is the cost of debt;
k_{pe}	is the cost of preferred equity; and
τ	is the effective tax rate.

PRICE-TO-EARNINGS MULTIPLES APPROACH

Related concepts in valuation are the uses of market multiples. An example is using the price-to-earnings multiple, which is a simple derivation of the constant growth model shown above, breaking it down into dividends per share (*DPS*) and earnings per share (*EPS*) components.

The derivation starts with the constant growth model:

$$P_0 = \frac{DPS_0(1 + g_n)}{k_e - g_n} = \frac{DPS_1}{k_e - g_n}$$

We then use the fact that the dividend per share next period (DPS_1) is the earnings per share current period multiplied by the payout ratio (PR), defined as the ratio of dividends per share to earnings per share, which is assumed to be constant, multiplied by one plus the growth rate ($1 + g$) of earnings:

$$DPS_1 = EPS_0[PR](1 + g_n)$$

Similarly, the earnings per share the following period is the same as the earnings per share this period multiplied by one plus the growth rate:

$$EPS_1 = EPS_0(1 + g_n)$$

Substituting the earnings per share model for the dividends per share in the constant growth model, we get the pricing relationship:

$$P_0 = \frac{EPS_0[PR](1 + g_n)}{k_e - g_n}$$

Because we are using price-to-earnings ratios, we can divide the pricing relationship by earnings per share to obtain an approximation of the price-to-earnings ratio (PE):

$$\frac{P_0}{EPS_1} = \frac{[PR]}{k_e - g_n} \approx PE_1$$

Assuming that the PE and EPS ratios are fairly stable over time, we can estimate the current pricing structure through forecasting the next term EPS we obtain:

$$P_0 = E\tilde{P}S_1[PE_1]$$

Issues of using PE ratios include the fact that PE ratios change across different markets. If a firm serves multiple markets, it is difficult to find an adequate weighted average PE ratio. PE ratios may not be stable through time and are most certainly not stable across firms. If more efficient firms are added to less efficiently run firms, the average PE ratio may be skewed. In addition, market overreaction and speculation, particularly among high-growth firms, provide an overinflated PE ratio. Furthermore, not all firms are publicly held, some firms may not have a PE ratio, and if valuation of individual projects is required, PE ratios may not be adequate because it is difficult to isolate a specific investment's profitability and its corresponding PE ratio. Similar approaches include using other proxy multiples, including Business Enterprise Value to Earnings, Price to Book, Price to Sales, and so forth, with similar methods and applications.

DISCOUNTING CONVENTIONS

In using discounted cash flow analysis, several conventions require consideration: continuous versus discrete discounting, midyear versus end-of-year convention, and beginning versus end-of-period discounting.

Continuous versus Discrete Periodic Discounting

The discounting convention is important when performing a discounted cash flow analysis. Using the same compounding period principle, future cash flows can be discounted using the effective annualized discount rate. For instance, suppose an annualized discount rate of 30 percent is used on a \$100 cash flow. Depending on the compounding periodicity, the calculated present value and future value differ (see Table 2A.1).

To illustrate this point further, a \$100 deposit in a 30 percent interest-bearing account will yield \$130 at the end of one year if the interest compounds once a year. However, if interest is compounded quarterly, the deposit value increases to \$133.55 due to the additional interest-on-interest compounding effects. For instance,

Value at the end of the first quarter = $\$100.00(1 + 0.30/4)^1 = \$107.50$

Value at the end of the second quarter = $\$107.50(1 + 0.30/4)^1 = \$115.56$

Value at the end of the third quarter = $\$115.56(1 + 0.30/4)^1 = \$124.23$

Value at the end of the fourth quarter = $\$124.23(1 + 0.30/4)^1 = \$133.55$

That is, the annualized discount rate for different compounding periods is its effective annualized rate, calculated as

$$\left(1 + \frac{discount}{periods}\right)^{periods} - 1$$

For the quarterly compounding interest rate, the effective annualized rate is

$$\left(1 + \frac{30.00\%}{4}\right)^{4} - 1 = 33.55\%$$

TABLE 2A.1 Continuous versus Periodic Discrete Discounting

Periodicity	Periods/Year	Interest Factor	Future Value	Present Value
Annual	1	30.00%	$130.00	$76.92
Quarterly	4	33.55	133.55	74.88
Monthly	12	34.49	134.49	74.36
Daily	365	34.97	134.97	74.09
Continuous	∞	34.99	134.99	74.08

Applying this rate for the year, we have $100(1 + 0.3355) = $133.55.

This analysis can be extended for monthly, daily, or any other periodicities. In addition, if the interest rate is assumed to be continuously compounding, the continuous effective annualized rate should be used, where

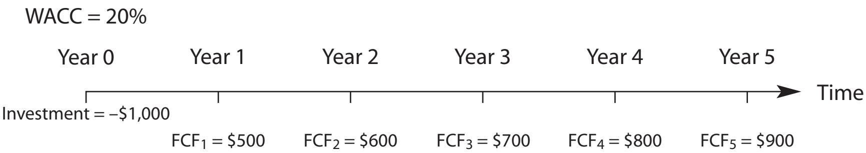
$$\lim_{periods \to \infty} \left(1 + \frac{discount}{periods}\right)^{periods} - 1 = e^{discount} - 1$$

For instance, the 30 percent interest rate compounded continuously yields $e^{0.3} - 1 = 34.99\%$. Notice that as the number of compounding periods increases, the effective interest rate increases until it approaches the limit of continuous compounding.

The annual, quarterly, monthly, and daily compounding is termed discrete periodic compounding, as compared to the continuous compounding approach using the exponential function. In summary, the higher the number of compounding periods, the higher the future value and the lower the present value of a cash flow payment. When applied to discounted cash flow analysis, if the discount rate calculated using a weighted average cost of capital is continuously compounding (e.g., interest payments and cost of capital are continuously compounding), then the net present value calculated may be overoptimistic if discounted discretely.

Full-Year versus Midyear Convention

In the conventional discounted cash flow approach, cash flows occurring in the future are discounted back to the present value and summed to obtain the net present value of a project. These cash flows are usually attached to a particular period in the future, measured usually in years, quarters, or months. The time line in Figure 2A.1 illustrates a sample series of cash flows over the next five years, with an assumed 20 percent discount rate. Because the cash flows



$$NPV = -\$1{,}000 + \frac{\$500}{(1+0.2)^1} + \frac{\$600}{(1+0.2)^2} + \frac{\$700}{(1+0.2)^3} + \frac{\$800}{(1+0.2)^4} + \frac{\$900}{(1+0.2)^5} = \$985$$

FIGURE 2A.1 Full-Year versus Mid-Year Discounting

are attached to an annual time line, they are usually assumed to occur at the end of each year. That is, $500 will be recognized at the end of the first full year, $600 at the end of the second year, and so forth. This is termed the full-year discounting convention.

However, under usual business conditions, cash flows tend to accrue throughout the entire year and do not arrive in a single lump sum at the end of the year. Instead, the midyear convention may be applied. That is, the $500 cash flow gets accrued over the entire first year and should be discounted at 0.5 years, rather than 1.0 years. Using this midpoint supposes that the $500 cash flow comes in equally over the entire year.

$$NPV = -\$1{,}000 + \frac{\$500}{(1+0.2)^{0.5}} + \frac{\$600}{(1+0.2)^{1.5}} + \frac{\$700}{(1+0.2)^{2.5}} + \frac{\$800}{(1+0.2)^{3.5}} + \frac{\$900}{(1+0.2)^{4.5}} = \$1{,}175$$

End-of-Period versus Beginning-of-Period Discounting

Another key issue in discounting involves the use of end-of-period versus beginning-of-period discounting. Suppose the cash flow series are generated on a time line such as in Figure 2A.2.

Further suppose that the valuation date is January 1, 2002. The $500 cash flow can occur either at the beginning of the first year (January 1, 2003) or at the end of the first year (December 31, 2003). The former requires the discounting of one year and the latter, the discounting of two

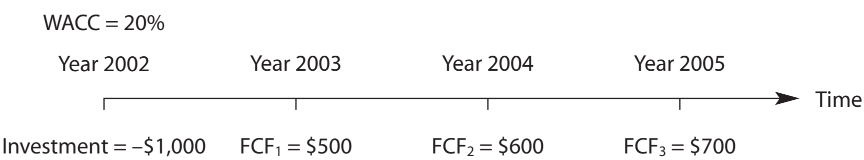


FIGURE 2A.2 End-of-Period versus Beginning-of-Period Discounting years. If the cash flows are assumed to roll in equally over the year—that is, from January 1, 2002, to January 1, 2003—the discounting should only be for 0.5 years.

In contrast, suppose that the valuation date is December 31, 2002, and the cash flow series occurs at January 1, 2003, or December 31, 2003. The former requires no discounting, while the latter requires a one-year discounting using an end-of-year discounting convention. In the midyear convention, the cash flow occurring on December 31, 2003, should be discounted at 0.5 years.

APPENDIX 2B

Discount Rate versus Risk-Free Rate

Generally, the weighted average cost of capital (WACC) would be used as the discount rate for the cash flow series. The only mitigating circumstance is when the firm wishes to use a hurdle rate that exceeds the WACC to compensate for the additional uncertainty, risks, and opportunity costs the firm believes it will face by investing in a particular project. As we will see, the use of a WACC is problematic, and in the real options world, the input is instead a U.S. Treasury spot rate of return with its maturity corresponding to the economic life of the project under scrutiny.

In general, the WACC is the weighted average of the cost components of issuing debt, preferred stock, and common equity: $WACC = \omega_d k_d(1 - \tau) + \omega_p k_p + \omega_e k_e$, where ω are the respective weights, τ is the corporate effective tax rate, and k are the costs corresponding to debt[1] d, preferred stocks[2] p, and common equity[3] e.

However, multiple other factors affect the cost of capital that need to be considered, including:

1. The company's capital structure used to calculate the relevant WACC discount rate may be inadequate, because project-specific risks are usually not the same as the overall company's risk structure.
2. The current and future general interest rates in the economy may be higher or lower, thus bond coupon rates may change in order to raise the capital based on fluctuations in the general interest rate. Therefore, an interest-rate-bootstrapping methodology should be applied to infer the future spot interest rates using forward interest rates.
3. Tax law changes over time may affect the tax shield enjoyed by debt repayments. Furthermore, different tax jurisdictions in different countries have different tax law applications of tax shields.
4. The firm's capital structure policy may have specific long-term targets and weights that do not agree with the current structure, and the firm may find itself moving toward that optimal structure over time.

5. Payout versus retention rate policy may change the dividend yield and thereby change the projected dividend growth rate necessary to calculate the cost of equity.
6. Investment policy of the firm, including the minimum required rate of return and risk profile.
7. Dynamic considerations in the economy and industry both *ex post* and *ex ante*.
8. Measurement problems on specific security cost structure.
9. Small business problems making it difficult to measure costs correctly.
10. Depreciation-generated funds and off-balance sheet items are generally not included in the calculations.
11. Geometric averages and not simple arithmetic averages should be used for intrayear WACC rates.[4]
12. Selection of market value versus book value weightings[5] in calculating the WACC.
13. The capital asset-pricing model (CAPM) is flawed.

THE CAPM VERSUS THE MULTIFACTOR ASSET-PRICING MODEL

The CAPM model states that under some simplifying assumptions, the rate of return on any asset may be expected to be equal to the rate of return on a riskless asset plus a premium that is proportional to the asset's risk relative to the market. The CAPM is developed in a theoretical and hypothetical world with multiple assumptions[6] that do not hold true in reality, and therefore it is flawed by design.[7]

The alternative is to use a multifactor model that adequately captures the systematic risks experienced by the firm. In a separate article, the author used a nonparametric multifactor asset-pricing model and showed that the results are more robust. However, the details exceed the scope of this book.

Other researchers have tested the CAPM and found that a single factor, beta, does not sufficiently explain expected returns. Their empirical research finds support for the inclusion of both size (measured using market value) and leverage variables. The two leverage variables found to be significant were the book-to-market ratio and the price-to-earnings ratio. However, when used together, the book-to-market ratio and size variable absorb the effects of the price-to-earnings ratio. With empirical support that beta alone is insufficient to capture risk, their model relies on the addition of the natural logarithm of both the book-to-market ratio and the size of the firm's market equity as

$$E[R_{i,t}] - [R_{f,t}] = \beta_{i,t}\,(E[R_{m,t}] - R_{f,t}) + \delta_{i,t}\ln(BME_{i,t}) + \gamma_{i,t}\ln(ME_{i,t})$$

where $R_{i,t}$, $R_{m,t}$, and $R_{f,t}$ are the individual expected return for firm *i*, the expected market return, and the risk-free rate of return at time *t*, respectively. $BME_{i,t}$ and $ME_{i,t}$ are the book-to-market ratio and the size of the total market equity value for firm *i* at time *t*, respectively.

Other researchers have confirmed these findings, that a three-factor model better predicts expected returns than the single-factor CAPM. Their main conjecture is that asset-pricing is rational and conforms to a three-factor model that does not reduce to the standard single-factor CAPM. One of the major problems with the single-factor CAPM is that of determining a good proxy for the market, which should truly represent all traded securities. In addition, the expected return on the market proxy typically relies on *ex post* returns and does not truly capture expectations. Therefore, the multifactor model is an attempt to recover the expected CAPM results without all the single-factor model shortcomings. A variation of the three-factor model is shown as

$$E[R_{i,t}] - [R_{f,t}] = \beta_{i,t}\,(E[R_{m,t}] - R_{f,t}) + \xi_{i,t}\ln(SMB_{i,t}) + \psi_{i,t}\ln(HML_{i,t})$$

where $SMB_{i,t}$ is the time series of differences in average returns from the smallest and largest capitalization stocks. $HML_{i,t}$ is the time series of differences in average returns from the highest to the lowest book-to-market ratios, after ranking the market portfolios into differing quartiles.

We can adapt this multifactor model to accommodate any market and any industry. The factors in the foregoing model can be sector- or industry-specific. The macroeconomic variables used will have to be highly correlated to historical returns of the firm. If sufficient data are available, a multifactor regression model can be generated, and variables found to be statistically significant can then be used. Obviously, there is potential for abuse and misuse of the model.[8] If used correctly, the model will provide a wealth of information on the potential risks that the project or asset holds. However, in the end, the jury is still out on what constitutes a good discount rate model.

Chapter 6 introduces an alternative method of discount rate determination using internal comparables, Monte Carlo simulation, and real options volatility estimates. This new approach is discussed in the Risk versus Uncertainty section of Chapter 6.

CHAPTER 3

Real Options Analysis

INTRODUCTION

This chapter introduces the fundamental essence of real options, providing the reader several simplified but convincing examples of why a real options approach provides more insights than traditional valuation methodologies do. A lengthy but simplified example details the steps an analyst might go through in evaluating a project. The example expounds on the different decisions that will be made depending on which methodology is employed, and introduces the user to the idea of adding significant value to a project by looking at the different optionalities that exist, sometimes by even creating strategic optionalities within a project, thereby enhancing its overall value to the firm.

THE FUNDAMENTAL ESSENCE OF REAL OPTIONS

The use of traditional discounted cash flow alone is inappropriate in valuing certain strategic projects involving managerial flexibility. Two finance professors, Michael Brennan and Eduardo Schwartz, provided an example on valuing the rights to a gold mine. In their example, a mining company owns the rights to a local gold mine. The rights provide the firm the option, and not the legal obligation, to mine the gold reserves supposedly abundant in said mine. Therefore, if the price of gold in the market is high, the firm might wish to start mining and, in contrast, stop and wait for a later time to begin mining should the price of gold drop significantly in the market. Suppose we set the cost of mining as *X* and the payoff on the mined gold as *S,* taking into consideration the time value of money. We then have the following payoff schedule:

$$\begin{array}{lll} S - X & \text{if and only if} & S > X \\ 0 & \text{if and only if} & S \leq X \end{array}$$

This payoff is identical to the payoff on a call option on the underlying asset, the value of the mined gold. If the cost exceeds the value of the underlying asset, the option is left to expire worthless, without execution; otherwise, the option will be exercised. That is, mine if and only if *S* exceeds *X;* otherwise, do not mine.

As an extension of the gold mine scenario, say we have a proprietary technology in development or a patent that currently and in the near future carries little or no cash flow but nonetheless is highly valuable due to the potential strategic positioning it holds for the firm that owns it. A traditional discounted cash flow method will grossly underestimate the value of this asset. A real options approach is more suitable and provides better insights into the actual value of the asset. The firm has the option to either develop the technology if the potential payoff exceeds the cost or abandon its development should the opposite be true.

For instance, assume a firm owns a patent on some technology with a 10-year economic life. To develop the project, the present value of the total research and development costs is $250 million, but the present value of the projected sum of all future net cash flows is only $200 million. In a traditional discounted cash flow sense, the net present value will be –$50 million, and the project should be abandoned. However, the proprietary technology is still valuable to the firm given that there's a probability it will become more valuable in the future than projected or that future projects can benefit from the technology developed. If we apply real options to valuing this simplified technology example, the results will be significantly different. By assuming the nominal rate on a 10-year risk-free U.S. Treasury note is 6 percent and simulating the projected cash flows, we calculate the value of the research and development initiative to be $2 million. This implies that the value of flexibility is $52 million or 26 percent of its static NPV value.[1] By definition, a research and development initiative involves creating something new and unique or developing a more enhanced product. The nature of most research and development initiatives is that they are highly risky and involve a significant investment up front, with highly variable potential cash flows in the future that are generally skewed toward the low end. In other words, most research and development projects fail to meet expectations and generally produce lower incremental revenues than deemed profitable. Hence, in a traditional discounted cash flow sense, research and development initiatives are usually unattractive and provide little to no incentives. However, a cursory look at the current industry would imply otherwise. Research and development initiatives abound, implying that senior management sees significant intrinsic value in such initiatives. So there arises a need to quantify such strategic values.

THE BASICS OF REAL OPTIONS

Real options, as its name implies, use options theory to evaluate physical or real assets, as opposed to financial assets or stocks and bonds. In reality, real options have been in the past very useful in analyzing distressed firms and firms engaged in research and development with significant amounts of managerial flexibility under significant amounts of uncertainty. Only in the past decade has real options started to receive corporate attention in general.

A SIMPLIFIED EXAMPLE OF REAL OPTIONS IN ACTION

Suppose a client is currently researching and developing new pharmaceutical products, and the initial outlay required for initiating this endeavor is \$100 million. The projected net benefits, using free cash flow as a proxy, resulting from this research and development effort brings about positive cash flows of \$8 million, \$12 million, \$15 million, \$12 million, \$11 million, and \$10 million for the first six years, starting next year. Furthermore, assume that there is a terminal value of \$155 million in year six.[2] These cash flows result from routine business functions associated with the firm's research and development efforts (assuming the firm is a specialized firm engaged strictly in research and development). Panel A in Table 3.1 shows a simple discounted cash flow series resulting in a discounted net present value of \$24.85 million using a given 12 percent market risk-adjusted weighted average cost of capital (WACC).

Assume that the research and development efforts are successful and that in three years, there is a potential to invest more funds to take the product to market. For instance, in the case of the pharmaceutical firm, suppose the first two to three years of research have paid off, and the firm is now ready to produce and mass-market the newly discovered drug. Panel B shows the series of cash flows relevant to this event, starting with an initial outlay of another \$382 million in year three, which will in turn provide the positive free cash flows of \$30 million, \$43 million, and \$53 million in years four through six. In addition, a terminal value of \$454 is calculated using the Gordon constant growth model for the remaining cash flows based on economic life considerations. The net present value is calculated as –\$24.99 million for this second phase. The total net present value for Panels A and B is therefore –\$0.14 million, indicating that the project is not viable. Using this traditional net present value calculation underestimates the value of the research and development effort significantly.

TABLE 3.1 Comparing Real Options and Discounted Cash Flow

Panel A

($ millions)							
Time	0	1	2	3	4	5	6
Initial Outlay	$(100.00)						
Cash Flow		$8.00	$12.00	$15.00	$12.00	$11.00	$ 10.00
Terminal Value							$155.00
Net Cash Flow	$(100.00)	$8.00	$12.00	$15.00	$12.00	$11.00	$165.00
Discount Rate	0%	12%	12%	12%	12%	12%	12%
Present Value	$(100.00)	$7.14	$ 9.57	$10.68	$ 7.63	$ 6.24	$ 83.59
Net Present Value	$ 24.85						

Panel B

($ millions)							
Time	0	1	2	3	4	5	6
Initial Outlay				$(382.00)			
Cash Flow					$30.00	$43.00	$ 53.00
Terminal Value							$454.00
Net Cash Flow				$(382.00)	$30.00	$43.00	$507.00
Discount Rate				5.50%	12%	12%	12%
Present Value				$(325.32)	$19.07	$24.40	$256.86
Net Present Value	$(24.99)						
Total NPV	$ (0.14)						
Calculated Call Value	$ 73.27						
Value of the Investment	$ 98.12						

There are a few issues that need to be considered. The first is the discount rate used on the second initial outlay of $382 million. The second is the optionality of the second series of cash flow projections.

All positive cash flow projections are discounted at a constant 12 percent WACC, but the second initial outlay is discounted at 5.5 percent, as seen in Panel B in Table 3.1. In reality, the discount rate over time should theoretically change slightly due to different interest rate expectations as risks change over time. An approach is to use a recursive interest rate bootstrap based on market-forward rates adjusted for risk; but in our simple analysis, we assume that the 12 percent does not change much over time. The three-year spot Treasury risk-free rate of 5.5 percent is used on the second investment outlay because this cash outflow is projected at present and is assumed to be susceptible only to private risks and not market risks; hence, the outlay should be discounted at the risk-free rate. If the cost outlay is discounted at the 12 percent WACC, the true value of the investment will be overinflated. To prepare for this payment in the future, the firm can set aside the funds equal to \$382 million for use in three years. The firm's expected rate of return is set at the corresponding maturity spot Treasury risk-free rate, and any additional interest income is considered income from investing activities. The 12 percent market risk-adjusted weighted average cost of capital should not be used because the firm wants a 12 percent rate of return on its research and development initiatives by taking on risk of failure where the future cash flows are highly susceptible to market risks; but the \$382 million is not under similar risks at present. In financial theory, we tend to separate market risks (unknown future revenues and free cash flow streams that are susceptible to market fluctuations) at a market risk-adjusted discount rate—in this case, the 12 percent WACC—and private risks (the second investment outlay that may change due to internal firm cost structures and not due to the market, meaning that the market will not compensate the firm for its cost inefficiencies in taking the drug to market) at a risk-free rate of 5.5 percent. In other words, cash flows should first be discounted for time value of money (5.5 percent) and then discounted for risk (6.5 percent). Market risk should be discounted for time and risk (12 percent) while private risk should only be discounted for time (5.5 percent).

Next, the optionality of the second cash flow series can be seen as a call option. The firm has the option to invest and pursue the product to market phase but not the obligation to do so. If the projected net present value in three years indicates a negative amount, the firm may abandon this second phase; or the firm may decide to initiate the second phase should the net present value prove to be positive and adequately compensate the risks borne. So, if we value the second phase as a call option, the total net present value of the entire undertaking, phase one and two combined, would be a positive \$98.12 million (calculated by adding the call value of \$73.27 million and the phase one net present value of \$24.85 million). This is the true intrinsic strategic value of the project, because if things do not look as rosy in the future, the firm does not have the obligation to take the drug to market but can always shelve the product for later release, sell its patent rights, or use the knowledge gained for creating other drugs in the future. If the firm neglects this ability to not execute the second phase, it underestimates the true value of the project.

ADVANCED APPROACHES TO REAL OPTIONS

Clearly, the foregoing example is a simple single-option condition. In more protracted and sophisticated situations, more sophisticated models have to be used. These include closed-form exotic options solutions, partial-differential equations through the optimization of objective functions subject to constraints through dynamic programming, trinomial and multinomial lattice models, binomial lattices, and stochastic simulations. This book goes into some of these more advanced applications in later chapters, along with their corresponding technical appendixes, and shows how they can be applied in actual business cases. However, for now, we are interested only in the high-level understanding of what real options are and how even thinking in terms of strategic optionality helps management make better decisions and obtain insights that would be unavailable otherwise.

WHY ARE REAL OPTIONS IMPORTANT?

An important point is that the traditional discounted cash flow approach assumes a single decision pathway with fixed outcomes, and all decisions are made in the beginning without the ability to change and develop over time. The real options approach considers multiple decision pathways as a consequence of high uncertainty coupled with management's flexibility in choosing the optimal strategies or options along the way when new information becomes available. That is, management has the flexibility to make mid-course strategy corrections when there is uncertainty involved in the future. As information becomes available and uncertainty becomes resolved, management can choose the best strategies to implement. Traditional discounted cash flow assumes a single static decision, while real options assume a multidimensional dynamic series of decisions, where management has the flexibility to adapt given a change in the business environment.

> ***Traditional approaches assume a static decision-making ability, while real options assume a dynamic series of future decisions where management has the flexibility to adapt given changes in the business environment.***

Another way to view the problem is that there are two points to consider: (1) the initial investment starting point where strategic investment decisions have to be made; and (2) the ultimate goal, the optimal decision that can ever be made to maximize the firm's return on investment and shareholder's wealth. In the traditional discounted cash flow approach, joining these two points is a straight line, whereas the real options approach looks like a map with multiple routes to get to the ultimate goal, where each route is conjoint with others. The former implies a one-time decision-making process, while the latter implies a dynamic decision-making process wherein the investor learns over time and makes different updated decisions as time passes and events unfold.

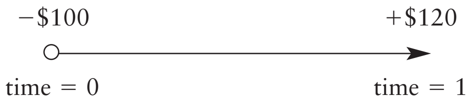
As previously outlined, traditional approaches coupled with discounted cash flow analysis have their pitfalls. Real options provide additional insights beyond the traditional analyses. At its least, real options provide a sobriety test of the results obtained using discounted cash flow and, at its best, provide a robust approach to valuation when coupled with the discounted cash flow methodology. The theory behind options is sound and reasonably applicable.

Some examples of real options using day-to-day terminology include:

- Option to abandon.
- Option to wait and see.
- Option to delay.
- Option to expand.
- Option to contract.
- Option to choose.
- Option to switch resources.
- Option for phased stage-gate and sequential investments.

Notice that the names used to describe the more common real options are rather self-explanatory, unlike the actual model names such as the "Barone-Adesi-Whaley approximation model for an American option to expand." This is important because when it comes to explaining the process and results to management, the easier it is for them to understand, the higher the chances of acceptance of the methodology and results. We will, with greater detail, revisit this idea of making a series of black-box analytics transparent and expositionally easy in Chapter 12.

Traditional approaches to valuing projects associated with the value of a firm, including any strategic options the firm possesses, or flexible management decisions that are dynamic and have the capacity to change over time, are flawed in several respects. Projects valued using the traditional discounted cash flow model often provide a value that grossly understates the true fair market value of the asset. This is because projects may provide a low or zero cash flow in the near future but nonetheless be valuable to the firm. In addition, projects can be viewed in terms of owning the option to execute the rights, not owning the rights per se, because the owner can execute the option or allow it to expire should the opportunity cost outweigh the benefits of execution. The recommended options approach takes into consideration this option to exercise and prices it accordingly. Compared to traditional approaches, real options provide added elements of robustness to the analysis. Its inputs in the option-pricing model can be constructed via multiple alternatives, thus providing a method of stress testing or sensitivity testing of the final



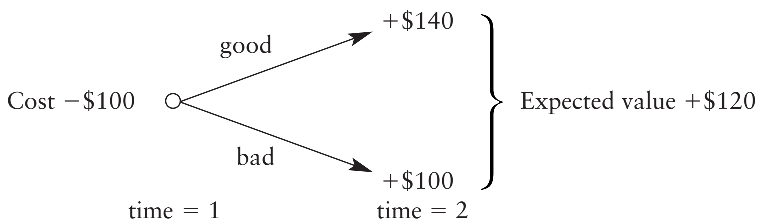
$$\text{Net Present Value} = \frac{120}{(1.15)^1} - 100 = \$4.3$$

FIGURE 3.1 Why Optionality Is Important

results. The corollary analysis resulting from real options also provides a ready means of sobriety checks without having to perform the entire analysis again from scratch using different assumptions. Finally, Monte Carlo simulation can be adapted to create thousands of possible outcomes, whose results can be used in a real options analysis, thereby increasing the model's robustness.

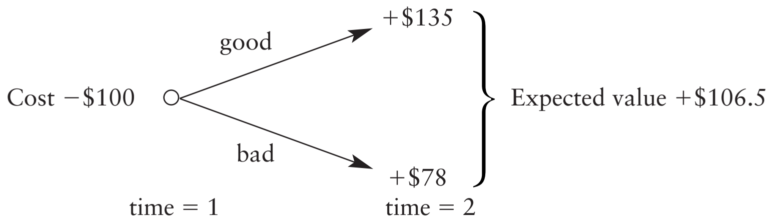
The following example provides a simplified analogy to why optionality is important and should be considered in corporate capital investment strategies. Suppose you have an investment strategy that costs $100 to initiate and you anticipate that on average, the payoff will yield $120 in exactly one year. Assume a 15 percent weighted average cost of capital and a 5 percent risk-free rate, both of which are annualized rates. As Figure 3.1 illustrates, the net present value of the strategy is $4.3, indicating a good investment potential because the benefits outweigh the costs.

However, if we wait and see before investing, when uncertainty becomes resolved, we get the profile shown in Figure 3.2, where the initial investment outlay occurs at time one and positive cash inflows are going to occur only at time two. Let's say that your initial expectations were correct and that the average or expected value came to be $120 with good market demand providing a $140 cash flow and in the case of bad demand, only $100. If we had the option to wait a year, then we could better estimate the trends in demand and we would have seen the payoff profile bifurcating into two scenarios. Should the scenario prove unfavorable, we would have the option to aban-



$$\text{Net Present Value} = \frac{140}{(1.15)^2} - \frac{100}{(1.05)^1} = \$10.6$$

FIGURE 3.2 If We Wait until Uncertainty Becomes Resolved



$$\text{Net Present Value} = \frac{135}{(1.15)^2} - \frac{100}{(1.05)^1} = \$6.8$$

FIGURE 3.3 Realistic Payoff Schedule

don the investment because the costs are identical to the cash inflow (−$100 versus +$100), and we would rationally not pursue this avenue. Hence, we would pursue this investment only if a good market demand is observed for the product, and our net present value for waiting an extra year will be $10.6. This analysis indicates a truncated downside where there is a limited liability because a rational investor would never knowingly enter a sure-loss investment strategy. Therefore, the value of flexibility is $6.3.

However, a more realistic payoff schedule should look like Figure 3.3. By waiting a year and putting off the investment until year two, you are giving up the potential for a cash inflow now, and the leakage or opportunity cost by not investing now is the $5 less you could receive ($140 – $135). However, by putting off the investment, you are also defraying the cost of investing in that the cost outlay will only occur a year later. The calculated net present value in this case is $6.8.

COMPARING TRADITIONAL APPROACHES WITH REAL OPTIONS

Figures 3.4 through 3.9 show a step-by-step analysis comparing a traditional analysis with that of real options, from the analyst's viewpoint. The analysis starts off with a discounted cash flow model in analyzing future cash flows. The analyst then applies sensitivity and scenario analysis. This is usually the extent of traditional approaches. As the results are relatively negative, the analyst then decides to add some new analytics. Monte Carlo simulation is then used, as well as real options analysis. The results from all these analytical steps are then compared and conclusions are drawn. This is a good comparative analysis of the results and insights obtained by using the new analytics. In this example, the analyst has actually added significant value to the overall project by creating optionalities within the project by virtue of actively

Comparing Traditional Approaches and Real Options with Simulation

A. Discounted Cash Flow

The extended example below shows the importance of waiting. That is, suppose a firm needs to make a rather large capital investment decision but at the same time has the Option to Wait and Defer on making the decision until later. The firm may be involved in pharmaceutical research and development activities, IT investment activities, or simply in marketing a new product that is yet untested in the market.

Suppose the analyst charged with performing a financial analysis on the project estimates that the most probable level of net revenues generated through the implementation of the project with an economic life of 5 years is presented in the time line below. Further, s/he assumes that the implementation cost is $200 million and the project's risk-adjusted discount rate is 20%, which also happens to be the firm's weighted average cost of capital. The calculated net present value (NPV) is found to be at a loss of -$26.70M.

	t = 0	t = 1	t = 2	t = 3	t = 4	t = 5	time
discount rate = 20%	–$200M	$30M	$36M	$70M	$80M	$110M	

calculated NPV = –$26.70M

B. Sensitivity Analysis on Discounted Cash Flow

Even though the NPV shows a significant negative amount, the analyst feels that the investment decision can be better improved through more rigor. Hence, s/he decides to perform a sensitivity analysis. Since in this simplified example, we only have three variables (discount rate, cost, and future net revenue cash flows), the analyst increases each of these variables by 10% to note the sensitivity of calculated NPV to these changes.

	t = 0	t = 1	t = 2	t = 3	t = 4	t = 5	time
discount rate = 20%	–$220M	$30M	$36M	$70M	$80M	$110M	

increase cost by 10% (from -$200M to -220M)

calculated NPV goes from –$26.70M to –$46.70M

	t = 0	t = 1	t = 2	t = 3	t = 4	t = 5	time
discount rate = 20%	–$200M	$33M	$40M	$77M	$88M	$121M	

increase projected revenues by 10%

calculated NPV goes from –$26.70M to –$9.37M

	t = 0	t = 1	t = 2	t = 3	t = 4	t = 5	time
discount rate = 22%	–$200M	$30M	$36M	$70M	$80M	$110M	

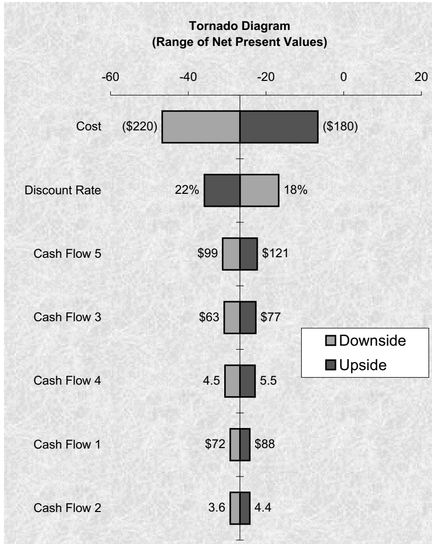
increase discount rate to 22%

calculated NPV goes from –$26.70M to –$35.86M

The entire set of possible sensitivities are presented in the table below, arranged in descending order based on the range of potential outcomes (indication of the variable's sensitivity). A Tornado Diagram is also created based on this sensitivity table.

Expected NPV				Input		
Variable	*Downside*	*Upside*	*Range*	*Downside*	*Upside*	*Base Case*
Cost	($46.70)	($6.70)	$40.00	($220)	($180)	($200)
Discount Rate	($16.77)	($35.86)	$19.09	18%	22%	20%
Cash Flow 5	($31.12)	($22.28)	$8.84	$99	$121	$110
Cash Flow 3	($30.75)	($22.65)	$8.10	$63	$77	$70
Cash Flow 4	($30.56)	($22.85)	$7.72	$72	$88	$80
Cash Flow 1	($29.20)	($24.20)	$5.00	$27	$33	$30
Cash Flow 2	($29.20)	($24.20)	$5.00	$32	$40	$36

FIGURE 3.4 Discounted Cash Flow Model



C. Scenario Analysis

Next, scenarios were generated. The analyst creates three possible scenarios and provides a subjective estimate of the probabilities each scenario will occur. For instance, the worst case scenario is 50% of the nominal scenario's projected revenues, while the best case scenario is 150% of the nominal scenario's projected revenues.

Worst Case Scenario

discount rate = 20%

20% probability of occurrence

t = 0	t = 1	t = 2	t = 3	t = 4	t = 5	time
–$200M	$15M	$18M	$35M	$40M	$55M	

calculated NPV = –$113.25M

Nominal Case Scenario

discount rate = 20%

50% probability of occurrence

t = 0	t = 1	t = 2	t = 3	t = 4	t = 5	time
–$200M	$30M	$36M	$70M	$80M	$110M	

calculated NPV = –$26.70M

Best Case Scenario

discount rate = 20%

30% probability of occurrence

t = 0	t = 1	t = 2	t = 3	t = 4	t = 5	time
–$200M	$45M	$54M	$105M	$120M	$165M	

calculated NPV = $59.94M

Expected NPV = 0.20 (–$113.25M) + 0.50 (–$26.70M) + 0.30 ($59.94M) = –$18.04M

NPVs for each of the scenarios are calculated, and an Expected NPV is calculated to be -$18.04M based on the probability assumptions. The problem here is obvious. The range of possibilities is too large to make any inferences. That is, which figure should be believed? The -$18.04 or perhaps the nominal case of -$26.70? In addition, the upside potential and downside risks are fairly significantly different from the nominal or expected cases. What are the chances that any of these will actually come true? What odds or bets or faith can one place in the results? The analyst then decides to perform some Monte Carlo simulations to answer these questions.

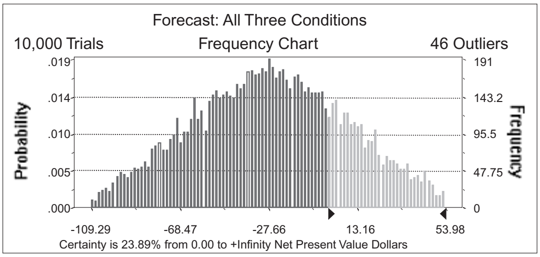
FIGURE 3.5 Tornado Diagram and Scenario Analysis

D. Simulation

There are two ways to perform a Monte Carlo simulation in this example. The first is to take the scenario analysis above and simulate around the calculated NPVs. This assumes that the analyst is highly confident of his/her future cash flow projections and that the worst-case scenario is indeed the absolute minimum the firm can attain and the best-case scenario is exactly at the top of the range of possibilities. The second approach is to use the most likely or nominal scenario and simulate its inputs based on some management-defined ranges of possible cost and revenue structures.

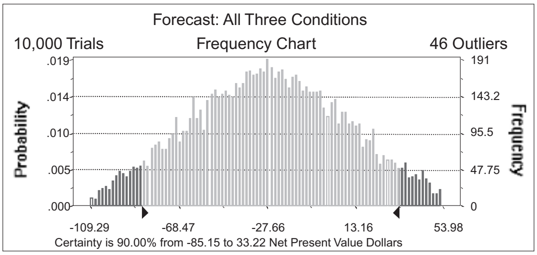
(i) Simulating around scenarios

The analyst simulates around the three scenarios using a Triangular Distribution with the worst-case, nominal-case and best-case scenarios as input parameters into the simulation model.



Mean	-27.06
Standard Deviation	35.31
Range Minimum	-112.21
Range Maximum	57.43
Range Width	169.64

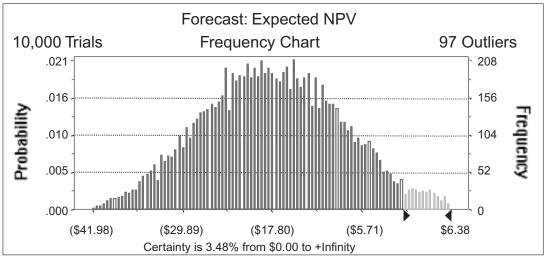
We see that the range is fairly large because the scenarios were rather extreme. In addition, there is only a 23.89% chance that the project will break even or have an NPV > 0.



The 90% statistical confidence interval is between -\$85.15M and \$33.22M, which is also rather wide. Given such a huge swing in possibilities, we are much better off with performing a simulation using the second method, that is, to look at the nominal case and simulate around that case's input parameters.

(ii) Simulating around the nominal scenario

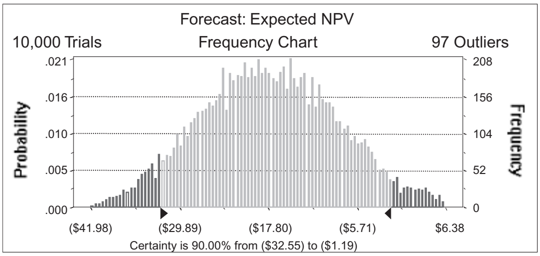
Since in the scenario analysis, the analyst created two different scenarios (worst case and best case) based on a 50% fluctuation in projected revenues from the base case, here we simply look at the base case and by simulation, generate 10,000 scenarios. Looking back at the Tornado diagram, we noticed that discount rate and cost were the two key determining factors in the analysis; the second approach can take the form of simulating these two key factors. The analyst simulates around the nominal scenario assuming a normal distribution for the discount rate with a mean of 20% and a standard deviation of 2% based on historical data on discount rates used in the firm. The cost structure is simulated assuming a uniform distribution with a minimum of -\$180M and a maximum of -\$220M based on input by management. This cost range is based on management intuition and substantiated by similar projects in the past. The results of the simulation are shown below.



Mean	-25.06
Standard Deviation	14.3
Range Minimum	-69.54
Range Maximum	38.52
Range Width	108.06

Here we see that the range is somewhat more manageable and we can make more meaningful inferences. Based on the simulation results, there is only a 3.48% chance that the project will break even.

FIGURE 3.6 Simulation

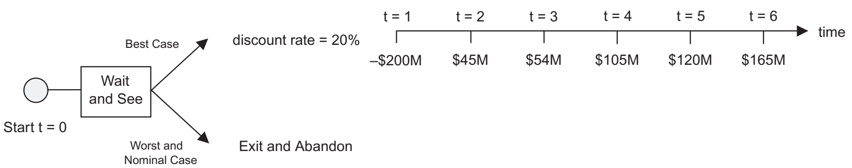


The 90% statistical confidence interval is between –$32.55M and –$1.19M.

Most of the time, the project is in negative NPV territory, suggesting a rather grim outlook for the project. However, the project is rather important to senior management and they wish to know if there is some way to add value to this project or make it financially justifiable to invest in. The answer lies in using Real Options.

E. Real Options
We have the option to wait or defer investing until a later date. That is, wait until uncertainty becomes resolved and then decide on the next course of action afterwards. Invest in the project only if market conditions indicate a good scenario and decide to abandon the project if the market condition is akin to the nominal or worst-case scenarios as they both bear negative NPVs.

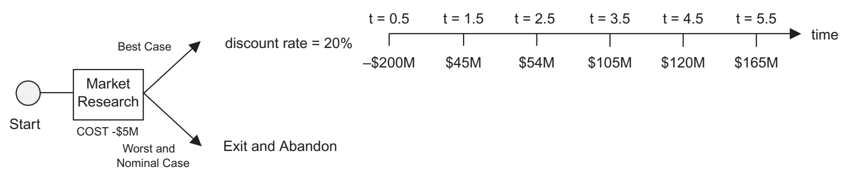
(i) Option to Wait I (Passive Wait and See Strategy)
Say we decide to wait one year and assuming that we will gather more valuable information within this time frame, we can then decide whether to execute the project or not at that time. Below is a decision tree indicating our decision path.



Calculated NPV after waiting for one year on new information = $49.95M

We see here that the NPV is positive since if after waiting for a year, the market demand is nominal or sluggish, then management has the right to pull the plug on the project. Otherwise, if it is a great market which meets or exceeds the best-case scenario, management has the option to execute the project, thereby guaranteeing a positive NPV. The calculated NPV is based on the forecast revenue stream and is valued at $49.95M.

(ii) Option to Wait II (Active Market Research Strategy)
Instead of waiting passively for the market to reveal itself over the one-year period as expected previously, management can decide on an active strategy of pursuing a market research strategy. If the market research costs $5M to initiate and takes 6 months to obtain reliable information, the firm saves additional time without waiting for the market to reveal itself. Here, if the market research indicates a highly favorable condition where the best-case scenario revenue stream is to be expected, then the project will be executed after 6 months. The strategy path and time lines are shown below.

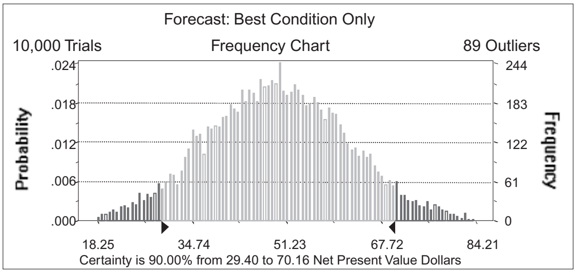


Calculated NPV after active market research = $49.72M
(after accounting for the -$5M in market research costs)

The calculated NPV here is $49.72M, relatively close to the passive waiting strategy. However, the downside is the $5M which also represents the greatest possible loss, which is also the premium paid to obtain the option to execute given the right market conditions.

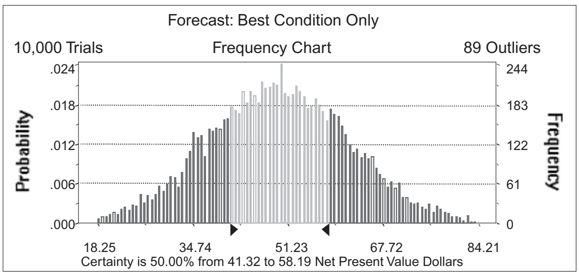
FIGURE 3.7 Real Options Analysis (Active versus Passive Strategies)

In retrospect, management could find out the maximum it is willing to pay for the market research in order to cut down the time it has to wait before making an informed decision. That is, at what market research price would the first option to wait be the same as the second option to wait? Setting the difference between $49.95M and $49.72M as the reduction in market research cost brings down the initial $5M to $4.77M. In other words, the maximum amount the firm should pay for the market research should be no more than $4.77M; otherwise, it is simply wise to follow the passive strategy and wait for a year.



Mean	
Standard Deviation	49.73
Range Minimum	12.43
Range Maximum	-0.25
Range Width	94.57
	94.82

The resulting distribution range is less wide, providing a more meaningful inference. Based on the simulation results, the 90% confidence interval has the NPV between $29.40M and $70.16M. The range, which means almost 100% of the time, the NPV takes on a positive value.



The 50% confidence interval has the NPV between $41.32M and $58.19M. We can interpret this range as the expected value range since 50% of the time, the real NPV will fall within this range, with a mean of $49.73M.

F. Observations

We clearly see that by using the three Scenarios versus an Expected Value approach, we obtain rather similar results in terms of NPV but through simulation, the Expected Value approach provides a much tighter distribution and the results are more robust as well as easier to interpret. Once we added in the Real Options approach, the risk has been significantly reduced and the return dramatically increased. The overlay chart below compares the simulated distributions of the three approaches. The blue series is the Scenario approach incorporating all three scenarios and simulating around them. The green series is the Expected Value approach, simulating around the nominal revenue projections, and the red series is the Real Options approach where we only execute if the best condition is obtained.

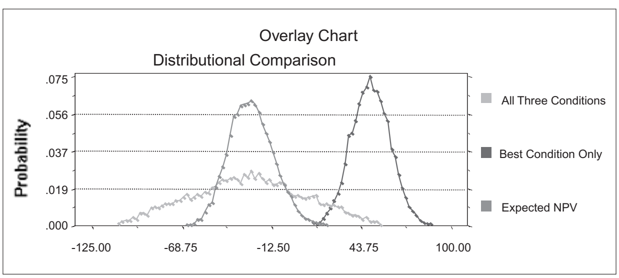
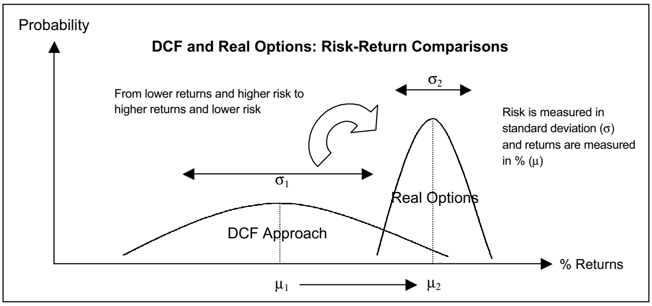


FIGURE 3.8 Analysis Observations

The example here holds true in most cases when we compare the approach used in a traditional Discounted Cash Flow (DCF) method to Real Options. Since we can define risk as uncertain fluctuations in revenues and the NPV level, all downside risks are mitigated in Real Options since you do not execute the project if the nominal or worst-case scenario occurs in time. In retrospect, the upside risks are maximized such that the returns are increased since the project will only be executed when the best-case scenario occurs. This hereby creates a win-win situation where risks are mitigated and returns are enhanced, simply by having the right strategic optionalities available, acting appropriately, and valuing the project in terms of its "real" or intrinsic value, which includes this opportunity to make midcourse corrections when new information becomes available.



In addition, what seems on the outset as an unprofitable project yielding an NPV of -$26.70M can be justified and made profitable since the project has in reality an Option to Wait or Defer until a later date. Once uncertainty becomes resolved and we have more available information, management can then decide whether to go forward based on market conditions. This call option could be bought through the use of active market research. By having this delay tactic, the firm has indeed truncated any downside risks but still protected its upside potential.

Next, if we look at the Minimax Approach, where we attempt to Minimize the Maximum regret of making a decision, the maximum level of regret for pursuing the project blindly using a DCF approach may yield the worst-case scenario of -$113.25M while using an Option to Wait but simultaneously pursuing an active marketing research strategy will yield a maximum regret of -$4.77M. This is because the levels of maximum regret occur under the worst possible scenario. If this occurs, investing in the project blindly will yield the worst case of -$113.25, but the maximum loss in the real options world is the limited liability of the premium paid to run the market research, adding up to only -$4.77M because the firm would never execute the project when the market is highly unfavorable.

In addition, the Value of Perfect Information can be calculated as the increase in value created through the Option to Wait as compared to the naïve Expected NPV approach. That is, the Value of having Perfect Information is $68M. We obtain this level of perfect information through the initiation of a marketing research strategy which costs an additional $4.77M. This means that the strategic Real Options thinking and decision-making process has created a leverage of 14.25 times. This view is analogous to a financial option where we can purchase a call option for, say, $5 with a specified exercise price for a specified time of an underlying common equity with a current market price of $100. With $5, the call purchaser has leveraged his purchasing power into $100, or 20 times. In addition, if the equity price rises to $150 (50% increase akin to our example above), the call holder will execute the option, purchase the stock at $100, turn around and sell it for $150, less the $5 cost and yield a net $45. The option holder has, under this execution condition, leveraged the initial $5 into a $45 profit, or 9 times the original investment.

Finally and more importantly is that we see by adding in a strategic option, we have increased the value of the project immensely. It is therefore wise for management to consider an optionality framework in the decision-making process. That is, to find the strategic options that exist in different projects or to create strategic options in order to increase the project's value.

FIGURE 3.9 Analysis Conclusions

pursuing and passively waiting for more information to become available prior to making any decisions.

Of course, several simplifying assumptions have to be made here, including the ability for the firm to simply wait and execute a year from now without any market or competitive repercussions. That is, the one-year delay will not allow a competitor to gain a first-to-market advantage or capture additional market share, where the firm's competitor may be willing to take the risk and invest in a similar project and gain the advantage while the firm is not willing to do so. In addition, the cost and cash flows are assumed to be the same whether the project is initiated immediately or in the future. Obviously, these more complex assumptions can be added into the analysis, but for illustration purposes, we assume the basic assumptions hold, where costs and cash flows remain the same no matter the execution date, and that competition is negligible. See Chapters 10 and 11 for cases where a dividend yield or opportunity cost (leakage or outflow) occurs if we wait too long, and how these can be modeled in real options.

SUMMARY

Having real options in a project can be highly valuable, both in recognizing where these optionalities exist and in introducing and strategically setting up options in the project. Strategic options can provide decision makers the opportunity to hedge their bets in the face of uncertainty. By having the ability to make midcourse corrections downstream when these uncertainties become known, decision makers have essentially hedged themselves against any downside risks. As seen in this chapter, a real options approach provides the decision maker not only a hedging vehicle but also significant upside leverage. In comparing approaches, real options analysis shows that not only can a project's risk be reduced but also returns can be enhanced by strategically creating options in projects.

CHAPTER 3 QUESTIONS

1. Can an option take on a negative value?
2. Why are real options sometimes viewed as strategic maps of convoluted pathways?
3. Why are real options seen as risk-reduction and value-enhancement strategies?
4. Why are the real options names usually self-explanatory and not based on names of mathematical models?
5. What is a Tornado diagram as presented in Figure 3.5's example?

CHAPTER 4

The Real Options Process

INTRODUCTION

This chapter introduces the reader to the real options process framework. This framework comprises eight distinct phases of a successful real options implementation, going from a qualitative management screening process to creating clear and concise reports for management. The process was developed by the author based on previous successful implementations of real options both in the consulting arena and in industry-specific problems. These phases can be performed either in isolation or together in sequence for a more robust real options analysis.

CRITICAL STEPS IN PERFORMING REAL OPTIONS ANALYSIS

Figure 4.1 at the end of the chapter shows the real options process up close. We can segregate the real options process into the following eight simple steps. These steps include:

- Qualitative management screening.
- Time-series and regression forecasting.
- Base case net present value analysis.
- Monte Carlo simulation.
- Real options problem framing.
- Real options modeling and analysis.
- Portfolio and resource optimization.
- Reporting and update analysis.

Qualitative Management Screening

Qualitative management screening is the first step in any real options analysis (Figure 4.1). Management has to decide which projects, assets, initiatives, or strategies are viable for further analysis, in accordance with the firm's mission, vision, goal, or overall business strategy. The firm's mission, vision, goal, or overall business strategy may include market penetration strategies, competitive advantage, technical, acquisition, growth, synergistic, or globalization issues. That is, the initial list of projects should be qualified in terms of meeting management's agenda. Often this is where the most valuable insight is created as management frames the complete problem to be resolved. This is where the various risks to the firm are identified and flushed out.

Time-Series and Regression Forecasting

The future is then forecasted using time-series analysis or multivariate regression analysis if historical or comparable data exist. Otherwise, other qualitative forecasting methods may be used (subjective guesses, growth rate assumptions, expert opinions, Delphi method, and so forth). See Chapter 9 for details on using the author's Risk Simulator software to run time-series forecasts.

Base Case Net Present Value Analysis

For each project that passes the initial qualitative screens and forecasts, a discounted cash flow model is created. This model serves as the base case analysis, where a net present value is calculated for each project. This also applies if only a single project is under evaluation. This net present value is calculated using the traditional approach of using the forecast revenues and costs, and discounting the net of these revenues and costs at an appropriate risk-adjusted rate.

Monte Carlo Simulation

Because the static discounted cash flow produces only a single-point estimate result, there is oftentimes little confidence in its accuracy given that future events that affect forecast cash flows are highly uncertain. To better estimate the actual value of a particular project, Monte Carlo simulation should be employed next. See Chapter 9 for details on running Monte Carlo simulations using the author's Risk Simulator software.

Usually, a sensitivity analysis is first performed on the discounted cash flow model. That is, setting the net present value as the resulting variable, we can change each of its precedent variables and note the change in the resulting variable. Precedent variables include revenues, costs, tax rates, discount rates, capital expenditures, depreciation, and so forth, which ultimately flow through the model to affect the net present value figure. By tracing back all these precedent variables, we can change each one by a preset amount and see the effect on the resulting net present value. A graphical representation can then be created, which is often called a Tornado chart, because of its shape, where the most sensitive precedent variables are listed first, in descending order of magnitude. Armed with this information, the analyst can then decide which key variables are highly uncertain in the future and which are deterministic. The uncertain key variables that drive the net present value and hence the decision are called critical success drivers. These critical success drivers are prime candidates for Monte Carlo simulation.[1] Because some of these critical success drivers may be correlated—for example, operating costs may increase in proportion to quantity sold of a particular product, or prices may be inversely correlated to quantity sold—a correlated Monte Carlo simulation may be required. Typically these correlations can be obtained through historical data. Running correlated simulations provides a much closer approximation to the variables' real-life behaviors. This step models, analyzes, and quantifies the various risks of each project. The result is a distribution of the NPVs and the project's volatility (see Appendix 7A for details).

Real Options Problem Framing

Framing the problem within the context of a real options paradigm is the next critical step. Based on the overall problem identification occurring during the initial qualitative management screening process, certain strategic optionalities would have become apparent for each particular project. The strategic optionalities may include, among other things, the option to expand, contract, abandon, switch, choose, and so forth. Based on the identification of strategic optionalities that exist for each project or at each stage of the project, the analyst can then choose from a list of options to analyze in more detail.[2] Real options are added to the projects to hedge downside risks and to take advantage of upside swings (see Chapter 11 for details).

Real Options Modeling and Analysis

Through the use of Monte Carlo simulation, the resulting stochastic discounted cash flow model will have a distribution of values. In real options, we assume that the underlying variable is the future profitability of the project, which is the future cash flow series. An implied volatility of the future free cash flow or underlying variable can be calculated through the results of a Monte Carlo simulation previously performed. Usually, the volatility is measured as the standard deviation of the logarithmic returns on the free cash flows stream. In addition, the present value of future cash flows for the base case discounted cash flow model is used as the initial underlying asset value in real options modeling. Using these inputs, real options analysis is performed to obtain the projects' strategic option values (see Chapters 7 to 11 for details).

Portfolio and Resource Optimization

Portfolio optimization is an optional step in the analysis. If the analysis is done on multiple projects, management should view the results as a portfolio of rolled-up projects because the projects are in most cases correlated with one another and viewing them individually will not present the true picture. As firms do not only have single projects, portfolio optimization is crucial. Given that certain projects are related to others, there are opportunities for hedging and diversifying risks through a portfolio. Because firms have limited budgets, have time and resource constraints, while at the same time have requirements for certain overall levels of returns, risk tolerances, and so forth, portfolio optimization takes into account all these to create an optimal portfolio mix. The analysis will provide the optimal allocation of investments across multiple projects.[3]

Reporting and Update Analysis

The analysis is not complete until reports can be generated.[4] Not only are results presented but also the process should be shown. Clear, concise, and precise explanations transform a difficult black-box set of analytics into transparent steps. Management will never accept results coming from black boxes if they do not understand where the assumptions or data originate and what types of mathematical or financial massaging takes place.

Real options analysis assumes that the future is uncertain and that management has the right to make midcourse corrections when these uncertainties become resolved or risks become known; the analysis is usually done ahead of time and thus, ahead of such uncertainty and risks. Therefore, when these risks become known, the analysis should be revisited to incorporate the decisions made or revising any input assumptions. Sometimes, for long-horizon projects, several iterations of the real options analysis should be performed, where future iterations are updated with the latest data and assumptions. See Chapter 12 for details on how to present real options analysis results to management.

SUMMARY

Understanding the steps required to undertake real options analyses is important because it provides insight not only into the methodology itself but also into how it evolves from traditional analyses, showing where the traditional approach ends and where the new analytics start. The eight phases discussed include performing a qualitative management screening process, forecasting the future, running base case net present value or discounted cash flow analysis, Monte Carlo simulation, real options framing, real options modeling, portfolio optimization, as well as reporting and update analysis.

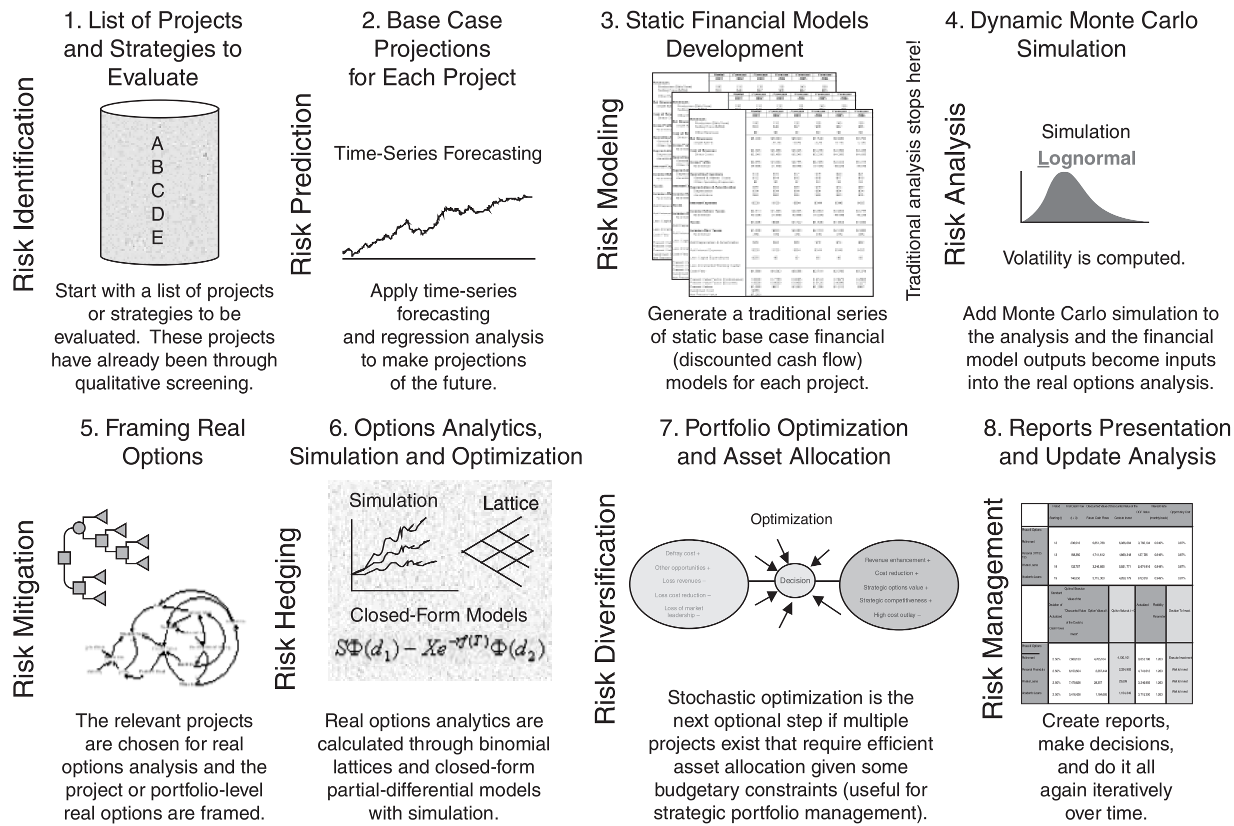


FIGURE 4.1 Real Options Process

CHAPTER 4 QUESTIONS

1. What is Monte Carlo simulation?
2. What is portfolio optimization?
3. Why is update analysis required in a real options analysis framework?
4. What is problem framing?
5. Why are reports important?

CHAPTER 5

Real Options, Financial Options, Monte Carlo Simulation, and Optimization

INTRODUCTION

This chapter discusses the differences between real options and financial options, understanding that real options theory stems from financial options but that there are key differences. These differences are important to note because they will inevitably change the mathematical structure of real options models. The chapter then continues with an introduction to Monte Carlo simulation and portfolio optimization, discussing how these two concepts relate to the overall real options analysis process. See Chapter 9 for getting started in using the author's Real Options Valuation Super Lattice Solver and Risk Simulator software for solving real options problems and running Monte Carlo simulations.

REAL OPTIONS VERSUS FINANCIAL OPTIONS

Real options apply financial options theory in analyzing real or physical assets. Therefore, there are certainly many similarities between financial and real options. However, there are key differences, as listed in Figure 5.1. For example, financial options have short maturities, usually expiring in several months. Real options have longer maturities, usually expiring in several years, with some exotic-type options having an infinite expiration date. The underlying asset in financial options is the stock price, as compared to a multitude of other business variables in real options. These variables may include free cash flows, market demand, commodity prices, and so forth. Thus, when applying real options analysis to analyzing physical assets, we have to be careful in discerning what the underlying variable is because the volatility measures used

FINANCIAL OPTIONS	REAL OPTIONS
Short maturity, usually in months.	Longer maturity, usually in years.
Underlying variable driving its value is equity price or price of a financial asset.	Underlying variables are free cash flows, which in turn are driven by competition, demand, management.
Cannot control option value by manipulating stock prices.	Can increase strategic option value by management decisions and flexibility.
Values are usually small.	Major million and billion dollar decisions.
Competitive or market effects are irrelevant to its value and pricing.	Competition and market drive the value of a strategic option.
Have been around and traded for more than three decades.	A recent development in corporate finance within the last decade.
Usually solved using closed-form partial differential equations and simulation/variance reduction techniques for exotic options.	Usually solved using closed-form equations and binomial lattices with simulation of the underlying variables, not on the option analysis.
Marketable and traded security with comparables and pricing info.	Not traded and proprietary in nature, with no market comparables.
Management assumptions and actions have no bearing on valuation.	Management assumptions and actions drive the value of a real option.

FIGURE 5.1 Financial Options versus Real Options

in options modeling pertain to the underlying variable. In financial options, due to insider trading regulations, options holders cannot, at least in theory, manipulate stock prices to their advantage. However, in real options, because certain strategic options can be created by management, their decisions can increase the value of the project's real options. Financial options have relatively less value (measured in tens or hundreds of dollars per option) than real options (thousands, millions, or even billions of dollars per strategic option).

Financial options have been traded for several decades, but the real options phenomenon is only a recent development, especially in the industry at large. Both types of options can be solved using similar approaches, including closed-form solutions, partial-differential equations, finite-differences, binomial lattices, and simulation; but industry acceptance for real options has been in the use of binomial lattices. This is because binomial lattices are much more easily explained to and accepted by management because the methodology is much simpler to understand. Chapters 6 and 7 provide step-by-step details on how to create and solve binomial and multinominal lattices. Finally, financial options models are based on market-traded securities and visible asset prices making their construction easier and more objective. Real options tend to be based on non-market-traded assets, and financially traded proxies are seldom available. Hence management assumptions are key in valuing real options and relatively less important in valuing financial options. Given a particular project, management can create strategies that will provide itself options in the future. The value of these options can change depending on how they are constructed.

In several basic cases, real options are similar to financial options. Figure 5.2 shows the payoff charts of a call option and a put option. On all four charts, the vertical axes represent the value of the strategic option and the horizontal axes represent the value of the underlying asset. The kinked bold line represents the payoff function of the option at termination, effectively the project's net present value, because at termination, maturity effectively becomes zero and the option value reverts to the net present value (underlying asset less implementation costs). The dotted curved line represents the payoff function of the option prior to termination, where there is still time before maturity and hence uncertainty still exists and option value is positive. The curved line is the net present value, including the strategic option value. Both lines effectively have a horizontal floor value, which is effectively the premium on the option, where the maximum value at risk is the premium or cost of obtaining the option, indicating the option's maximum loss as the price paid to obtain it.

The position of a long call or the buyer and holder of a call option is akin to an *expansion option*. This is because an expansion option usually costs something to create or set up, which is akin to the option's premium or purchase price. If the underlying asset does not increase in value over time, the maximum losses incurred by the holder of this expansion option will be

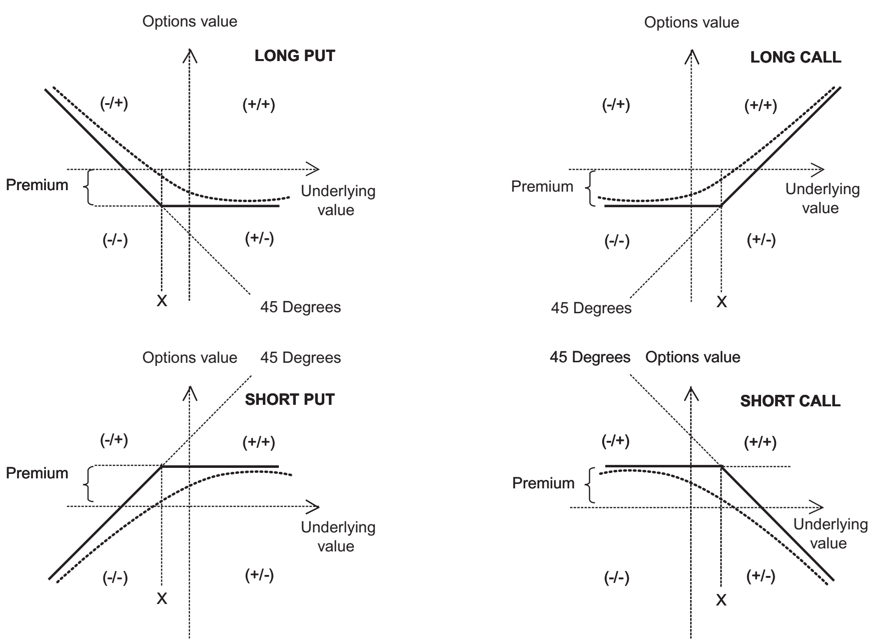


FIGURE 5.2 Option Payoff Charts

the cost of setting up this option (e.g., market research cost). When the value of the underlying asset increases sufficiently above the strike price (denoted *X* in the charts), the value of this expansion option increases. There is unlimited upside to this option, but the downside is limited to the premium paid for the option. The break-even point is where the bold line crosses the horizontal axis, which is equivalent to the strike price plus the premium paid.

The long put option position or the buyer and holder of a put option is akin to an *abandonment option*. This is because an abandonment option usually costs something to create or set up, which is akin to the option's premium or purchase price. If the value of the underlying asset does not decrease over time, the maximum losses incurred by the holder of this abandonment option will be the cost of setting up this option (seen as the horizontal bold line equivalent to the premium). When the value of the underlying asset decreases sufficiently below the strike price (denoted *X* in the charts), the value of this abandonment option increases. The option holder will find it more profitable to abandon the project currently in existence. There is unlimited upside to this option but the downside is limited to the premium paid for the option. The break-even point is where the bold line crosses the horizontal axis, which is equivalent to the strike price less the premium paid.[1]

The short positions or the writer and seller on both calls and puts have payoff profiles that are horizontal reflections of the long positions. That is, if you overlay both a long and short position of a call or a put, it becomes a zero-sum game. These short positions reflect the side of the issuer of the option. For instance, if the expansion and contraction options are based on some legally binding contract, the counterparty issuer of the contract would hold these short positions.

MONTE CARLO SIMULATION

Simulation is any analytical method that is meant to imitate a real-life system, especially when other analyses are too mathematically complex or too difficult to reproduce. Spreadsheet risk analysis uses both a spreadsheet model and simulation to analyze the effect of varying inputs based on outputs of the modeled system. One type of spreadsheet simulation is Monte Carlo simulation, which randomly generates values for uncertain variables over and over to simulate a real-life model. Chapter 9 details the use of the author's own Risk Simulator software and shows how Monte Carlo simulation works in a simple and practical way.

History Monte Carlo simulation was named after Monte Carlo, Monaco, where the primary attractions are casinos containing games of chance. Games of chance such as roulette wheels, dice, and slot machines exhibit random behavior. The random behavior in games of chance is similar to how Monte Carlo simulation selects variable values at random to simulate a model. When you roll a die, you know that a 1, 2, 3, 4, 5, or 6 will come up, but you don't know which for any particular trial. It is the same with the variables that have a known or estimated range of values but an uncertain value for any particular time or event (e.g., interest rates, staffing needs, revenues, stock prices, inventory, discount rates).

For each variable, you define the possible values with a probability distribution. The type of distribution you select depends on the conditions surrounding the variable. For example, some common distribution types are those shown in Figure 5.3.

During a simulation, the value to use for each variable is selected randomly from the defined possibilities.

Why Are Simulations Important? A simulation calculates numerous scenarios of a model by repeatedly picking values from the probability distribution for the uncertain variables and using those values for the event. As all those scenarios produce associated results, each scenario can have a forecast. Forecasts are events (usually with formulas or functions) that you define as important outputs of the model. These usually are events such as totals, net profit, or gross expenses.

An example of why simulation is important can be seen in the case illustration in Figures 5.4 and 5.5, termed the Flaw of Averages. It shows how an analyst may be misled into making the wrong decisions without the use of simulation. As the example shows, the obvious reason why this error occurs is that the distribution of historical demand is highly skewed while the cost structure is asymmetrical. For example, suppose you are in a meeting room, and your boss asks what everyone made last year. You take a quick poll and realize that the salary ranges from \$60,000 to \$150,000. You perform a quick calculation and find the average to be \$100,000. Then, your boss tells you that he made \$20 million last year! Suddenly, the average for the group becomes \$1.5 million. This value of \$1.5 million clearly in no way represents how much each of your peers made last year. In this case, the median may be more appropriate. Here you see that simply using the average will provide highly misleading results.[2]

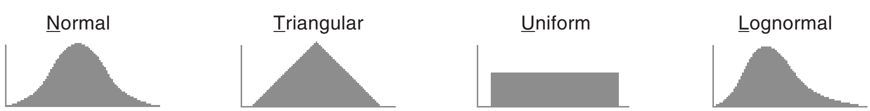


FIGURE 5.3 The Few Most Basic Distributions

The Flaw of Averages

Actual	5
Inventory Held	6
Perishable Cost	$100
Fed Ex Cost	$175
Total Cost	$100

Average 5.00

Historical Data (5 Yr)

Month	Actual
1	12
2	11
3	7
4	0
5	0
6	2
7	7
8	0
9	11
10	12
11	0
12	9
13	3
14	5
15	0
16	2
17	1
18	10
58	3
59	2
60	17

Your company is a retailer in perishable goods and you were tasked with finding the optimal level of inventory to have on hand. If your inventory exceeds actual demand, there is a $100 perishable cost while a $175 Fed Ex cost is incurred if your inventory is insufficient to cover the actual level of demand. These costs are on a per unit basis. Your first inclination is to collect historical demand data as seen on the right, for the past 60 months. You then take a simple average, which was found to be 5 units. Hence, you select 5 units as the optimal inventory level. You have just committed a major mistake called the Flaw of Averages!

The actual demand data are shown here on the right. Rows 19 through 57 are hidden to conserve space. Being the analyst, what must you then do?

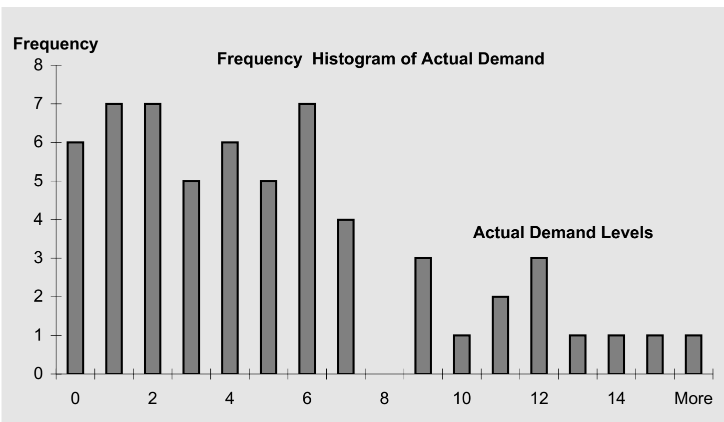


FIGURE 5.4 The Flaw of Averages

Fixing the Flaw of Averages with Simulation

Simulated Average Actual Demand	8.53	Simulated Demand Range	From 7.21 and 9.85
Inventory Held	9.00	Simulated Cost Range	From 178.91 to 149

Perishable Cost	$100
Fed Ex Cost	$175
Total Cost	$46.88

The best method is to perform a nonparametric simulation where we use the actual historical demand levels as inputs to simulate the most probable level of demand going forward, which we found as 8.53 units. Given this demand, the lowest cost is obtained through a trial inventory of 9 units, a far cry from the original Flaw of Averages estimate of 5 units.

Trial Inventory	Total Cost
1.00	$1,318
2.00	$1,143
3.00	$968
4.00	$793
5.00	$618
6.00	$443
7.00	$268
8.00	$93
9.00	$47
10.00	$147
11.00	$247
12.00	$347
13.00	$447
14.00	$547
15.00	$647
16.00	$747

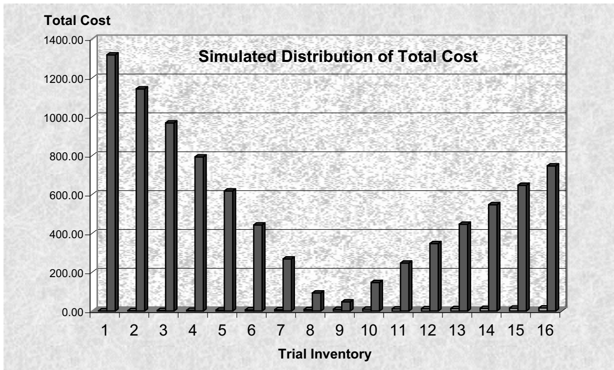


FIGURE 5.5 The Need for Simulation

Continuing with the example, Figure 5.5 shows how the right inventory level is calculated using simulation. The approach used here is called *nonparametric simulation.* It is nonparametric because in the simulation approach, no distributional parameters are assigned. Instead of assuming some preset distribution (normal, triangular, lognormal, or the like) and assumed parameters (mean, standard deviation, and so forth) as required in a Monte Carlo parametric simulation, nonparametric simulation uses the data themselves to tell the story. You can use Risk Simulator's custom distribution to perform nonparametric simulations.

Imagine you collect a year's worth of historical demand levels and write down the demand quantity on a golf ball for each day. Throw all 365 balls into a large basket and mix it. Pick a golf ball out at random and write down its value on a piece of paper, then place the ball back into the basket and mix the basket again. Do this 365 times, and calculate the average. This is a single grouped trial. Perform this entire process several thousand times, with replacement. The distribution of these averages represents the outcome of the simulation. The expected value of the simulation is simply the average value of these thousands of averages. Figure 5.5 shows an example of the distribution stemming from a nonparametric simulation. As you can see, the optimal inventory rate that minimizes carrying costs is nine units, far from the average value of five units previously calculated in Figure 5.4.

Figure 5.6 shows a simple example of performing a nonparametric simulation using Excel. There are limitations on what can be performed using Excel's functionalities. The example shown in Figure 5.6 assumes nine simple cases with varying probabilities of occurrence. The simulation can be set up in three simple steps. However, the number of columns and rows may be unmanageable because a large number of simulations are needed to obtain a good sampling distribution. The analysis can be modified easily for the flaw of averages example by simply listing out all the cases (the actual demand levels) with equal probabilities on each case. Obviously, performing large-scale simulations with Excel is not recommended. The optimal solution is to use a software like Risk Simulator to run simulations, as shown in Figure 5.7.

Figure 5.7 shows an example of using Risk Simulator in conjunction with an Excel spreadsheet. The highlighted cells are simulation assumption cells, forecast result cells, and decision variable cells. For more details, consult

Simulation (Probability Assumptions)

	Value	Probability
Step 1:	362995	55%
The Assumptions	363522	10%
	252094	10%
	122922	10%
	23572	3%
	305721	3%
	61877	3%
	147322	3%
	179360	3%

Here are the assumed values and their corresponding probabilities of occurrence. The sum of the probabilities have to add up to 100%.

We then translate the assumed values into a set of random numbers bounded by [0,1]. For instance, for a normal distribution, the probability of getting a number between 0.00 and 0.55 is 55% and between 0.56 and 0.65 is 10% and so forth. This is done in Step 2 below.

	Minimum	Maximum	Implied
Step 2:	0.00	0.55	362994.83
The Table Setup	0.56	0.65	363522.33
	0.66	0.75	252094
	0.76	0.85	122922.05
	0.86	0.88	23572.39
	0.89	0.91	305721.43
	0.92	0.94	61876.66
	0.95	0.97	147322.19
	0.98	1.00	179359.73

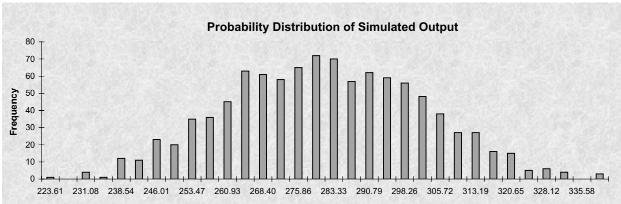
CELLS $D$16:$F$24

Simulate this for 100 trials and take the average. Then, repeat this for several thousand sets, taking the average on every set. Then, using these thousands of simulated sets, create a probability distribution and calculate its corresponding descriptive statistics (mean, standard deviation, confidence intervals, probabilities, et cetera).

VLOOKUP(RAND(),$D$16:$F$24,3)

Average	**297185**
90th%	**310390**

Step 3:	Trials	Set 1	Set 2	Set 3	Set 4	Set 5	Set 100	Set 1000	Set 1500	Set 2000	Set 5000
Simulate	1	147322	122922	252094	362995	362995	362995	252094	362995	61877	363522
	2	362995	362995	362995	362995	147322	61877	61877	362995	122922	179360
	3	252094	362995	362995	122922	362995	252094	61877	362995	362995	362995
	4	362995	362995	252094	362995	362995	362995	61877	179360	179360	122922
	5	252094	362995	363522	362995	363522	122922	363522	252094	147322	362995
	6	362995	362995	363522	122922	252094	363522	362995	179360	122922	179360
	7	122922	362995	363522	362995	362995	122922	122922	252094	61877	122922
	8	363522	362995	362995	122922	362995	122922	122922	122922	362995	61877
	9	362995	362995	362995	252094	252094	362995	362995	362995	179360	363522
	10	122922	122922	363522	362995	305721	362995	252094	61877	362995	362995
	11	305721	362995	362995	362995	252094	362995	252094	363522	362995	362995
	12	362995	362995	362995	362995	252094	362995	362995	252094	362995	122922
Rows 13	95	252094	362995	362995	363522	362995	122922	362995	362995	252094	61877
to 94 have	96	252094	252094	61877	362995	363522	122922	23572	122922	305721	362995
been hidden	97	362995	23572	362995	362995	122922	305721	362995	362995	23572	362995
to conserve	98	362995	362995	362995	147322	362995	252094	362995	362995	362995	252094
space.	99	122922	362995	362995	362995	362995	362995	362995	147322	362995	252094
	100	363522	252094	362995	362995	362995	362995	362995	362995	362995	362995
	Average	**275763**	**282681**	**318044**	**292146**	**300325**	**299948**	**298498**	**302302**	**296806**	**294590**



Descriptive Statistics

Mean	279.50
Median	279.34
Mode	313.66
Standard Deviation	20.42
Skew	0.05
5th Percentile	245.34
10th Percentile	253.16
90th Percentile	306.00
95th Percentile	312.71

FIGURE 5.6 Simulation in Excel

Monte Carlo Simulation on Financial Analysis

Project A

		2001	2002	2003	2004	2005		
							NPV	$126
Revenues		$1,010	$1,111	$1,233	$1,384	$1,573	IRR	15.68%
Opex/Revenue Multiple		0.09	0.10	0.11	0.12	0.13	Risk Adjusted Discount Rate	12.00%
Operating Expenses		$91	$109	$133	$165	$210	Growth Rate	3.00%
EBITDA		$919	$1,002	$1,100	$1,219	$1,363	Terminal Value	$8,692
FCF/EBITDA Multiple		0.20	0.25	0.31	0.40	0.56	Terminal Risk Adjustment	30.00%
Free Cash Flows	($1,200)	$187	$246	$336	$486	$760	Discounted Terminal Value	$2,341
Initial Investment	($1,200)						Terminal to NPV Ratio	18.52
Revenue Growth Rates		10.00%	11.00%	12.21%	13.70%	15.58%	Payback Period	3.89
							Simulated Risk Value	$390

Project B

		2001	2002	2003	2004	2005		
							NPV	$149
Revenues		$1,200	$1,404	$1,683	$2,085	$2,700	IRR	33.74%
Opex/Revenue Multiple		0.09	0.10	0.11	0.12	0.13	Risk Adjusted Discount Rate	19.00%
Operating Expenses		$108	$138	$181	$249	$361	Growth Rate	3.75%
EBITDA		$1,092	$1,266	$1,502	$1,836	$2,340	Terminal Value	$2,480
FCF/EBITDA Multiple		0.10	0.11	0.12	0.14	0.16	Terminal Risk Adjustment	30.00%
Free Cash Flows	($400)	$109	$139	$183	$252	$364	Discounted Terminal Value	$668
Initial Investment	($400)						Terminal to NPV Ratio	4.49
Revenue Growth Rates		17.00%	19.89%	23.85%	29.53%	38.25%	Payback Period	2.83
							Simulated Risk Value	$122

Project C

		2001	2002	2003	2004	2005		
							NPV	$29
Revenues		$950	$1,069	$1,219	$1,415	$1,678	IRR	15.99%
Opex/Revenue Multiple		0.13	0.15	0.17	0.20	0.24	Risk Adjusted Discount Rate	15.00%
Operating Expenses		$124	$157	$205	$278	$395	Growth Rate	5.50%
EBITDA		$827	$912	$1,014	$1,136	$1,283	Terminal Value	$7,935
FCF/EBITDA Multiple		0.20	0.25	0.31	0.40	0.56	Terminal Risk Adjustment	30.00%
Free Cash Flows	($1,100)	$168	$224	$309	$453	$715	Discounted Terminal Value	$2,137
Initial Investment	($1,100)						Terminal to NPV Ratio	74.73
Revenue Growth Rates		12.50%	14.06%	16.04%	18.61%	22.08%	Payback Period	3.88
							Simulated Risk Value	$53

Project D

		2001	2002	2003	2004	2005		
							NPV	$26
Revenues		$1,200	$1,328	$1,485	$1,681	$1,932	IRR	21.57%
Opex/Revenue Multiple		0.08	0.08	0.09	0.09	0.10	Risk Adjusted Discount Rate	20.00%
Operating Expenses		$90	$107	$129	$159	$200	Growth Rate	1.50%
EBITDA		$1,110	$1,221	$1,355	$1,522	$1,732	Terminal Value	$2,648
FCF/EBITDA Multiple		0.14	0.16	0.19	0.23	0.28	Terminal Risk Adjustment	30.00%
Free Cash Flows	($750)	$159	$200	$259	$346	$483	Discounted Terminal Value	$713
Initial Investment	($750)						Terminal to NPV Ratio	26.98
Revenue Growth Rates		10.67%	11.80%	13.20%	14.94%	17.17%	Payback Period	3.38
							Simulated Risk Value	$56

	Implementation Cost	Sharpe Ratio	Weight	Project Cost	Project NPV	Risk Parameter	Payback Period	Technology Level	Tech Mix
Project A	$1,200	0.02	5.14%	$62	$6	29%	3.89	5	0.26
Project B	$400	0.31	25.27%	$101	$38	15%	2.83	3	0.76
Project C	$1,100	0.19	34.59%	$380	$10	21%	3.88	2	0.69
Project D	$750	0.17	35.00%	$263	$9	17%	3.38	4	1.40
Total	$3,450	0.17	100.00%	$806	$63	28%	3.49	3.5	3.11

Constraints:

	Lower Barrier	Upper Barrier	
Budget	$0	$900	(10 percentile at top 900)
Payback Mix	0.10	1.00	
Technology Mix	0.40	4.00	
Per Project Mix	5%	35%	

FIGURE 5.7 Monte Carlo Simulation

Chapter 9 on getting started with and using the Risk Simulator Monte Carlo simulation software. Remember that there is also a complimentary limited edition trial version of Risk Simulator simulation software on CD-ROM included at the back of this book, complete with example spreadsheets and the Real Options Valuation Super Lattice Solver trial software.

Obviously there are many uses of simulation, and we are barely scratching the surface with these examples. One additional use of simulation deserves mention: simulation can be used in forecasting. Specifically, an analyst can forecast future cash flows, cost, revenues, prices, and so forth using simulation. Figure 5.8 shows an example of how stock prices can be forecasted using simulation. This example is built upon a stochastic process called the Geometric Brownian Motion.[3] Using this assumption, we can simulate the price path of a particular stock. Three stock price paths are shown here, but in reality, thousands of paths are generated, and a probability distribution of the

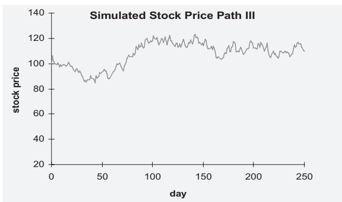
Conceptualizing the Lognormal Distribution

A Simple Simulation Example

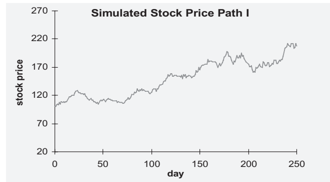
We need to perform many simulations to obtain a valid distribution

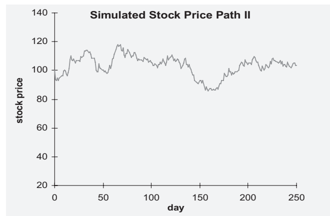
Mean	15%
Sigma	30%
Timing	Daily
Starting Value	100

Here we see the effects of performing a simulation of stock price paths following a Geometric Brownian Motion model for daily closing prices. Three sample paths are seen here, in reality, thousands of simulations are performed and their distribution properties are analyzed. Frequently, the average closing prices of these thousands of simulations are analyzed, based on these simulated price paths.

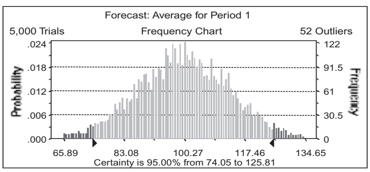


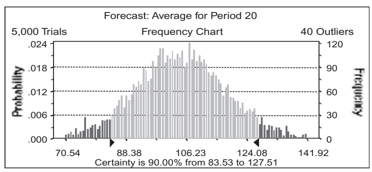
time days	normal deviates	value simulated
0	NA	100.0000
1	0.0873	100.2259
2	-0.4320	99.4675
3	-0.1389	99.2652
4	-0.4583	98.4649
5	1.7807	101.9095
6	-1.4406	99.2212
7	-0.5577	98.2357
8	0.5277	99.2838
9	-0.4844	98.4345
10	-0.2307	98.0634
11	0.8688	99.7532
12	2.1195	83.9088
13	-1.9756	100.1461
14	1.3734	102.8517
15	-0.8790	101.2112
16	-0.7610	99.8203
17	0.3168	100.4824
18	-0.0511	100.4452
19	0.0653	100.6301
20	-0.6073	99.5368
21	0.6900	100.9091
22	-0.7012	99.6353
23	1.4784	102.5312
24	-0.9195	100.8184
25	-0.3343	100.2411
26	-2.3395	95.9465
27	-1.7831	92.8103
28	-0.3247	92.2958
29	0.5053	93.2409
30	0.0386	93.3652
247	1.0418	100.9205
248	-0.7052	99.6388
249	0.1338	99.9521
250	0.0451	100.0978

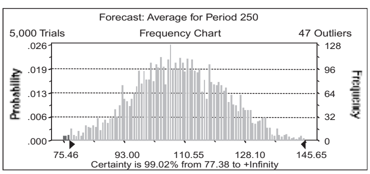




Rows 31 through 246 have been hidden to conserve space.

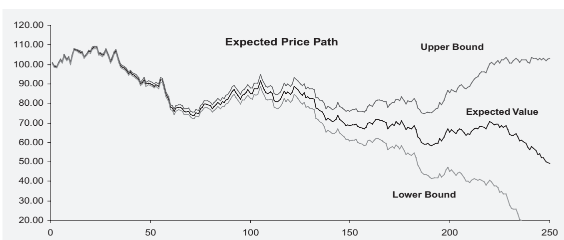






The thousands of simulated price paths are then tabulated into probability distributions. Here are three sample price paths at three different points in time, for periods 1, 20, and 250. There will be a total of 250 distributions for each time period, which corresponds to the number of trading days a year.

We can also analyze each of these time-specific probability distributions and calculate relevant statistically valid confidence intervals for decision-making purposes.



We can then graph out the confidence intervals together with the expected values of each forecasted time period.

Notice that as time increases, the confidence interval widens because there will be more risk and uncertainty as more time passes.

FIGURE 5.8 Lognormal Simulation

outcomes can then be created. That is, for a particular time period in the future—say, on day 100—we can determine the probability distribution of prices on that day. We can apply similar concepts to forecasting demand, cost, and any other variables of interest. Risk Simulator has a forecasting module capable of running time-series forecasts, regression analysis, nonlinear extrapolation, and stochastic processes. See Chapter 9 for details.

SUMMARY

This chapter reviews the similarities between financial options and real options. The most important difference is that the latter revolves around physical assets that are usually not traded in the market, as compared to highly volatile financial assets that are actively traded in the market with shorter maturities for financial options. Monte Carlo simulation is also introduced as an important and integral approach when performing real options. The example on the flaw of using simple averages shows that without the added insights of probabilities and simulation, wrong decisions will be made.

CHAPTER 5 QUESTIONS

1. What do you believe are the three most important differences between financial options and real options?
2. In the Flaw of Averages example, a nonparametric simulation approach is used. What does nonparametric simulation mean?
3. In simulating a sample stock price path, a stochastic process called Geometric Brownian Motion is used. What does a stochastic process mean?
4. What are some of the restrictive assumptions used in the Black-Scholes equation?
5. Using the example in Figure 5.8, simulate a sample revenue path in Excel, based on a Geometric Brownian Motion process, where $\delta S_t = S_{t-1}[\mu\delta t + \sigma\varepsilon\sqrt{\delta t}]$. Assume a 50 percent annualized volatility (σ), mean drift rate (μ) of 2 percent, and a starting value (S_0) of $100 on January 2002. Create a monthly price path simulation for the period January 2002 to December 2004. Use the function: "=NORMSINV(RAND( ))" in Excel to recreate the simulated standard normal random distribution value ε.

PART Two

Application

CHAPTER 6

Behind the Scenes

INTRODUCTION

This chapter and the following two chapters introduce the reader to some common types of real options and a step-by-step approach to analyzing them. The methods introduced include closed-form models, partial-differential equations, and binomial lattices through the use of risk-neutral probabilities. The advantages and disadvantages of each method are discussed in detail. In addition, the theoretical underpinnings surrounding the binomial equations are demystified here, leading the reader through a set of simplified discussions on how certain binomial equations are derived, without the use of fancy mathematics. This chapter and the next chapter are key to understanding the fundamental theories underlying option valuation and should be reviewed first before embarking on Chapters 9 to 11 where real options cases are solved using the author's Super Lattice Solver software and Risk Simulator software.

REAL OPTIONS: BEHIND THE SCENES

Multiple methodologies and approaches are used in financial options analysis to calculate an option's value. These range from using closed-form equations like the Black-Scholes model and its modifications, Monte Carlo path-dependent simulation methods, lattices (for example, binomial, trinomial, quadranomial, and multinomial lattices), variance reduction and other numerical techniques, to using partial-differential equations, and so forth. However, the most widely used mainstream methods are the closed-form solutions, partial-differential equations, and the binomial lattices.

Closed-form solutions are models like the Black-Scholes, where there exist equations that can be solved given a set of input assumptions. They are exact, quick, and easy to implement with the assistance of some basic programming knowledge but are difficult to explain because they tend to apply highly technical stochastic calculus mathematics. They are also very specific in nature, with limited modeling flexibility. Closed-form solutions are exact for European options but are only approximations for American options. Many exotic and Bermudan options cannot be solved using closed-form solutions. The same limitations apply to path-dependent simulations. Only simple European options can be solved using simulations. Many exotic, American, and Bermudan options cannot be solved using simulation.

Real options can be calculated in different ways, including the use of path-dependent simulation, closed-form models, partial-differential equations, and multinomial and binomial approaches.

Binomial lattices, in contrast, are easy to implement and easy to explain. Lattices can solve all types of options, including American, Bermudan, European, and many types of exotic options, as will be seen in later chapters. They are also highly flexible but require significant computing power and lattice steps to obtain good approximations, as we will see later in this chapter. It is important to note, however, that in the limit, results obtained through the use of binomial lattices tend to approach those derived from closed-form solutions. The results from closed-form solutions may be used in conjunction with the binomial lattice approach when presenting to management a complete real options solution. In this chapter, we explore these mainstream approaches and compare their results as well as when each approach may be best used, when analyzing the more common types of financial and real options.

American options are exercisable at any time prior to and including maturity. European options are exercisable only at maturity and not before. Bermudan options are exercisable at any time prior to and including maturity except during specific vesting or blackout periods. Options are also divided into plain-vanilla types (simple combinations of calls and puts) and exotic types (all others).

Here is an example to illustrate the point of binomial lattices approaching the results of a closed-form solution. Let us look at a European Call Option as calculated using the Generalized Black-Scholes model[1] specified below:

$$Call = Se^{-q(T)}\,\Phi\left[\frac{\ln(S/X)+(rf-q+\sigma^2/2)T}{\sigma\sqrt{T}}\right] - Xe^{-rf(T)}\,\Phi\left[\frac{\ln(S/X)+(rf-q-\sigma^2/2)T}{\sigma\sqrt{T}}\right]$$

Let us assume that both the stock price (S) and the strike price (X) are \$100, the time to expiration (T) is one year, with a 5 percent risk-free rate (rf) for the same duration, while the volatility (σ) of the underlying asset is 25 percent with no dividends (q). The Generalized Black-Scholes calculation yields \$12.3360, while using a binomial lattice we obtain the following results:

N = 10 steps	\$12.0923
N = 20 steps	\$12.2132
N = 50 steps	\$12.2867
N = 100 steps	\$12.3113
N = 1,000 steps	\$12.3335
N = 10,000 steps	\$12.3358
N = 50,000 steps	\$12.3360

Notice that even in this oversimplified example, as the number of time-steps (N) gets larger, the value calculated using the binomial lattice approaches the closed-form solution. Do not worry about the computation at this point as we will detail the stepwise calculations in a moment. Suffice it to say, many steps are required for a good estimate using binomial lattices. It has been shown in past research that 100 to 1,000 time-steps are usually sufficient for a good valuation.

We can define time-steps as the number of branching events in a lattice. For instance, the binomial lattice shown in Figure 6.1 has three time-steps, starting from time 0. The first time-step has two nodes (S_0u and S_0d), while the second time-step has three nodes (S_0u^2, S_0ud, and S_0d^2), and so on. Therefore, as we have seen previously, to obtain 1,000 time-steps, we need to calculate 1, 2, 3 . . . 1,001 nodes, which is equivalent to calculating 501,501 nodes. If we intend to perform 10,000 simulation trials on the options calculation, we will need approximately 5×10^9 nodal calculations, equivalent to 299 Excel spreadsheets or 4.6 GB of memory space. Definitely a daunting task, to say the least, and we clearly see here the need for using software to facilitate such calculations.[2] One noteworthy item is that the lattice is something called a *recombining* lattice, where at time-step 2, the middle node (S_0ud) is the same as time-step 1's lower bifurcation of S_0u and upper bifurcation of S_0d.

Figure 6.2 is an example of a two time-step binomial lattice that is nonrecombining. That is, the center nodes in time-step 2 are different (S_0ud' is not the same as S_0du'). In this case, the computational time and resources are even higher due to the exponential growth of the number of nodes—specifically, 2^0 nodes at time-step 0, 2^1 nodes at time-step 1, 2^2 nodes at time-step 2, and so forth, until $2^{1,000}$ nodes at time-step 1,000 or approximately 2×10^{301} nodes, taking your computer potentially years to calculate the entire binomial lattice. Recombining and nonrecombining binomial lattices yield the same

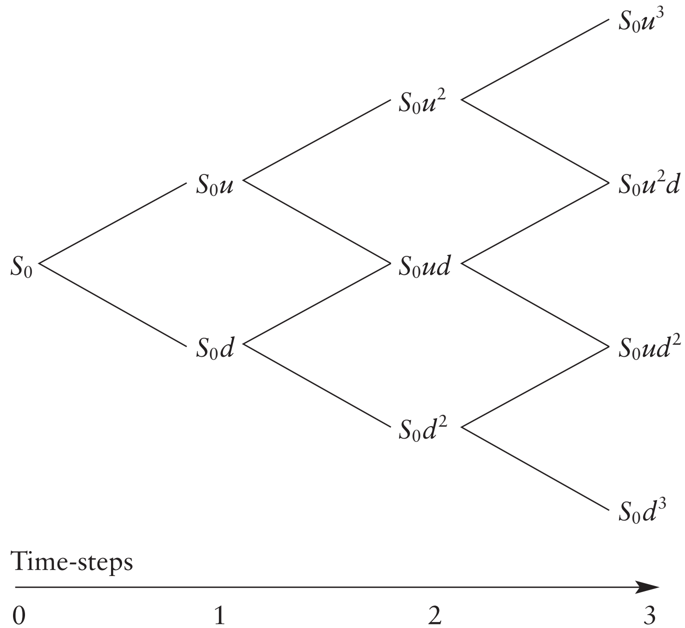


FIGURE 6.1 Three Time-Steps (Recombining Lattice)

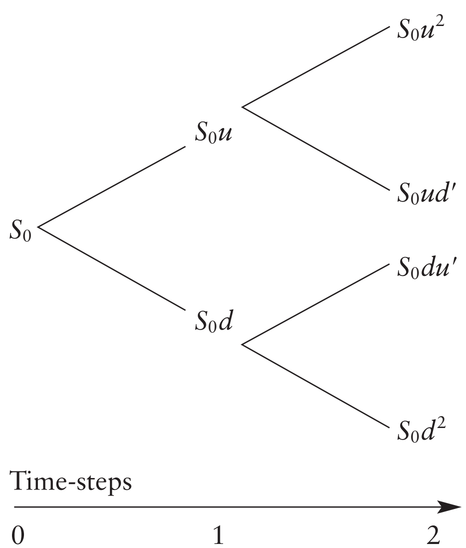


FIGURE 6.2 Two Time-Steps (Nonrecombining Lattice)

results at the limit, so it is definitely easier to use recombining lattices for most of our analysis. However, there are exceptions where nonrecombining lattices are required, especially when there are two or more stochastic underlying variables or when volatility of the single underlying variable changes over time. Appendix 7I details the use of nonrecombining lattices with multiple volatilities, and the use of multiple recombining lattices to recreate a nonrecombining lattice. The examples in this appendix show that the results from a recombining lattice are exactly the same for nonrecombing lattices. Chapters 9 and 10 show how multiple underlying assets can be solved using the Multiple Asset Super Lattice Solver and the Multinominal Super Lattice Solver software. However, for illustration purposes, we will continue with single underlying asset with constant volatility examples solved using recombining lattices throughout this chapter and the next.

As you can see, closed-form solutions certainly have computational ease compared to binomial lattices. However, it is more difficult to explain the exact nature of a fancy stochastic calculus equation than it would be to explain a binomial lattice that branches up and down. Because both methods tend to provide the same results at the limit anyway, for ease of exposition, the binomial lattice should be presented for management discussions. There are also other issues to contend with in terms of advantages and disadvantages of each technique. For instance, closed-form solutions are mathematically elegant but very difficult to derive and are highly specific in nature. Tweaking a closed-form equation requires facility with sophisticated stochastic mathematics. Binomial lattices, however, although sometimes computationally stressful, are easy to build and require no more than simple algebra, as we will see later. Binomial lattices are also very flexible in that they can be tweaked easily to accommodate most types of real options problems.

We continue the rest of the book with introductions to various types of common real options problems and their associated solutions, using closed-form models, partial-differential equations, and binomial lattices, wherever appropriate. We further use, for simplicity, recombining lattices with only five time-steps in most cases. The reader can very easily extend these five time-step examples into thousands of time-steps using the same algorithms.

BINOMIAL LATTICES

In the binomial world, several basic similarities are worth mentioning. No matter the types of real options problems you are trying to solve, if the binomial lattice approach is used, the solution can be obtained in one of two ways. The first is the use of risk-neutral probabilities, and the second is the use of market-replicating portfolios. Throughout this book, the former approach is used. An example of the market-replicating portfolio approach is shown in Appendix 7C for the sake of completeness. The use of a replicating portfolio is more difficult to understand and apply, but the results obtained from replicating portfolios are identical to those obtained through risk-neutral probabilities. So it does not matter which method is used; nevertheless, application and expositional ease should be emphasized.

Market-replicating portfolios' predominant assumptions are that there are no arbitrage opportunities and that there exist a number of traded assets in the market that can be obtained to replicate the existing asset's payout profile. A simple illustration is in order here. Suppose you own a portfolio of publicly traded stocks that pay a set percentage *dividend* per period. You can, in theory, assuming no trading restrictions, taxes, or transaction costs, purchase a second portfolio of several *non-dividend-paying* stocks, bonds, and other instruments, and replicate the payout of the first portfolio of *dividend-paying* stocks. You can, for instance, sell a particular number of shares per period to replicate the first portfolio's dividend payout amount at every time period. Hence, if both payouts are identical although their stock compositions are different, the value of both portfolios should then be identical. Otherwise, there will be arbitrage opportunities, and market forces will tend to make them equilibrate in value. This makes perfect sense in a financial securities world where stocks are freely traded and highly liquid. However, in a real options world where physical assets and firm-specific projects are being valued, financial purists would argue that this assumption is hard to accept, not to mention the mathematics behind replicating portfolios are also more difficult to apply.

Compare that to using something called a risk-neutral probability approach. Simply stated, instead of using a risky set of cash flows and discounting them at a risk-adjusted discount rate akin to the discounted cash flow models, one can instead easily risk-adjust the probabilities of specific cash flows occurring at specific times. Thus, using these risk-adjusted probabilities on the cash flows allows the analyst to discount these cash flows (whose risks have now been accounted for) at the risk-free rate. This is the essence of binomial lattices as applied in valuing options. The results obtained are identical.

Let's now see how easy it is to apply risk-neutral valuation in a binomial lattice setting. In any options model, there is a minimum requirement of at least two lattices. The first lattice is always the lattice of the underlying asset, while the second lattice is the option valuation lattice. No matter what real options model is of interest, the basic structure almost always exists, taking the form:

Inputs: S, X, σ, T, rf, b

$$u = e^{\sigma\sqrt{\delta t}} \text{ and } d = e^{-\sigma\sqrt{\delta t}} = \frac{1}{u}$$

$$p = \frac{e^{(rf-b)(\delta t)} - d}{u - d}$$

The basic inputs are the present value of the underlying asset (S), present value of implementation cost of the option (X), volatility of the natural logarithm of the underlying free cash flow returns in percent (σ), time to expiration in years (T), risk-free rate or the rate of return on a riskless asset (rf), and continuous dividend outflows in percent (b). In addition, the binomial lattice approach requires two additional sets of calculations, the up and down factors (u and d) as well as a risk-neutral probability measure (p). We see from the foregoing equations that the up factor is simply the exponential function of the cash flow returns volatility multiplied by the square root of time-steps or stepping time (δt). Time-steps or stepping time is simply the time scale between steps. That is, if an option has a one-year maturity and the binomial lattice that is constructed has 10 steps, each time-step has a stepping time of 0.1 years. The volatility measure is an annualized value; multiplying it by the square root of time-steps breaks it down into the time-step's equivalent volatility. The down factor is simply the reciprocal of the up factor. In addition, the higher the volatility measure, the higher the up and down factors. This reciprocal magnitude ensures that the lattices are recombining because the up and down steps have the same magnitude but different signs; at places along the future path these binomial bifurcations must meet.

The second required calculation is that of the risk-neutral probability, defined simply as the ratio of the exponential function of the difference between risk-free rate and dividend, multiplied by the stepping time less the down factor, to the difference between the up and down factors. This risk-neutral probability value is a mathematical intermediate and by itself has no particular meaning. One major error real options users commit is to extrapolate these probabilities as some kind of subjective or objective probabilities that a certain event will occur. Nothing is further from the truth. There is no economic or financial meaning attached to these risk-neutralized probabilities save that it is an intermediate step in a series of calculations. Armed with these values, you are now on your way to creating a binomial lattice of the underlying asset value, shown in Figure 6.3.

Binomial lattices can be solved through the use of risk-neutral probabilities and market-replicating portfolios. In using binomial and multinomial lattices, the higher the number of steps, the higher the level of granularity, and hence, the higher the level of accuracy.

Starting with the present value of the underlying asset at time zero (S_0), multiply it with the up (u) and down (d) factors as shown in Figure 6.3 to create a binomial lattice. Remember that there is one bifurcation at

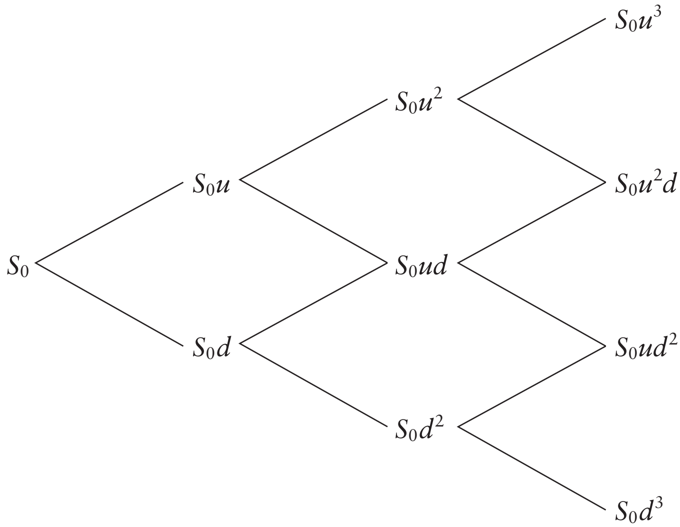


FIGURE 6.3 Binomial Lattice of the Underlying Asset Value

each node, creating an up and a down branch. The intermediate branches are all recombining. This evolution of the underlying asset shows that if the volatility is zero, in a deterministic world where there are no uncertainties, the lattice would be a straight line, and a discounted cash flow model will be adequate because the value of the option or flexibility is also zero. In other words, if volatility (σ) is zero, then the up ($u = e^{\sigma\sqrt{\delta t}}$) and down ($d = e^{-\sigma\sqrt{\delta t}}$) jump sizes are equal to one. It is because there are uncertainties and risks, as captured by the volatility measure, that the lattice is not a straight horizontal line but comprises up and down movements. It is this up and down uncertainty that generates the value in an option. The higher the volatility measure, the higher the difference between the up and down factors as previously defined, the higher the potential value of an option as higher uncertainties exist and the potential upside for the option increases.

Chapter 7 goes into more detail on how certain real options problems can be solved. Each type of problem is introduced with a short business case. Then a closed-form equation is used to value the strategic option. A binomial lattice is then used to confirm the results. The cases conclude with a summary of the results and relevant interpretations. In each case, a limited number of lattice steps are used to facilitate the exposition of the stepwise methodology. The reader can very easily extend the analysis to incorporate more lattice steps as necessary.

THE LOOK AND FEEL OF UNCERTAINTY

In most financial analyses, the first step is to create a series of free cash flows, which can take the shape of an income statement or statement of cash flows. The resulting free cash flows are depicted on a time line, similar to that shown in Figure 6.4. These cash flow figures are in most cases forecasts of the unknown future. In this simple example, the cash flows are assumed to follow a straight-line growth curve. Similar forecasts can be constructed using historical data and fitting these data to a time series model or a regression analysis. Whatever the method of obtaining said forecasts or the shape of the growth curve, these are single-point estimates of the unknown future. Performing a discounted cash flow analysis on these static cash flows provides an accurate value of the project assuming all the future cash flows are known with certainty—that is, no uncertainty exists, and hence, there exists zero volatility around the forecast values.

However, in reality, business conditions are hard to forecast. Uncertainty exists, and the actual levels of future cash flows may look more like those in Figure 6.5. That is, at certain time periods, actual cash flows may be above, below, or at the forecast levels. For instance, at any time period, the actual cash flow may fall within a range of figures with a certain percent probability. As an example, the first year's cash flow may fall anywhere between $480

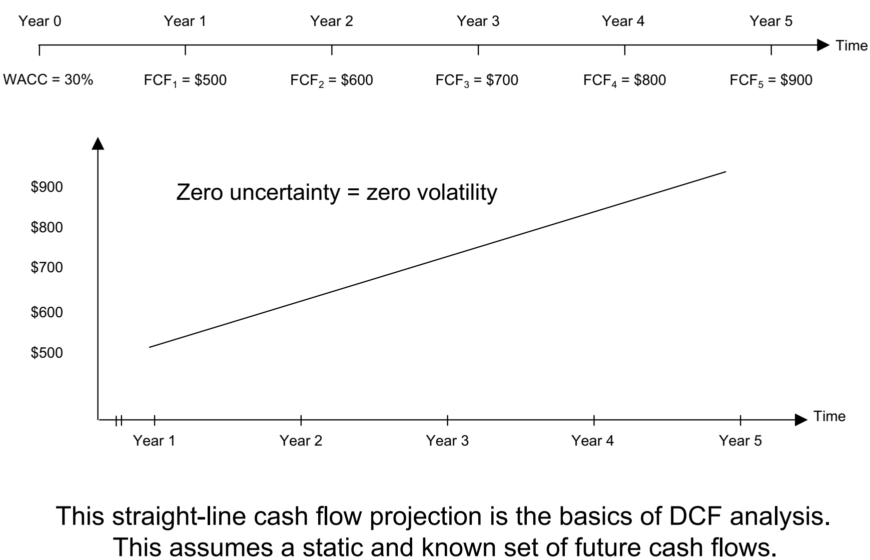


FIGURE 6.4 Straight-Line Discounted Cash Flow

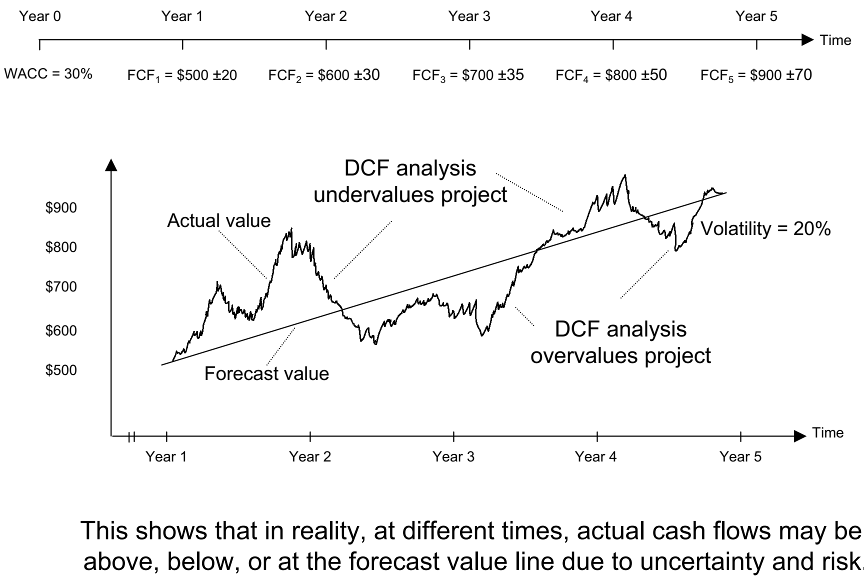


FIGURE 6.5 Discounted Cash Flow with Simulation

and \$520. The actual values are shown to fluctuate around the forecast values at an average volatility of 20 percent.[3] Certainly this example provides a much more accurate view of the true nature of business conditions, which are fairly difficult to predict with any amount of certainty.

Figure 6.6 shows two sample actual cash flows around the straight-line forecast value. The higher the uncertainty around the actual cash flow levels, the higher the volatility. The darker line with 20 percent volatility fluctuates more wildly around the forecast values. These values can be quantified using Monte Carlo simulation. For instance, Figure 6.6 also shows the Monte Carlo simulated probability distribution output for the 5 percent volatility line, where 95 percent of the time, the actual values will fall between \$510 and \$698. Contrast this to a 95 percent confidence range of between \$405 and \$923 for the 20 percent volatility case. This implies that the actual cash flows can fluctuate anywhere in these ranges, where the higher the volatility, the higher the range of uncertainty. A point of interest to note is that the y-axis on the time-series chart is the x-axis of the frequency distribution chart. Thus, a highly volatile cash flow will have a wider y-axis range on the first chart and a wider x-axis range on the second chart. The width of the frequency distribution chart is measured by the standard deviation, a way to measure volatility. Also, the area in the frequency chart is the relevant probabilities of occurrence. Hence, standard deviation, volatility, and probability are all

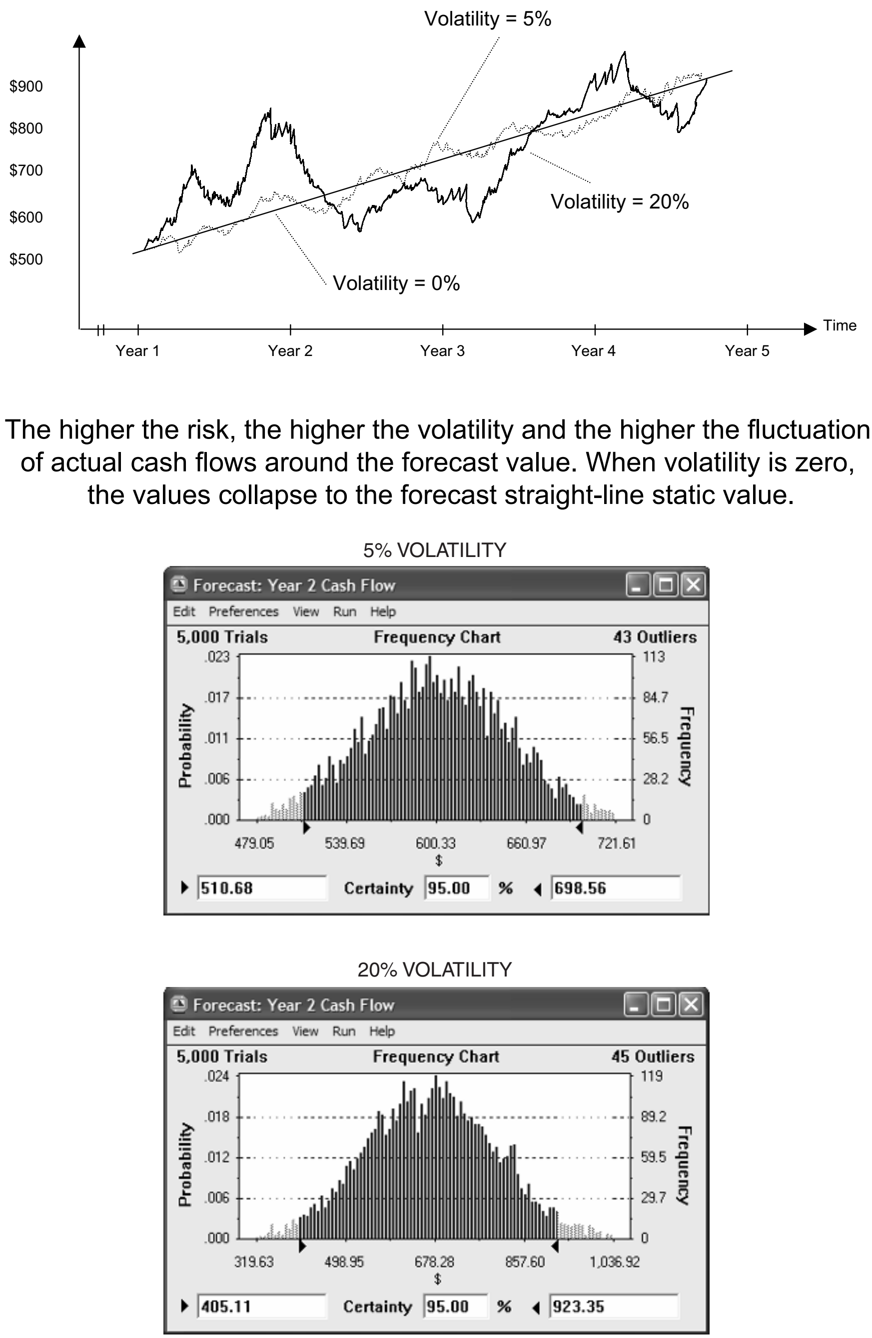


FIGURE 6.6 The Face of Uncertainty

related and we can impute one from the other, as will be seen later in this chapter and in Appendix 7A (calculating volatility).

A FIRM'S REAL OPTIONS PROVIDE VALUE IN THE FACE OF UNCERTAINTY

As seen previously, Monte Carlo simulation can be applied to quantify the levels of uncertainty in cash flows. However, simulation does not consider the strategic alternatives that management may have. For instance, simulation accounts for the range and probability that actual cash flows can be above or below predicted levels but does not consider what management can do if such conditions occur.

Consider Figure 6.7 for a moment. The area above the mean predicted levels, assuming that management has a strategic option to expand into different markets or products, or develop a new technology, means that executing such an option will yield considerable value. Conversely, if management has the option to abandon a particular technology, market, or development initiative when operating conditions deteriorate, possessing and executing such an

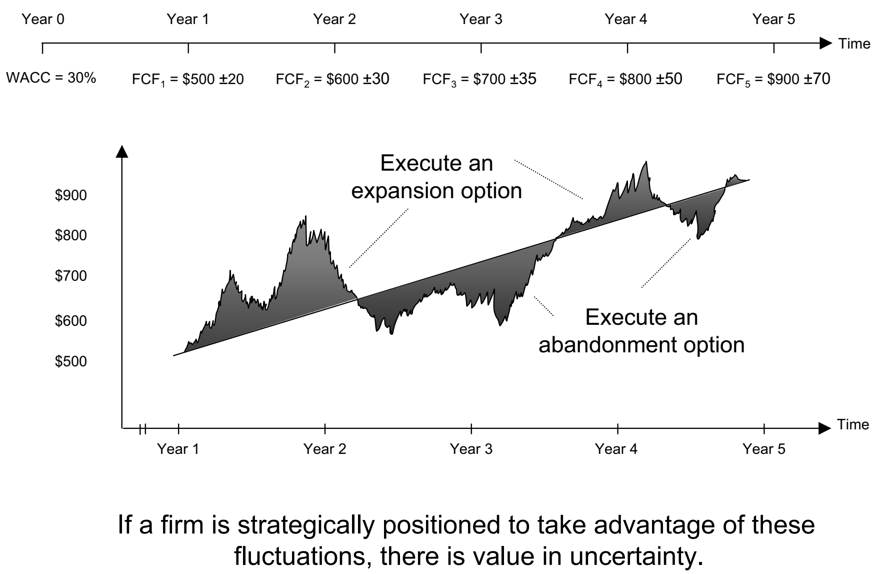


FIGURE 6.7 The Real Options Intuition

abandonment or switching strategy may be valuable. This assumes that management not only has the flexibility to execute these options but also has the willingness to follow through with these strategies when the appropriate time comes. Often, when faced with an abandonment decision, even when it is clearly optimal to abandon a particular project, management may still be inclined to keep the project alive in the hopes that conditions would revert and make the project profitable once again. In addition, management psychology and project attachment may come into play. When the successful execution of a project is tied to some financial remuneration, reputation, or personal strive for merit and achievement, abandoning a project may be hard to do even when it is clearly the optimal decision.

The value of a project's real options requires several assumptions. First, a financial model can be built where the model's operating, technological, market, and other factors are subject to uncertainty and change. These uncertainties have to drive a project or initiative's value. Furthermore, there exists managerial flexibility or strategic options that management can execute along the way as these uncertainties become resolved over time, actions, and events. Finally, management must not only be able but also willing to execute these options when it becomes optimal to do so. That is, we have to assume that management is rational and execute strategies where the additional value generated is at least commensurate with the risks undertaken. Ignoring such strategic value will grossly underestimate the value of a project. Real options not only provide an accurate accounting of this flexibility value but also indicate the conditions under which executing certain strategies becomes optimal.

> ***Real options have strategic value only when***
> ***(i) A financial model can be built.***
> ***(ii) There is uncertainty.***
> ***(iii) Uncertainty drives project value.***
> ***(iv) Management has flexibility and strategic options.***
> ***(v) Management is rational in executing these strategic options.***

Projects that are at-the-money or out-of-the-money—that is, projects with static net present values that are negative or close to breaking even—are most valuable in terms of applying real options. Because real options analysis captures strategic value that is otherwise overlooked in traditional analyses, the additional value obtained may be sufficient to justify projects that are barely profitable.

BINOMIAL LATTICES AS A DISCRETE SIMULATION OF UNCERTAINTY

As uncertainty drives the value of projects, we need to further the discussion on the nature of uncertainty. Figure 6.8 shows a cone of uncertainty, where we can depict uncertainty as increasing over time. Notice that risk may or may not increase over time, but uncertainty does increase over time. For instance, it is usually much easier to predict business conditions a few months in advance, but it becomes more and more difficult the further one goes into the future, even when business risks remain unchanged. This is the nature of the cone of uncertainty. If we were to attempt to forecast future cash flows while attempting to quantify uncertainty using simulation, a well-prescribed method is to simulate thousands of cash flow paths over time, as shown in Figure 6.8. Based on all the simulated paths, a probability distribution can be constructed at each time period. The simulated pathways were generated using a Geometric Brownian Motion with a fixed volatility. A Geometric Brownian Motion can be depicted as

$$\frac{\delta S}{S} = \mu(\delta t) + \sigma\varepsilon\sqrt{\delta t}$$

where a percent change in the variable S (denoted $\delta S/S$) is simply a combination of a deterministic part ($\mu(\delta t)$) and a stochastic part ($\sigma\varepsilon\sqrt{\delta t}$). Here, μ is a drift term or growth parameter that increases at a factor of time-steps

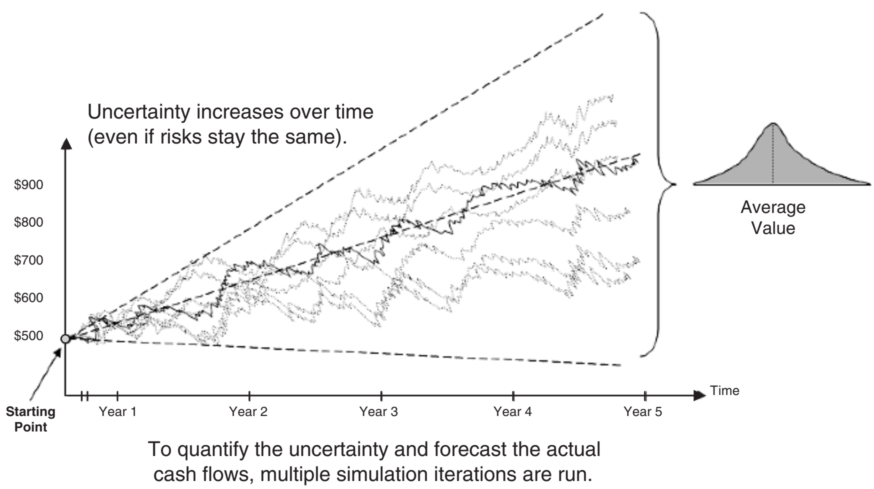


FIGURE 6.8 The Cone of Uncertainty

δt, while σ is the volatility parameter, growing at a rate of the square root of time, and ε is a simulated variable, usually following a normal distribution with a mean of zero and a variance of one. Note that the different types of Brownian Motions are widely regarded and accepted as standard assumptions necessary for pricing options. Brownian Motions are also widely used in predicting stock prices.

Notice that the volatility (σ) remains constant throughout several thousand simulations. Only the simulated variable (ε) changes every time.[4] This is an important aspect that will become clear when we discuss the intuitive nature of the binomial equations required to solve a binomial lattice, because one of the required assumptions in options modeling is the reliance on Brownian Motion. Although the risk or volatility (σ) in this example remains constant over time, the level of uncertainty increases over time at a factor of ($\sigma\sqrt{\delta t}$). That is, the level of uncertainty grows at the square root of time and the more time passes, the harder it is to predict the future. This is seen in the cone of uncertainty, where the width of the cone increases over time even when volatility remains constant.

Based on the cone of uncertainty, which depicts uncertainty as increasing over time, we can clearly see the similar triangular shape of the cone of uncertainty and a binomial lattice as shown in Figure 6.9. In essence, a binomial

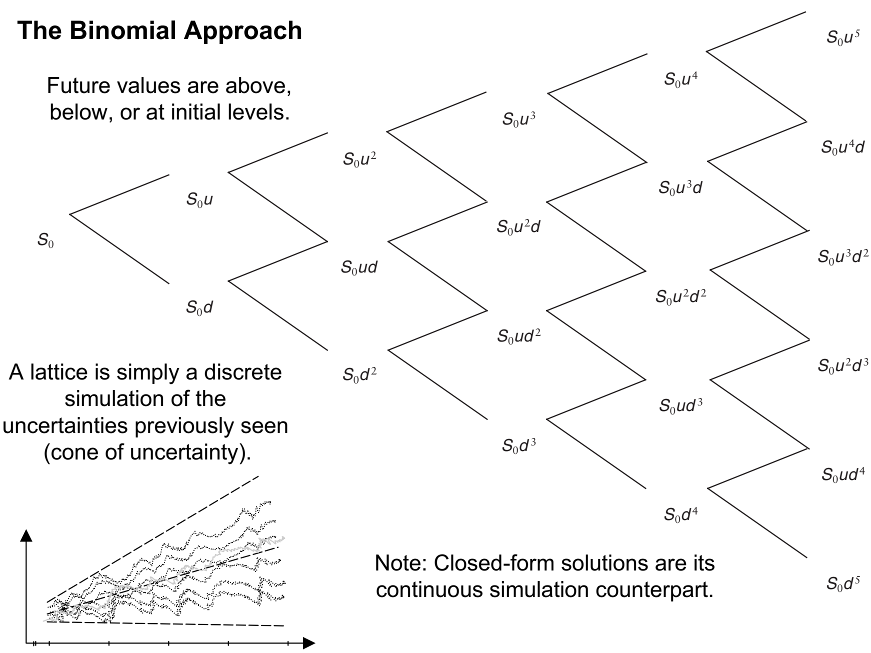


FIGURE 6.9 Discrete Simulation Using Binomial Lattices

lattice is simply a discrete simulation of the cone of uncertainty. Whereas a Brownian Motion is a continuous stochastic simulation process, a binomial lattice is a discrete simulation process.

At the limit, where the number of steps approach infinity, the time-steps approach zero and the results stemming from a binomial lattice approach those obtained from a Brownian Motion process. Solving a Brownian Motion in a discrete sense yields the binomial equations, while solving it in a continuous sense yields closed-form equations like the Black-Scholes and its ancillary models. The following few sections show the simple intuitive discrete derivation of the Brownian Motion process to obtain the binomial equations.

A binomial lattice is a type of discrete simulation, whereas a Brownian Motion stochastic process is a continuous simulation.

As a side note, multinomial models that involve more than two bifurcations at each node, such as the trinomial (three-branch) models or quadranomial (four-branch) models or pentanomial (five-branch) models, require a similar Brownian Motion process or other stochastic processes such as a mean-reverting or jump-diffusion process, and hence are mathematically more difficult to solve. No matter how many branches are at each node, these models provide exactly the same results at the limit, the difference being that the more branches at each node, the faster the results are reached. For instance, a binomial model may require a hundred steps to solve a particular real options problem, while a trinomial model probably only requires half the number of steps. However, due to the complexity involved in solving trinomial lattices as compared to the easier mathematics required for binomial lattices, most real options problems are more readily solved using binomials. For the sake of completeness, Appendix 7H provides an example of how to solve a trinomial lattice. Chapters 9 and 10 illustrate the trinomial, quadranomial, and pentanomial lattices in action, using the author's Multinomial Super Lattice Solver software.

To continue the exploration into the nature of binomial lattices, Figure 6.10 shows the different binomial lattices with different volatilities. This means that the higher the volatility, the wider the range and spread of values between the upper and lower branches of each node in the lattice. Because binomial lattices are discrete simulations, the higher the volatility, the wider the spread of the distribution. This can be seen on the terminal nodes, where the range between the highest and lowest values at the terminal nodes is higher for higher volatilities than the range of a lattice with a lower volatility.

At the extreme, where volatility equals zero, the lattice collapses into a straight line. This straight line is akin to the straight-line cash flow model

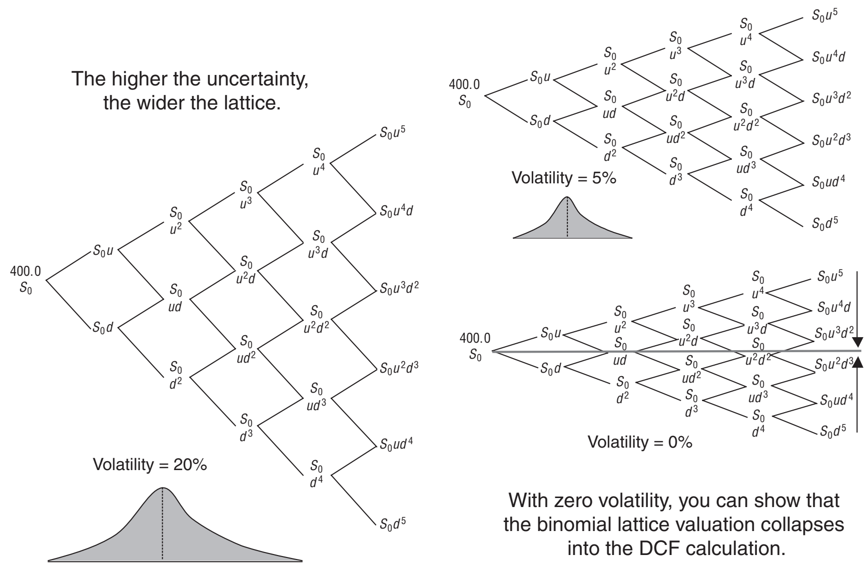


FIGURE 6.10 Volatility and Binomial Lattices

shown in Figure 6.4. We will further show through an example that for a binomial lattice calculation involving cash flows with zero volatility, the results approach those calculated using a discounted cash flow model's net present value approach. This is important because if there is zero uncertainty and risk, meaning that all future cash flows are known with absolute certainty, then there is no strategic real options value. The discounted cash flow model will suffice because business conditions are fraught with uncertainty, and hence volatility exists and can be captured using a binomial lattice. Therefore, the discounted cash flow model can be seen as a special case of a real options model, when uncertainty is negligible and volatility approaches zero. Hence, discounted cash flow is not necessarily wrong at all; it only implies zero uncertainty in the future forecast of cash flows.

RISK VERSUS UNCERTAINTY, VOLATILITY VERSUS DISCOUNT RATES

Risk versus Uncertainty

Up to this point, the terms *risk* and *uncertainty* have been loosely used and defined. However, it is crucial to understand that there are significant differences when we apply these terms to real options. Risk and uncertainty are very different species of animals but they are of the same family and genus; however, the lines of demarcation are often blurred. A distinction is critical at this juncture before proceeding and worthy of segue.

To loosely illustrate their differences, suppose I am senseless enough to take a sky-diving trip with a good friend, who also is equally senseless, and we board a plane headed for the Palm Springs desert. After a 30-minute sky-diving crash course, we are airborne at 12,000 feet and, watching our lives flash before our eyes, we realize that in our haste we forgot to pack our parachutes on board. However, there is an old, dusty, and dilapidated emergency parachute on the plane (with a few holes in it). At that point, both my friend and I have the same level of uncertainty—the uncertainty of whether the old parachute will open and if it does not, whether we will fall to our deaths. However, being the risk-averse, nice guy I am, I decide to let my buddy take the plunge. Clearly, he is the one taking the plunge and the same person taking the risk. I bear no risk at this time while my friend bears all the risk. However, we both have the same level of uncertainty as to whether the parachute will actually fail. In fact, we both have the same level of uncertainty as to the outcome of the day's trading on the New York Stock Exchange—which has absolutely no impact on whether we live or die that day. Only when he jumps and the parachute opens will the *uncertainty become resolved through the passage of time, events, and action*. However, even when the uncertainty is resolved with the opening of the parachute, the risk still exists as to whether he will land safely on the ground below. Once he exits the plane, he no longer has the risk of the plane crashing but I still bear the risk of going down with the plane—we have in essence traded risks but our uncertainties are still the same, that is, he and I both are uncertain if the pilot will land the plane safely albeit my life is on the line while he's watching on the ground as the plane crashes and burns.

> ***Uncertainty is different from risk. Uncertainty becomes resolved through the passage of time, events, and action. Risk is something one bears and is the outcome of uncertainty. Risk may remain constant but uncertainty will increase over time. The terms* uncertainty *and* risk *are sometimes used interachangeably and can be divided into the known, unknown, and unknowable.***

Therefore, *risk is something one bears and is the outcome of uncertainty*. Just because there is uncertainty, there could very well be no risk. If the only thing that bothers a U.S.-based firm's CEO is the fluctuation in the foreign exchange market of the Zambian Kwacha, then I might suggest shorting some Kwachas and shifting his portfolio to U.S.-based debt. This uncertainty if it does not affect the firm's bottom line in any way is only uncertainty and not risk. This book is concerned with risk by performing uncertainty analysis—the same uncertainty that brings about risk by its mere existence as it impacts the value of a particular project. It is further assumed that the end user of this uncertainty analysis uses the results appropriately, whether the analysis is for identifying, adjusting, or selecting projects with respect to their risks, and so forth. Otherwise, running millions of fancy simulation trials and letting the results "marinate" will be useless. By running simulations on the foreign exchange market of the Zambian Kwacha, an analyst sitting in a cubicle somewhere in downtown San Francisco will in no way reduce the risk of the Kwacha in the market or the firm's exposure to the same. Only by using the results from an uncertainty simulation analysis and finding ways to hedge or mitigate the quantified fluctuation and downside risks of the firm's foreign exchange exposure through the derivatives market could the analyst be construed as having performed risk analysis and risk management.

To further illustrate the differences between risk and uncertainty, suppose we are attempting to forecast the stock price of Microsoft (MSFT). Suppose MSFT is currently priced at $25 per share, and historical prices place the stock at 21.89 percent volatility. Using a Brownian motion random walk stochastic process simulation, 10,000 possible paths are created and look something like in Figure 6.8 except that the intercept is at $25 for Year 0. Now suppose that for the next five years, MSFT does not engage in any risky ventures and stays exactly the way it is, and further suppose that the entire economic and financial world remains constant. This means that *risk* is fixed and unchanging, that is, volatility (σ) is unchanging for the next five years. However, the same cone of uncertainty in Figure 6.8 exists. That is, the width of the forecast intervals will still increase over time. For instance, Year 0's forecast is known and is $25. However, as we progress one day, MSFT will most probably vary between $24 and $26. One year later, the uncertainty bounds may be between $20 and $30. Five years into the future, the boundaries might be between $10 and $50. So, *uncertainties increase* while *risk remains the same*. Therefore, risk is not equal to uncertainty. This idea is of course applicable to any forecasting approach whereby it becomes more and more difficult to forecast the future albeit the same risk. Now, if risk changes over time, the bounds of uncertainty get more complicated (e.g., uncertainty bounds of sinusoidal waves with discrete event jumps) and will need to be modeled with heteroskedastic models (heteroskedasticity means volatility is assumed to be changing over time) or in the case of real options, using nonrecombining lattices (see Appendix 7I for details).

In other instances, risk and uncertainty are used interchangeably. For instance, suppose you play a coin-toss game—bet $0.50 and if heads come up you win $1 but you lose everything if tails appear. The risk here is you lose everything because the risk is that tails may appear. The uncertainty here is that tails may appear. Therefore, given that tails appear, I lose everything.

Hence, uncertainty brings with it risk. Uncertainty is the possibility of an event occurring and risk is the ramification of such an event occurring. Hence, people tend to use these two terms interchangeably.

In discussing uncertainty, there are three levels of uncertainties in the world: the *known*, the *unknown*, and the *unknowable*. The known is, of course, what we know will occur and are certain of its occurrence (contractual obligations or a guaranteed event); the unknown is what we do not know and can be simulated. These events will become known through the passage of time, events, and action (the uncertainty of whether a new drug or technology can be developed successfully will become known after spending years and millions on research programs—it will either work or not, and we will know this in the future), and these events carry with them risks but these risks will be reduced or eliminated over time. However, unknowable events carry both uncertainty and risk that the totality of the risk and uncertainty may not change through the passage of time, events, or actions. These are events such as when the next tsunami or earthquake will hit, or when another act of terrorism will occur around the world. When an event occurs, uncertainty becomes resolved but risk still remains (another one may or may not hit tomorrow). In discounted cash flow analysis, we care about the known factors. In real options analysis, we care about the unknown and unknowable factors. The unknowable factors are easy to hedge—get the appropriate insurance! That is, don't do business in a war-torn country, get away from politically unstable economies, buy or create hazard and business interruption strategies and insurance, and so forth. It is the unknown factors that real options will provide the most significant amount of value.

In real options analysis, when we say uncertainty, we simulate this uncertainty with Monte Carlo simulation. The interactions of these uncertainties will ramify themselves through a discounted cash flow model. The resulting cash flows used are the result of the distilled interactions of all uncertainties in the financial model, and are used to compute the volatility, or the risk of the project or asset. For instance, we can simulate the probability of technical success or market share or price of a product in the market as these are uncertainties. I do not know what the market share will be but the best indication is that it will fluctuate between 30 percent and 35 percent, so we simulate this uncertainty. Until we bring this uncertainty back into the model (i.e., 30 percent market share means a 10 percent reduction in total revenues, while a 35 percent market share means a 5 percent increase in total revenues, and so forth), we have no idea what its effects are. Applying simulation on all uncertainties and tying them back into the financial model will then yield the net effects of these uncertainties on cash flows. The risk of the project is this net interaction of all uncertainties on the cash flow, and is computed using volatility. This means that uncertainty is quantified using Monte Carlo simulation, which is then converted into risk when modeled in the DCF analysis, and the outcome is the project's volatility, the measurement of risk of the project, which is then used in a real options analysis, where these risks are hedged (abandonment, sequential compound options, and options to wait) or taken advantage of (expansion and execution options). The variable that ties all these things together is volatility—for technical details on computing volatility, see Appendix 7A.

Discount Rates

Another related item in the discussion of risk, uncertainty, and volatility is that of discount rates. In a discounted cash flow model, the old axiom of "high risk, high return" is seen through the use of a discount rate. That is, the higher the risk of a project, the higher the discount rate should be to risk-adjust this riskier project so that all projects are comparable. Of course, the infamous capital asset pricing model (CAPM) is often used to compute the appropriate discount rate for a discounted cash flow model (weighted average cost of capital, hurdle rates, multiple asset pricing models, and arbitrage pricing models are the other alternatives but are based on similar principles).

Recall that the CAPM uses a beta *(β)* coefficient—a measure of systematic undiversifiable risk relative to the capital markets—to compute the appropriate discount rate. The β coefficient is calculated simply as the covariance (*cov*) of the asset *(i)* and the market's *(m)* returns $cov(r_i,r_m)$, divided by the variance of the market returns $var(r_m)$. Further, covariance can be broken down into the products of correlation $\rho_{i,m}$, and the standard deviations of the asset (σ_i) and the market (σ_m) or

$$\beta = \frac{cov(r_i,r_m)}{var(r_m)} = \frac{\rho_{i,m}\sigma_i\sigma_m}{\sigma_m^2}$$

There are many problems with computing the CAPM discount rate using these traditional approaches. For instance, if the company is not publicly traded, there are no stock returns to calculate the β coefficient. The firm may also have projects that are not highly diversified, meaning that using the diversified market as a proxy for the risks inherent in a single project is unjustified. For instance, a large firm like Microsoft may have hundreds or thousands of small projects, business units, and investments in its corporate portfolio, and saying that the risk inherent in one of its projects can be wholly explained by the fluctuations of its stock prices in the market (stock price returns are used to calculate β) is very dangerous. Not to mention that stock prices change every few minutes, meaning that β changes every few minutes and is relatively unstable over different time horizons. Also, stock prices fluctuate in the market due to investor overreaction, advent of news and events, economic conditions, and many other factors not directly attributable to the risk of the single project being analyzed. Next, using the weighted average cost of capital (WACC) or a corporate hurdle rate—both of which rely on the CAPM as a basis—by themselves across all projects in a firm will be disastrous. It penalizes less risky projects by discounting them at a higher rate than may be required, while it biases the firm toward choosing riskier projects (as higher risks usually bring higher returns, discounting these risky projects at a lower rate will make the projects look more profitable than they really are).

This author recommends a new and novel approach to modify the traditional approaches to discount rate determination—not to replace the old methodologies per se, but to add to them and to enhance their validity—by applying two advanced analytical techniques: Monte Carlo simulation and real options analysis. Using the underlying theory of CAPM, we can estimate a project's discount rate through Monte Carlo simulation and real options analysis to obtain the correlation between, and the volatilities of the project and an internal corporate portfolio. Then, all other projects in the firm are combined to create the company's cumulative portfolio discounted cash flow. Sometimes, the company's cash flows from an annual report are used (as the company is made up of a portfolio of different projects, business units, and so forth, the total cash flows to a company is the project's internal market comparable). In other cases, a set of comparable internal projects can be selected as the market benchmark. This portfolio is used as the market comparable portfolio for the project. The volatility of the portfolio can be similarly calculated by applying Monte Carlo simulation, as applied to obtain the volatility of the project (see Appendix 7A). The correlation between the net cash flows of the project and portfolio are obtained using a nonparametric Spearman rank-based correlation coefficient. A nonparametric Spearman rank-based correlation is used instead of the regular parametric Pearson's correlation coefficient because the underlying distribution of the cash flows is probably non-normal. Also, the number of cash flow periods in the model may be less than 30, and normality cannot be automatically assumed. Finally, Pearson's correlation coefficient measures a linear relationship between two variables. The simulated cash flows may fluctuate extensively such that nonlinear relationships may exist, relationships that cannot be captured with the Pearson's correlation coefficient.

Using the newly calculated internal beta β^*, a revised discount rate (DR) can be calculated using

$$DR = WACC + Max[(\beta^* - 1)(WACC - R_{rf}),0]$$

This equation means that DR has a minimum value of WACC or hurdle rate if the β coefficient is less than or equal to 1.0. A coefficient less than or equal to 1.0 indicates that the risk of the specific project is less than or equal to the risk of the company's portfolio of projects (the internal market comparable). Under these circumstances, the WACC or company-specific hurdle rate should be used. For riskier projects (β exceeding 1.0 means that the project has a higher risk than the overall internal portfolio of projects), the excess risk above 1.0 should be compensated at the rate differential between WACC or hurdle rate and the risk-free rate.

To summarize, instead of using an external market-based beta coefficient that may not fully represent the risk of a specific project, an internal beta can be constructed based on the firm's portfolio of projects. If the firm uses a WACC or hurdle rate, then this rate should be applied to all projects as long as they have an internal beta of less than or equal to 1.0. For projects with higher risks, the relevant DR should be the WACC plus the excess returns (*alpha*) sufficient to compensate for this additional risk. Otherwise, riskier projects will be incorrectly chosen as they are discounted at a lower rate than the required rate of return. Of course this same approach can be used to compute the project-specific discount rate using the CAPM and not the WACC or hurdle rate by using the calculated internal beta.

Therefore, volatility can be used to impute the discount rate, and as we know that the discount rate is a measure of risk (high risk, high return), volatility is hence a measure of risk. Also, we simulated the individual uncertainties using Monte Carlo but the net interaction risk effects are captured as volatility. This means that uncertainty ramifies itself as risk only if there are tangible effects. Hence, uncertainty, risk, volatility, and discount rate can be imputed from one another and are very closely related.

Hard Options versus Soft Options: Adjusting for Nonmarketability of Real Options

Another related discussion about discount rates concerns the nonmarketability aspect of real options. That is, unlike financial options that are freely tradable, real options are in most cases not freely tradable or marketable. An investor can purchase a few calls and a few puts at various strike prices and create different trading strategies like straddles, strangles, butterflies, bull spreads, calendar spreads, and so forth, whereas a firm with real options cannot freely purchase two coal-fired power plants at a certain implementation cost and short another nuclear power plant at a different price to hedge the downside prices of electricity. Therefore, the term *hard options* is used to differentiate the freely traded financial options from *soft options* or options that are not liquid or marketable like real options. Therefore, financial purists may argue that valuing real options as is may be overestimating its value if we do not discount for its nonmarketability and nontransferability aspects. Several approaches can be used to discount for this nonliquid condition:

- ***Compute the real option and reduce it by a corresponding put option.*** Put options are options where the holder can freely sell the asset in the market at a prespecified price during a certain contractual term. The inability for a real option to have a liquid and marketable environment reduces its value by such a put value. Typically, the maturity is set lower than the life of the real option to reflect the actual period where marketability is a major factor, while the strike price is set at the asset value to reflect an instantaneous ability to sell the asset in the market at a moment's notice, at the asset price.

- ***Calculate the relevant carrying cost adjustment by artificially inserting an inflated dividend yield to convert the* hard *option calculation into a* soft *option, thereby discounting the value of the real option.*** This method is more difficult to apply and is susceptible to more subjectivity than using a put option.
- ***Compute Bermudan options instead of regular American options.*** In most cases, the value of an American option ≥ Bermudan option ≥ European option. Hence, instead of always relying on American-type options, compute Bermudan options and account for certain nonmarketable, vesting, cooling-off, or blackout periods. Doing so will reduce and adjust the value of the option to correctly account for the nonmarketability condition. This approach is preferred and is the best.

In practice, adjustments for soft options are usually never made because as long as the approach is comparable when comparing across multiple projects, real options analysis results are robust and correct. In most cases, the relative value among projects is more important than the absolute value of a certain project. However, to adjust for this nonmarketability soft option, simply use a higher dividend rate. In finding the relevant dividend rate to use, one method is to calculate the weighted average cost of capital, or the cost of money to the firm, less the risk-free rate, halve it and set it as the artificial dividend rate. To facilitate the computations throughout this book, this adjustment to dividend is not made. Instead, the Bermudan option will be introduced as a better and more objective alternative in Chapters 10 and 11. Adjustments using dividend rates are artificial and subjective but are still important as can be seen in Chapter 11's case on employee stock options where discounting for the nonmarketability aspects of such restricted options actually can reduce a firm's expenses by millions of dollars.

GRANULARITY LEADS TO PRECISION

Another key concept in the use of binomial lattices is the idea of steps and precision. For instance, if a five-year real options project is valued using five steps, each time-step size (δt) is equivalent to one year. Conversely, if 50 steps are used, then δt is equivalent to 0.1 years per step. Recall that the up and down step sizes were $e^{\sigma\sqrt{\delta t}}$ and $e^{-\sigma\sqrt{\delta t}}$, respectively. The smaller δt is, the smaller the up and down steps, and the more granular the lattice values will be.

An example is in order. Figure 6.11 shows the example of a simple European *financial* call option. Suppose the call option has an asset value of \$100 and a strike price of \$100 expiring in one year. Further, suppose that the corresponding risk-free rate is 5 percent and the calculated volatility of historical logarithmic returns is 25 percent. Because the option pays no dividends and is only exercisable at termination, a Black-Scholes equation will suffice. The call option value calculated using the Black-Scholes equation is \$12.3360, which is obtained by

$$Call = S\Phi\left[\frac{\ln(S/X)+(rf+\sigma^2/2)T}{\sigma\sqrt{T}}\right] - Xe^{-rf(T)}\Phi\left[\frac{\ln(S/X)+(rf-\sigma^2/2)T}{\sigma\sqrt{T}}\right]$$

$$Call = 100\Phi\left[\frac{\ln(100/100)+(0.05+0.25^2/2)1}{0.25\sqrt{1}}\right]$$
$$-100e^{-0.05(1)}\Phi\left[\frac{\ln(100/100)+(0.05-0.25^2/2)1}{0.25\sqrt{1}}\right]$$

$$Call = 100\Phi[0.325] - 95.13\Phi[0.075] = 100(0.6274) - 95.13(0.5298) = 12.3360$$

Note that the standard-normal (Φ) distribution can be computed in Excel using the function NORMSDIST.

A binomial lattice can also be applied to solve this problem, as seen in the example in Figures 6.12 and 6.13.

- Example of a European *financial* call option with an asset value (S) of \$100, a strike price (X) of \$100, a 1 year expiration (T), 5% risk free rate (r), and 25% volatility (σ) with no dividend payments
- Using the Black-Scholes equation, we obtain \$12.3360

$$Call = S\Phi\left[\frac{\ln(S/X)+(r+\sigma^2/2)T}{\sigma\sqrt{T}}\right] - Xe^{-rT}\Phi\left[\frac{\ln(S/X)+(r-\sigma^2/2)T}{\sigma\sqrt{T}}\right]$$

- Using a 5 step Binomial approach, we obtain \$12.79
  - Step I in the Binomial approach:

$$\text{Given } S = 100, X = 100, \sigma = 0.25, T = 1, rf = 0.05$$
$$u = e^{\sigma\sqrt{\delta t}} = 1.1183 \text{ and } d = e^{-\sigma\sqrt{\delta t}} = 0.8942$$
$$p = \frac{e^{rf(\delta t)} - d}{u - d} = 0.5169$$

FIGURE 6.11 European Option Example

The first step is to solve the binomial lattice equations, that is, to calculate the up step size, down step size, and risk-neutral probability. This assumes that the step size (δt) is 0.2 years (one-year expiration divided by five steps). The calculations proceed as follows:

$$u = e^{\sigma\sqrt{\delta t}} = e^{0.25\sqrt{0.2}} = 1.1183$$

$$d = e^{-\sigma\sqrt{\delta t}} = e^{-0.25\sqrt{0.2}} = 0.8942$$

$$p = \frac{e^{rf(\delta t)} - d}{u - d} = \frac{e^{0.05(0.2)} - 0.8942}{1.1183 - 0.8942} = 0.5169$$

Figure 6.12 illustrates the first lattice in the binomial approach. In a real options world, this lattice is created based on the evolution of the underlying asset's sum of the present values of future cash flows. However, in a financial option analysis, this is the \$100 initial stock price level. This \$100 value evolves over time due to the volatility that exists. For instance, the \$100 value becomes \$111.8 (\$100 × 1.118) on the upper bifurcation at the first time period and \$89.4 (\$100 × 0.894) on the lower bifurcation. This up and down compounding effect continues until the end terminal node, where given a 25 percent annualized volatility, stock prices can, after a period of five years, be anywhere between \$57.2 or \$174.9. Recall that if volatility is zero,

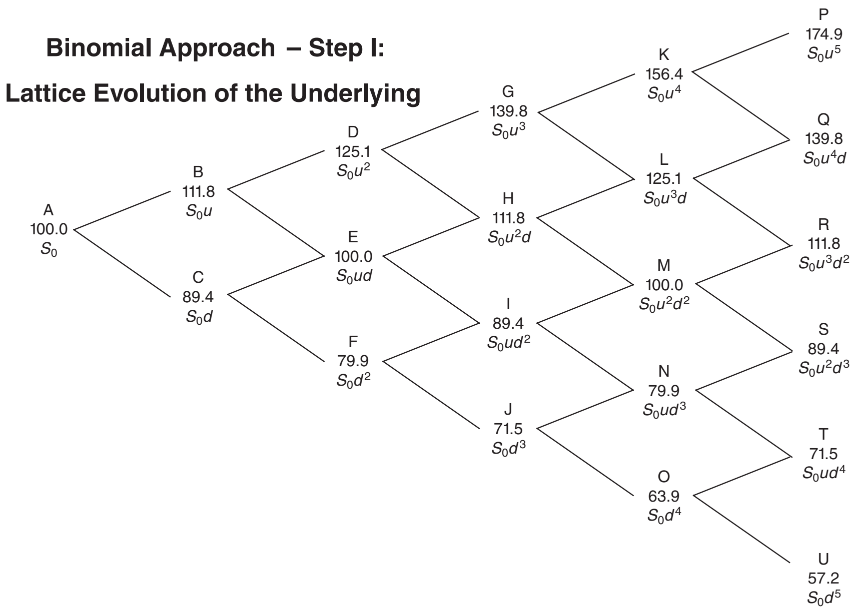


FIGURE 6.12 European Option Underlying Asset Lattice

then the lattice collapses into a straight line, where at every time-step interval, the value of the stock will be \$100. It is when uncertainty exists that stock prices can vary within this \$57.2 to \$174.9 interval. See Appendix 7I for computing the objective probabilities of certain nodes occurring.

Notice on the lattice in Figure 6.12 that the values are path-independent. That is, the value on node H can be attained through the multiplication of S_0u^2d, which can be arrived at by going through paths ABEH, ABDH, or ACEH. The value of path ABEH is $S \times u \times d \times u$, the value of path ABDH is $S \times u \times u \times d$, and the value of path ACEH is $S \times d \times u \times u$, all of which yields S_0u^2d.

Figure 6.13 shows the calculation of the European option's valuation lattice. The valuation lattice is calculated in two steps, starting with the terminal node and then the intermediate nodes, through a process called *backward induction*. For instance, the circled terminal node shows a value of \$74.9, which is calculated through the maximization between executing the option and letting the option expire worthless if the cost exceeds the benefits of execution. The value of executing the option is calculated as \$174.9 − \$100, which yields \$74.9. The value \$174.9 comes from Figure 6.12's (node P) lattice of the underlying, and \$100 is the cost of executing the option, leaving a value of \$74.9.

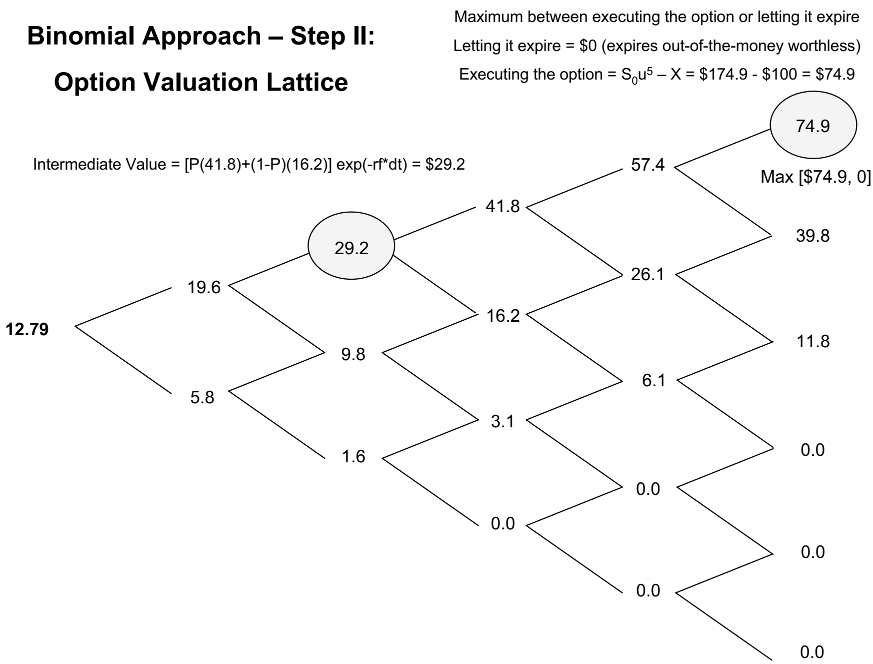


FIGURE 6.13 European Option Valuation Lattice

The second step is the calculation of intermediate nodes. The circled intermediate node illustrated in Figure 6.13 is calculated using a risk-neutral probability analysis. Using the previously calculated risk-neutral probability of 0.5169, a backward induction analysis is obtained through

$$[(p)up + (1 - p)down]\exp[(-riskfree)(\delta t)]$$

$$[(0.5169)41.8 + (1 - 0.5169)16.2]\exp[(-0.05)(0.2)] = 29.2$$

Using this backward induction calculation all the way back to the starting period, the option value at time zero is calculated as \$12.79.

Figure 6.14 shows a series of calculations using a Black-Scholes closed-form solution, binomial lattices with different time-steps, and Monte Carlo simulation. Notice that for the binomial lattice, the higher the number of time-steps, the more accurate the results become. At the limit, when the number of steps approaches infinity—that is, the time between steps (δt) approaches zero—the discrete simulation in a binomial lattice approaches that of a continuous simulation model, which is the closed-form solution. The famous Black-Scholes model is applicable here because there are no dividend payments and the option is only executable at termination. When the number of steps approaches 1,000, the results converge. However, in most cases, the level of accuracy becomes sufficient when the number of steps reaches anywhere from 100 to 1,000. Notice that the third method, using Monte Carlo simulation, also converges at 10,000 simulations with 100 steps.

Figure 6.15 shows another concept of binomial lattices. When there are more time-steps in a lattice, the underlying lattice shows more granularities and, hence, higher accuracy. The first lattice shows five steps and the second 20 steps (truncated at 10 steps due to space limitations). Notice the similar values that occur over time. For instance, the value 111.83 in the first lattice

- Comparison of approaches
  - Black-Scholes: \$12.3360
  - Binomial:
    - N = 5 steps \$12.7946 OVERESTIMATES
    - N = 10 steps \$12.0932
    - N = 20 steps \$12.2132
    - N = 50 steps \$12.2867 UNDERESTIMATES
    - N = 100 steps \$12.3113
    - N = 1,000 steps \$12.3335
    - N = 10,000 steps \$12.3358
    - N = 50,000 steps \$12.3360 EXACT VALUE
  - Simulation: (10,000 simulations: \$12.3360)

FIGURE 6.14 More Time-Steps, Higher Accuracy

5 TIME-STEPS

100.00	111.83	125.06	139.85	156.39	174.90
	89.42	100.00	111.83	125.06	139.85
		79.96	89.42	100.00	111.83
			71.50	79.96	89.42
				63.94	71.50
					57.18

20 TIME-STEPS

100.00	105.75	111.83	118.26	125.06	132.25	139.85	147.89	156.39	165.39	174.90
	94.56	100.00	105.75	111.83	118.26	125.06	132.25	139.85	147.89	156.39
		89.42	94.56	100.00	105.75	111.83	118.26	125.06	132.25	139.85
			84.56	89.42	94.56	100.00	105.75	111.83	118.26	125.06
				79.96	84.56	89.42	94.56	100.00	105.75	111.83
					75.62	79.96	84.56	89.42	94.56	100.00
						71.50	75.62	79.96	84.56	89.42
							67.62	71.50	75.62	79.96
								63.94	67.62	71.50
									60.46	63.94
										57.18

FIGURE 6.15 More Steps, More Granularity, More Accuracy

occurs at step 1 versus step 2 in the second lattice. All the values in the first lattice recur in the second lattice, but the second lattice is more granular in the sense that more intermediate values exist. As seen in Figure 6.14, the higher number of steps means a higher precision due to the higher granularity.

AN INTUITIVE LOOK AT THE BINOMIAL EQUATIONS

The following discussion provides an intuitive look into the binomial lattice methodology. Although knowledge of some stochastic mathematics and Martingale processes is required to fully understand the complexities involved even in a simple binomial lattice, the more important aspect is to understand how a lattice works, intuitively, without the need for complicated math.

Recall that there are two sets of key equations to consider when calculating a binomial lattice. These equations, shown in Figure 6.16, consist of an *up/down* equation (which is simply the discrete simulation's step size in a binomial lattice used in creating a lattice of the underlying asset) and a *risk-neutral probability* equation (used in valuing a lattice through backward induction). These two sets of equations are consistently applied to all real options binomial modeling regardless of its complexity.[5] In Figure 6.16, we see that the up step size (u) is shown as $u = e^{\sigma\sqrt{\delta t}}$, and the down step size (d)

$$u = e^{\sigma\sqrt{\delta t}} \text{and } d = e^{-\sigma\sqrt{\delta t}} = \frac{1}{u}$$

$$p = \frac{e^{(rf-b)(\delta t)} - d}{u-d}$$

- The first equation is simply a discrete simulation step size used in the first lattice of the underlying.
- The second equation is a risk-neutral probability calculation.

FIGURE 6.16 The Lattice Equations

is shown as $d = e^{-\sigma\sqrt{\delta t}}$, where σ is the volatility of logarithmic cash flow returns and δt is the time-step in a lattice. The risk-neutral probability (p) is shown as

$$p = \frac{e^{(rf-b)\delta t} - d}{u-d}$$

where rf is the risk-free rate in percent, and b is the continuous dividend payout in percent.

The intuition behind the lattice equations is somewhat more cumbersome but is nonetheless important. An analyst must not only have the mathematical aptitude but also the ability to explain what goes on behind the scenes when calculating a real options model. Figures 6.17 and 6.18 provide an intuitive look and feel of the derivation of the binomial lattice equations in a very simplified and intuitive format, as opposed to using cumbersome financial mathematics.

As Figure 6.17 shows, in the deterministic case where uncertainty is not built into a financial valuation model, future cash flows can be forecast using regression analysis on historical data, using time-series analysis, or using management assumptions. However, in a stochastic case when uncertainty exists and is built into the model, several methods can be applied, including simulating a Brownian Motion. As seen earlier, Brownian Motion processes are used in financial forecasting and option pricing models.

Starting with an Exponential Brownian Motion, where

$$\frac{\delta S}{S} = e^{\mu(\delta t) + \sigma\varepsilon\sqrt{\delta t}}$$

we can segregate the process into a deterministic and a stochastic part, where we have

$$\frac{\delta S}{S} = e^{\mu(\delta t)}e^{\sigma\varepsilon\sqrt{\delta t}}$$

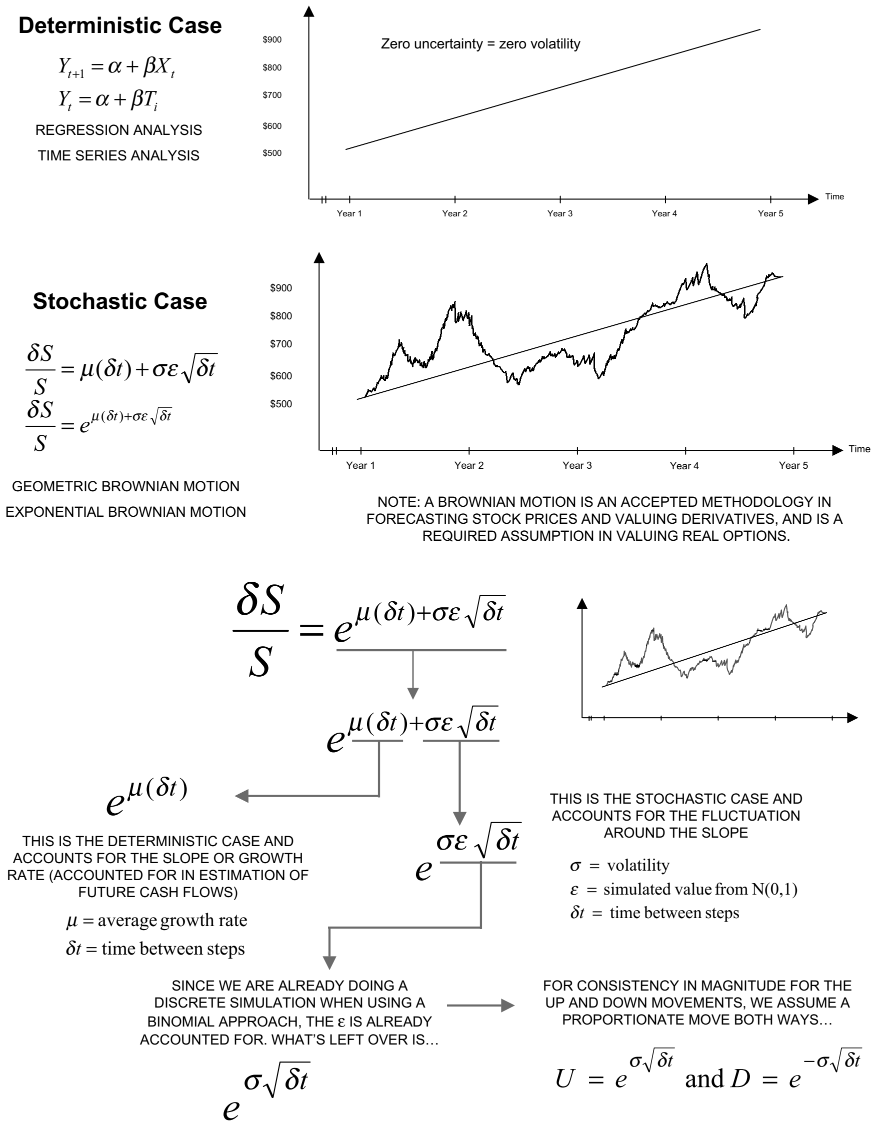


FIGURE 6.17 Up and Down Lattice Equations

The deterministic part of the model ($e^{\mu(\delta t)}$) accounts for the slope or growth rate of the Brownian process. If you recall, in real options analysis, the underlying asset variable (usually denoted *S* in options modeling) is the sum of the present values of future free cash flows, which means that the growth rates or slope in cash flows from one period to the next have already been intuitively accounted for in the discounted cash flow analysis.[6] Hence, we only have to account for the stochastic term ($e^{\sigma\varepsilon\sqrt{\delta t}}$), which has a highly variable simulated term (ε).

The stochastic term ($e^{\sigma\varepsilon\sqrt{\delta t}}$) has a volatility component (σ), a time component (δt), and a simulated component (ε). Again, recall that the binomial lattice approach is a discrete simulation model; we no longer need to resimulate at every time period, and the simulated variable (ε) drops out. The remaining stochastic term is simply $e^{\sigma\sqrt{\delta t}}$.

Finally, in order to obtain a recombining binomial lattice, the up and down step sizes have to be symmetrical in magnitude. Hence, if we set the up step size as $e^{\sigma\sqrt{\delta t}}$, we can set the down step size as its reciprocal, or $e^{-\sigma\sqrt{\delta t}}$.

These up and down step sizes are used in the creation of a lattice evolution of the underlying asset, the first step in a real options binomial modeling approach. Notice that the values on the lattice evolution of the underlying depend on nothing more than the volatility and time-steps between nodes. Each up and down jump size is identical no matter how far out on the lattice you go, but the cumulative effects of these jumps increase over time. That is, the up (u) value in Figure 6.16 is the same no matter which node you are on. However, the further out one goes, the cumulative effects (u^3 or u^2d, etc.) increase at the rate of $e^{\sigma\sqrt{\delta t}}$ or $e^{-\sigma\sqrt{\delta t}}$. This means that the higher the volatility, the wider the range of observed values on the lattice. In addition, the lower the value of the time-steps, the more granular and detailed the lattice becomes, as shown in Figure 6.15.

The second equation for the binomial model is that of a risk-neutral probability. The risk-neutral probability is defined in Figure 6.18 as

$$p = \frac{e^{(rf-b)\delta t} - d}{u - d}$$

Figure 6.18 shows an intuitive derivation of the risk-neutral probability, and Figure 6.19 explains what a risk-neutral probability is and what it does. Start with a simple example of a coin toss, where heads would yield a \$1 payoff and tails would yield a \$0 payoff. Assuming you start with a fair coin, the expected payoff for this game would be \$0.50 = 50%(\$1) + 50%(\$0). That is, the game has a value of \$0.50, where if you were risk-neutral, you would be indifferent between betting \$0.50 on the game and walking away. If you are risk-taker, you would be willing to bet more than \$0.50 on the game, and a risk-adverse person would probably only enter into the game if the cost of entry is less than the \$0.50 expected payoff.

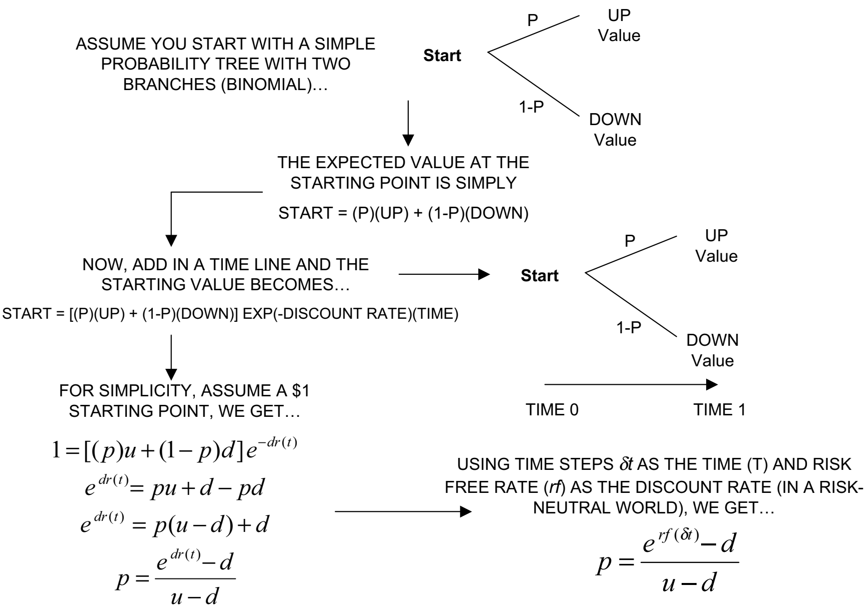


FIGURE 6.18 Risk-Neutral Probability Equation

Figure 6.18 shows a similar problem using a decision node with two bifurcations and their associated probabilities of occurrence. The expected value of the binomial tree is calculated the same way as the coin toss game just described, where the expected value of the starting point is simply (p) *up* + $(1-p)$ *down*. Now, if a time line is added to the analysis—that is, if the game takes time t (e.g., a whole year) to complete—the game payoffs should be discounted for the time value of money. If the payouts are not guaranteed values but have some risk associated with their levels, then they should be discounted at a market risk-adjusted discount rate. That is, the expected starting present value of the payoffs should be $[(p)\,up + (1-p)\,down]exp(-discount\ rate)(time)$.[7] If we define dr as discount rate, t as time, u as the payoff in the event of an up condition, and d for the payoff in the event of a down condition on the binomial branch, the starting present value of this problem can be shown as $Start = [(p)u + (1-p)d]e^{-dr(t)}$.

For simplicity, if we assume that the starting value is unity, a basic and well-accepted assumption that is used in option pricing models, then we can rewrite the starting value as $1 = [(p)u + (1-p)d]e^{-dr(t)}$. Multiplying both sides with the reciprocal of $e^{-dr(t)}$ yields $(p)u + (1-p)d = e^{dr(t)}$. Expanding and regrouping the terms yield $p(u-d) + d = e^{dr(t)}$, and solving for p yields

$$p = \frac{e^{dr(t)} - d}{u - d}$$

This risk-neutral probability is simply the solution for the probabilities on a binomial lattice. As in the binomial lattice paradigm, the time is simply the time-steps between nodes; we can denote t as δt. In addition, as will be explained later, this probability p is used in a risk-neutral world, a world where risks have already been accounted for; hence, the discount rate dr is simply the risk-free rate rf. Replacing these values, we get the binomial equation

$$p = \frac{e^{rf(\delta t)} - d}{u - d}$$

However, when there are continuous streams of dividend present, this risk-free rate is modified to risk-free rate less the dividend yield ($rf - b$).

FROLICKING IN A RISK-NEUTRAL WORLD

A risk-neutral world simply means that a certain variable is stripped of its risks. In our example, the certain variable is the cash flow payouts. These cash flow payouts can be stripped off their risks or, in common finance language, discounted of risks by risk-adjusting in two ways. The first method is simply to risk-adjust the cash flow payouts themselves. This implies the use of a discounted cash flow method, applying the appropriate market risk-adjusted discount rate, which is typically higher than the risk-free rate. The second method is to adjust the probabilities that lead to the payouts, then, using the original cash flows, discount them by the risk-free rate, not a market risk-adjusted rate as risk has already been accounted for by the adjusted probabilities and should not be double-counted. This implies the use of risk-neutral probabilities in the binomial world. Both approaches yield the same results when applied appropriately. As discussed earlier in this chapter, volatility, discount rates, and probabilities can be imputed from one another. In the risk-neutral approach, the risk-neutral probability is imputed from volatility. And because a relevant discount rate is implied from this volatility, discounting the cash flow with the risk-free rate to account for time value of money and then discounting the risk (volatility) using the risk-neutral probability (imputed from volatility) yields the same result as discounting the cash flow with a single market-risk-adjusted discount rate (which is nothing but a risk-free rate plus a risk premium).

Figure 6.19 illustrates both these risk-adjustment methods. For instance, if the discount rate is 22.08 percent and the payoff occurs after one year, the expected present value of the coin-toss game is $[50\%(\$1) + 50\%(\$0)]$ $exp[(-22.08\%)(1)] = \$0.40$. This \$0.40 is the risk-adjusted value of the game in present dollars, as compared to the \$0.50 if the payoffs are immediate. This is intuitive because a payoff that is risky and may or may not happen in a year is certainly worth less than a payoff that is certain and occurs

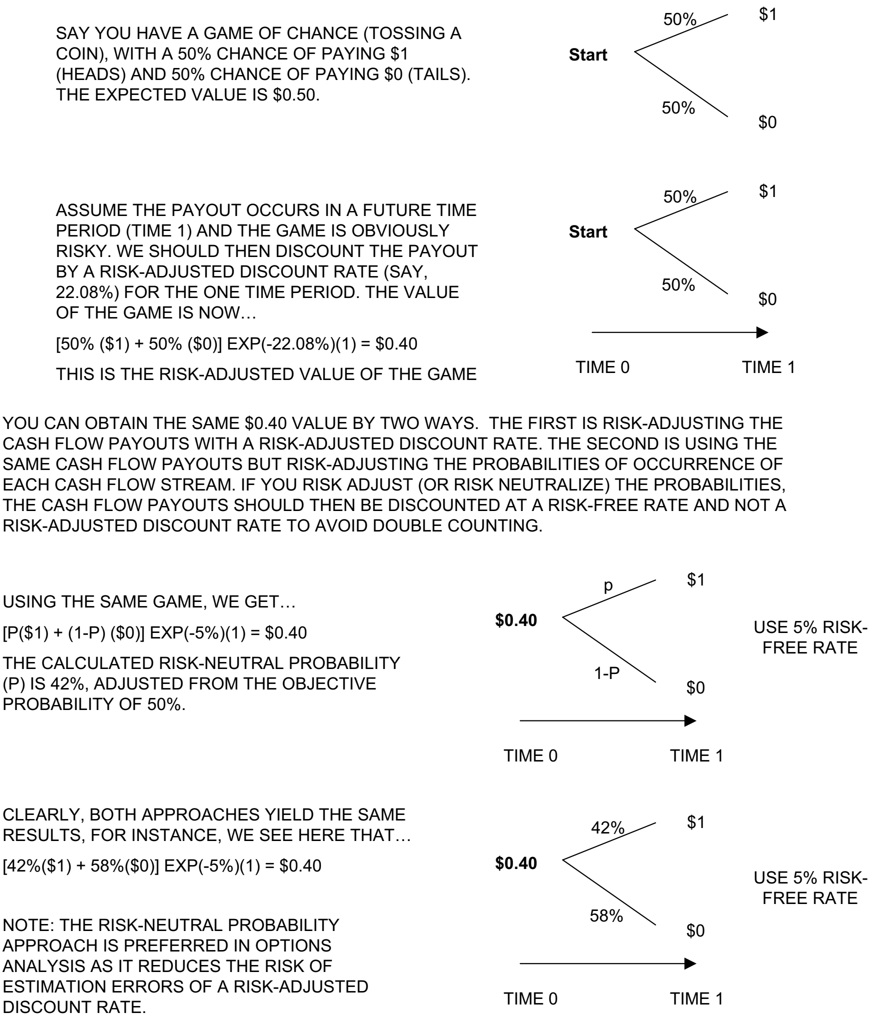


FIGURE 6.19 A Risk-Neutral World

immediately. A player should be willing to enter into a bet only if the cost of entry is lower than what the payoff is worth. This method is akin to the discounted cash flow approach where the cash flows are adjusted for risk and time by discounting them by the appropriate risk-adjusted discount rate.

Figure 6.19 also illustrates the second method, using risk-neutral probabilities. Using the same game parameters, the risk-neutral probabilities can be calculated. That is, as the expected value is calculated as $0.40, we can get p by imputing the expected value using $[(p)\$1 + (1-p)\$0]exp[(-5\%)(1)]$, where the risk-neutral probability p is calculated as 42 percent, compared to the original objective probability of 50 percent. By adjusting the probabilities for risk, the cash flow payoffs should then be discounted using the risk-free rate of 5 percent. Notice that using this imputed 42 percent risk-neutral probability, we can also calculate the expected present value of the cash flows through $[42\%(\$1) + 58\%(\$0)]exp[(-5\%)(1)] = \$0.40$, the same value obtained through discounting the cash flows.

The upshot is that a risky series of cash flows should be adjusted for risk, and two methods exist to perform the risk adjustment. The cash flow series themselves can be adjusted through a risk-adjusted discount rate; or the probabilities leading to the cash flows can be adjusted and the resulting adjusted cash flows can be discounted using a risk-free rate. The former approach is well known and widely used in discounted cash flow models and the latter for solving binomial lattices. The latter is preferred for real options analysis as it avoids having to estimate project-specific discount rates at different nodes along the binomial lattice or within the context of a decision tree analysis.

For instance, if a decision tree analysis is used (which by itself is insufficient for solving real options), then different discount rates have to be estimated at each decision node at different times because different projects at different times have different risk structures. Estimation errors will then be compounded on a large decision tree analysis. Binomial lattices using risk-neutral probabilities avoid this error. In addition, risk-free rates are objective and easy to obtain, and because volatility is obtained from a robust Monte Carlo simulation approach, the imputed risk-neutral probability is more accurate, compared to guessing at the discount rate. Also, the discount rate requires a market benchmark that may or may not exist in the real options world (e.g., the beta coefficient is covariance divided by the variance of an external or comparable market, to compute the CAPM discount rate).

One major conclusion that can be drawn using binomial lattices is that because risk-adjusting cash flows provides the same results as risk-adjusting the probabilities leading to those cash flows, the results stemming from a discounted cash flow analysis are identical to those generated using a binomial lattice. The only condition that is required is that the volatility of the cash flows be zero—in other words, the cash flows are assumed to be known with certainty. Because zero uncertainty exists, there is zero strategic option value, meaning that the net present value of a project is identical to its expanded net present value. Figure 6.20 illustrates this point.

Given the levels of cash flow series in Figure 6.20, the net present value is calculated to be \$1,426 after being discounted using a weighted average cost of capital of 35 percent. This is essentially the first approach where cash flows are risk-adjusted by this 35 percent market risk-adjusted discount rate.

The second approach is the use of a binomial lattice. Notice that the starting point on a binomial lattice is the present value of future cash flows; we

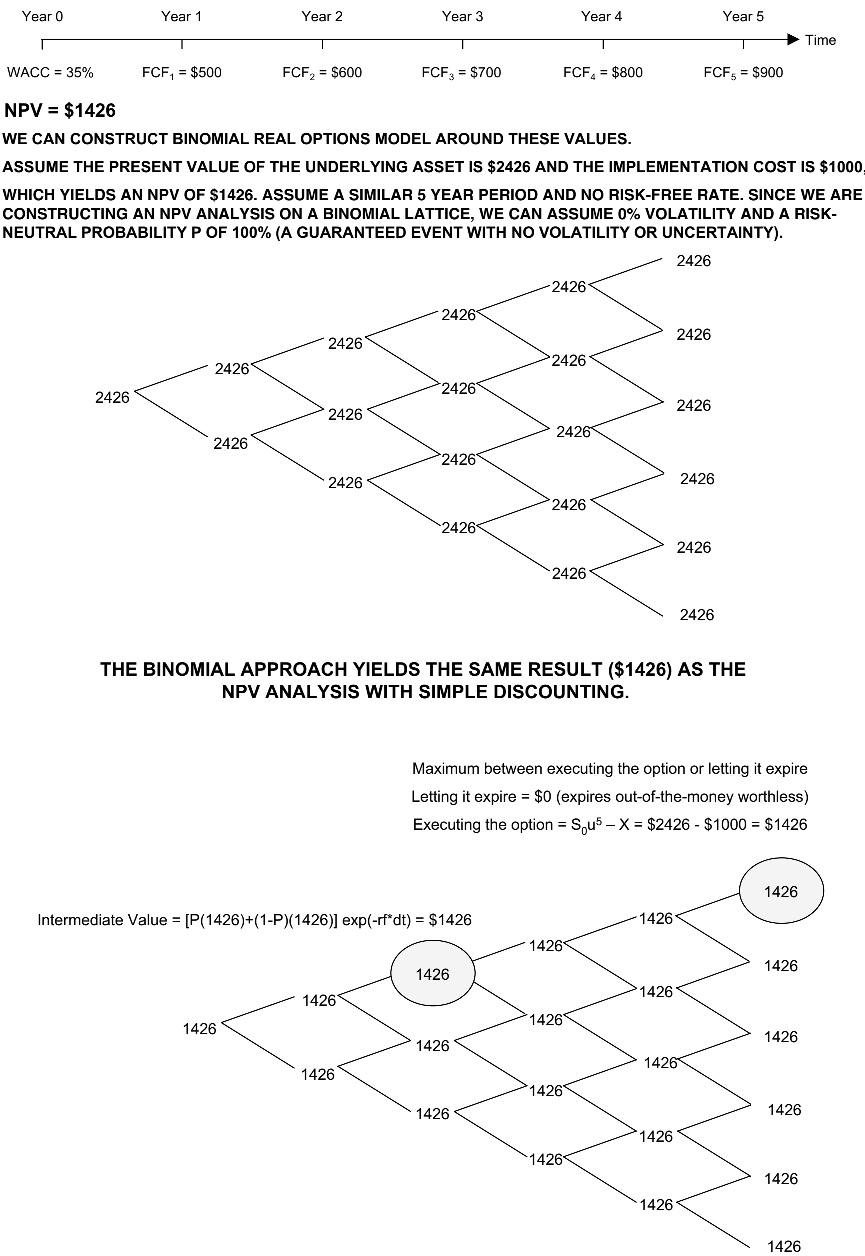


FIGURE 6.20 Solving a DCF Model with a Binomial Lattice

arbitrarily set it as \$2,426, with a corresponding \$1,000 implementation cost. This is acceptable as long as the net present value yields *\$1,426* (*\$2,426 − \$1,000*). Starting with this \$2,426 value, the binomial equations are calculated. First, the up and down step sizes are calculated using $u = e^{\sigma\sqrt{\delta t}} = e^{0\%\sqrt{1}} = 1$ and $d = e^{-\sigma\sqrt{\delta t}} = e^{-0\%\sqrt{1}} = 1$ because volatility is assumed to be 0 percent, and five steps are used for the five years, resulting in a time-step δt of 1. In addition, the risk-neutral probability is 100 percent. Recall from Figure 6.10 that a zero volatility lattice collapses into a straight line, there is no up or down step, hence, the risk-neutral probability is 100 percent. The first binomial lattice shown in Figure 6.20 illustrates this situation, where the asset evolutions in all future states are identical to the starting value.

The second lattice shows the valuation of the binomial model. The terminal nodes are simply the maximization between executing the option or letting it expire. The value of executing the option is \$2,426 − \$1,000 at every terminal node, and the value of letting the option expire is \$0. All intermediate nodes carry the value of the option going forward, similar to the European call option. For simplicity, assume a negligible risk-free rate. That is, the value of $[(p)\$1{,}426 + (1-p)\$1{,}426]exp[(-0\%)(1)] = \$1{,}426$ at each intermediate node, going back to the starting value. In this highly simplified and special case, the calculated net present value is identical to the value calculated using a binomial lattice approach. In essence, a real options analysis is, at its most basic level, similar in nature to the net present value analysis.

Figure 6.21 illustrates this condition. In a traditional financial analysis, we usually calculate the net present value, which is nothing but benefits less cost (first equation)—that is, benefits equal the sum of the present values of future net cash flows after taxes, discounted at some market risk-adjusted cost of capital; and cost equals the sum of the present values of investment costs discounted at the risk-free rate.

Management is usually knowledgeable of net present value and the way it is calculated. Conventional wisdom is such that if benefits outweigh costs—that is, when the net present value is positive—one would be inclined to accept a particular project. This is simple and intuitive enough. However, when we turn to options theory and look at a simple call option, it is also nothing but benefits less cost (second equation), with a slight modification.

The difference is the introduction of the $\Phi(d)$ multipliers behind benefits and costs. Obviously, the multipliers are nothing but the respective probabilities of occurrence, obtained through the discrete simulation process in binomial lattices. Hence, in real options theory, one can very simply define the value of an option as nothing more than benefits less costs, taking into account the risk or probabilities of occurrence for each variable, similar to using the Black-Scholes model. In fact, the Black-Scholes model can be simplified to the second option equation in Figure 6.21. Therefore, if there is no uncertainty and the volatility is zero, which means that the probability of occur-

$$NPV = Benefits - Cost$$
$$Option = Benefits\ \Phi(d_1) - Cost\ \Phi(d_2)$$
$$eNPV = NPV + Options\ Value$$

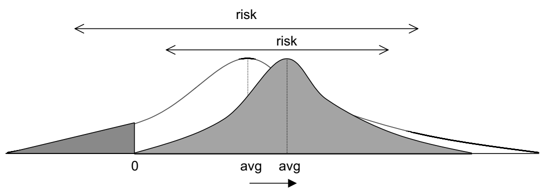


FIGURE 6.21 Real Options and Net Present Value

rence is 100 percent, indicating that the forecast values are guaranteed to occur, as in the special case, then the real options value collapses into the net present value when both $\Phi(d) = 100\%$. It is easy to understand that option value in this case is far superior to the net present value analysis if uncertainty exists and volatility is not equal to zero, and hence, both $\Phi(d)$ are not equal to 100 percent. Finally, we can say that the expanded net present value ($eNPV$) or total strategic value shown as the third equation is the sum of the deterministic base case net present value and the strategic options value. The options value takes into account the value of flexibility, that is, the ability to execute on a strategic option but not the obligation to do so; the *eNPV* accounts for both base-case analysis and the added value of flexibility. Figure 6.21 illustrates that options can be used to hedge downside risk and to capitalize on the upside uncertainties. Thus, truncating the left tail of the distribution moves the mean (expected returns) to the right and reduces the standard deviation and width (risk). Hence, real options provide risk reduction and value enhancement to projects and assets.

SUMMARY

The binomial approach, partial-differential equations, and closed-form solutions are the mainstream approaches used in solving real options problems. The binomial approach is favored due to its mathematical simplicity and ease of exposition. It helps make the black box more transparent and, in turn, the results more palatable to senior management. In addition, the mathematics involved in calculating a binomial lattice—that is, the use of up/down jumps as well as risk-neutral probabilities—can be easily and intuitively explained without the use of often intractable stochastic mathematical techniques applied in partial-differential equations and closed-form solutions.

CHAPTER 6 QUESTIONS

1. Why does solving a real options problem using the binomial lattices approach the results generated through closed-form models?
2. Is real options analysis a special case of discounted cash flow analysis, or is discounted cash flow analysis a special case of real options analysis?
3. Explain what a risk-neutral probability means.
4. What is the difference between a recombining lattice and a nonrecombining lattice?
5. Using the example in Figures 6.12 through 6.14, create and value the same European option using 10 time-steps. Verify that your answers match those given in Figure 6.14.

CHAPTER 7

Real Options Models

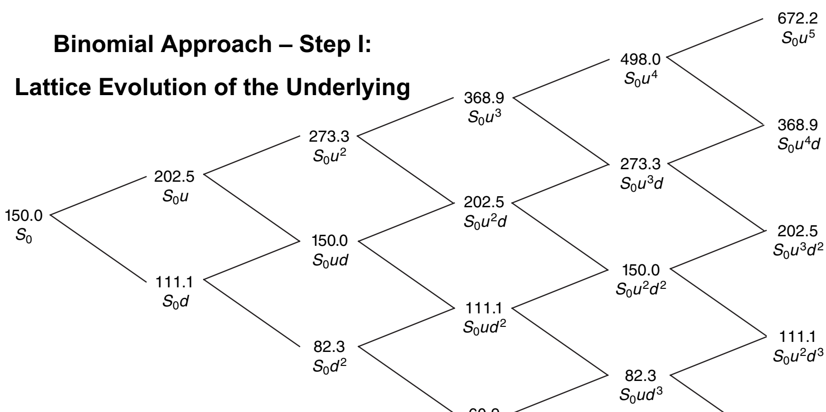
INTRODUCTION

This chapter provides step-by-step examples of solving real options models. The common types of real options solved include abandonment, expansion, contraction, chooser, switching, compound, changing strikes, and volatility options. Chapters 8 to 11 will discuss more advanced types of options including switching, timing, multiphased sequential compound, complex custom, and barrier options. The examples in this chapter are useful as building blocks for solving more complicated real options models. The examples used here are intentionally kept simple, for expositional purposes. More advanced technical examples are provided in the appendixes. These examples are again revisited in Chapters 9 to 11 and solved using the enclosed Real Options Valuation Super Lattice Solver software and Risk Simulator software.

OPTION TO ABANDON

Suppose a pharmaceutical company is developing a particular drug. However, due to the uncertain nature of the drug's development progress, market demand, success in human and animal testing, and FDA approval, management has decided that it will create a strategic abandonment option. That is, at any time period within the next five years of development, management can review the progress of the research and development effort and decide whether to terminate the drug development program. After five years, the firm would have either succeeded or completely failed in its drug development initiative, and there exists no option value after that time period. If the program is terminated, the firm can potentially sell off its intellectual property rights of the drug in question to another pharmaceutical firm with which it has a contractual agreement. This contract with the other firm is exercisable at any time within this time period, at the whim of the firm owning the patents. This option is obtained by the firm paying an amount up front to this counterparty. The question is: How much is this downside insurance or abandonment option worth in fair-market terms?

Using a traditional discounted cash flow model, the present value of the expected future cash flows discounted at an appropriate market risk-adjusted discount rate is found to be \$150 million. Using Monte Carlo simulation, the implied volatility of the logarithmic returns on future cash flows (see Appendix 7A) is found to be 30 percent. The risk-free rate on a riskless asset for the same time frame is 5 percent, and the abandonment contract stipulates that you will get \$100 million if the patents are sold within the next five years. For simplicity, assume that this \$100 million salvage value is fixed for the next five years. You attempt to calculate how much this abandonment option is worth and how much this drug development effort on the whole is worth to the firm. You decide to use a closed-form approximation of an American put option because the option to abandon the drug development can be exercised at any time up to the expiration date. You also decide to confirm the value of the closed-form analysis with a binomial lattice calculation. Figures 7.1 and 7.2 show the results of your analysis using a binomial approach. Using the Bjerksund closed-form American put option approximation equation (available in the Super Lattice Solver software), you calculate the value of the American option to abandon as \$6.9756 million. However, using the binomial approach, you calculate the value of the abandonment option as \$6.6412 million using 5 time-steps and \$7.0878 million using 1,000 time-steps, thereby verifying the results obtained.[1] An example of the first lattice, the lattice of the underlying asset, is shown in Figure 7.1.



Salvage value = \$100

Given: $S = 150,\ \sigma = 0.30,\ T = 5,\ rf = 0.05$

$u = e^{\sigma\sqrt{\delta t}} = 1.3499$ and $d = e^{-\sigma\sqrt{\delta t}} = 0.7408$

$$p = \frac{e^{rf(\delta t)} - d}{u - d} = 0.51$$

FIGURE 7.1 Abandonment Option (Underlying Asset Lattice)

All the required calculations and steps in Figure 7.1 are based on the up factor, down factor, and risk-neutral probability analysis previously alluded to in Chapter 6. The up factor is calculated to be 1.3499, and the down factor is 0.7408. Hence, starting with the underlying value of $150, we multiply this value with the up and down factors to obtain $202.5 and $111.1, respectively. Readers can verify for themselves the rest of the lattice calculations in Figure 7.1. The second step is to calculate the option valuation lattice as shown in Figure 7.2, using the values calculated in Figure 7.1's lattice evolution of the underlying asset.

Creating the option valuation lattice proceeds in two steps, the valuation of the terminal nodes and the valuation of the intermediate nodes using a process called *backward induction*. If you recall from the first lattice, the values are created in a forward multiplication of up and down factors, from left to right. For this second lattice, the calculation proceeds in a backward manner, starting from the terminal nodes. That is, the nodes at the end of the lattice are valued first, going from right to left.

In Figure 7.2, we see that the sample circled terminal node (denoted A) reveals a value of $672.2, which can be obtained through the value maximization of abandonment versus continuation. At the end of five years, the firm has the option to both sell off and abandon its existing drug program or to continue developing. Obviously, management will choose the strategy that

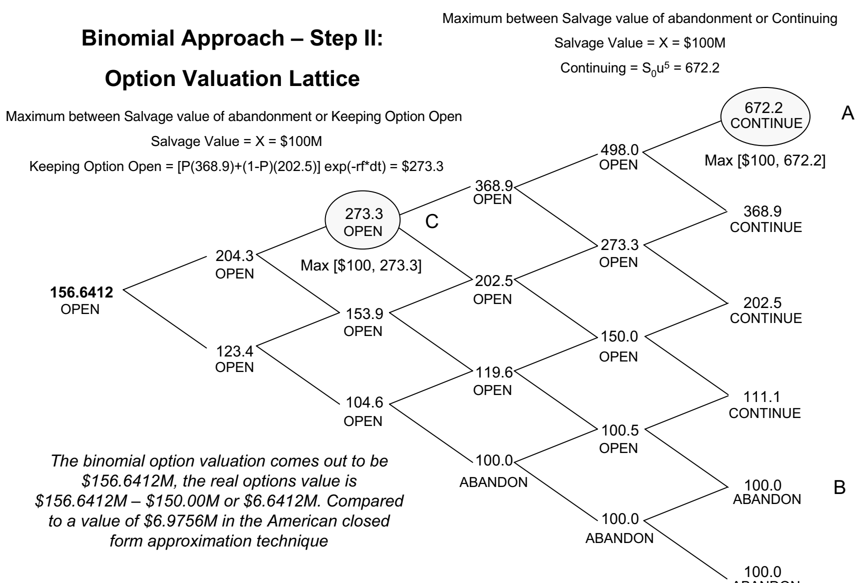


FIGURE 7.2 Abandonment Option (Valuation Lattice)

maximizes profitability. The value of abandoning the drug program is equivalent to selling the patent rights at the predetermined \$100 million value. The value of continuing with development can be found in Figure 7.1's lattice evolution of the underlying asset at the same node (S_0u^5), which is \$672.2 million. The profit-maximizing decision is to continue development; hence, we have the value \$672.2 million on that node (denoted A). Similarly, for the terminal node B in Figure 7.2, we see that the value of abandoning at that time is \$100 million as compared to \$60.9 in Figure 7.1. Hence, the decision at that node is to abandon the project, and the profit-maximizing value of that node becomes the abandonment value of \$100 million. This is very easy to understand because if the underlying asset value of pursuing the drug development is high (node A), it is wise to continue with the development. Otherwise, if circumstances force the value of the development effort down to such a low level as specified by node B, then it is more optimal to abandon the project and cut the firm's losses (development is failing, competitor has already developed the drug, the market is shrinking, and so forth). This of course assumes that management will execute the optimal profit-maximizing behavior of abandoning the project when it is optimal to do so rather than hanging on to it.

Moving on to the intermediate nodes, we see that node C is calculated as \$273.3 million. At this particular node, the firm again has two options, to abandon at that point or not to abandon, thereby keeping the option to abandon open and available for the future in the hopes that when things seem less rosy, the firm has the ability to execute the option and abandon the development program. The value of abandoning is again the \$100 million in salvage value. The value of continuing is simply the discounted weighted average of potential future option values using the risk-neutral probability.

Because the risk adjustment is performed on the probabilities of future option cash flows, the discounting can be done using the risk-free rate. That is, for the value of keeping the option alive and open, we have $[(P)(\$368.9) + (1 - P)(\$202.5)]exp[(-rf)(\delta t)] = \$273.3 \text{ million}$, which is higher than the abandonment value. This assumes a 5 percent risk-free rate *rf*, a time-step δt of 1 (five years divided into five time-steps means each time-step is equivalent to one year), and a risk-neutral probability P of 0.51. Using this backward induction technique, the lattice is calculated back to the starting point to obtain the value of \$156.6412 million. Because the value obtained through a discounted cash flow is \$150 million, we can say that the difference of \$6.6412 million additional value is due to the abandonment option.

By having a safety net or way out for management given dire circumstances, the project is worth more than its static value of \$150 million. The \$150 million is the static NPV without flexibility, the \$6.6412 million is the real options value, and the combined value of \$156.6412 million is the total strategic value or ENPV (expanded NPV) or NPV+O (NPV with real options flexibility), the correct total value of this drug development program. Clearly, modifications to the lattice analysis can be done to further mirror actual business conditions. For instance, the abandonment salvage value can change over time, which can simply be instituted through changing the salvage amount at the appropriate times with respect to the nodes on the lattice. This could be an inflation adjustment, a growth or decline in the value of the intellectual property over time, etc. Chapter 10 illustrates some of these modifications to the abandonment option (e.g., blackouts, Bermudan options, changing parameters, and so forth) using the Super Lattice Solver software.

OPTION TO EXPAND

Suppose a growth firm has a static valuation of future profitability using a discounted cash flow model (that is, the present value of the expected future cash flows discounted at an appropriate market risk-adjusted discount rate) is found to be $400 million. Using Monte Carlo simulation, you calculate the implied volatility of the logarithmic returns on the projected future cash flows to be 35 percent. The risk-free rate on a riskless asset for the next five years is found to be yielding 7 percent. Suppose that the firm has the option to expand and double its operations by acquiring its competitor for a sum of $250 million at any time over the next five years. What is the total value of this firm assuming you account for this expansion option?

You decided to use a closed-form approximation of an American call option because the option to expand the firm's operations can be exercised at any time up to the expiration date. You also decide to confirm the value of the closed-form analysis with a binomial lattice calculation. Figures 7.3 and 7.4 show the results of your analysis using a binomial approach. Using the Barone-Adesi-Whaley closed-form American call approximation equation, you estimate the gross benchmark value of the American option to expand as $626.6 million.[2] However, using the binomial approach, you calculate the value of the expansion option as $638.3 million using 5 time-steps and $638.8 using 1,000 time-steps, thereby verifying the results obtained. The results from the lattice approach are more accurate and should be used instead of the closed-form models. The reader can easily verify these results using the enclosed Super Lattice Solver software CD-ROM to run the binomial analysis for 1,000 steps.

All the required calculations and steps in Figure 7.3 are based on the up factor, down factor, and risk-neutral probability analysis previously alluded to in Chapter 6. The up factor is calculated to be 1.4191, and the down factor is 0.7047 as shown in Figure 7.3. Hence, starting with the underlying value of $400, we multiply this value with the up and down factors to obtain $567.6 and $281.9, respectively. Readers can verify the rest of the lattice calculations in Figure 7.3. Notice the similarities between the evolution lattice

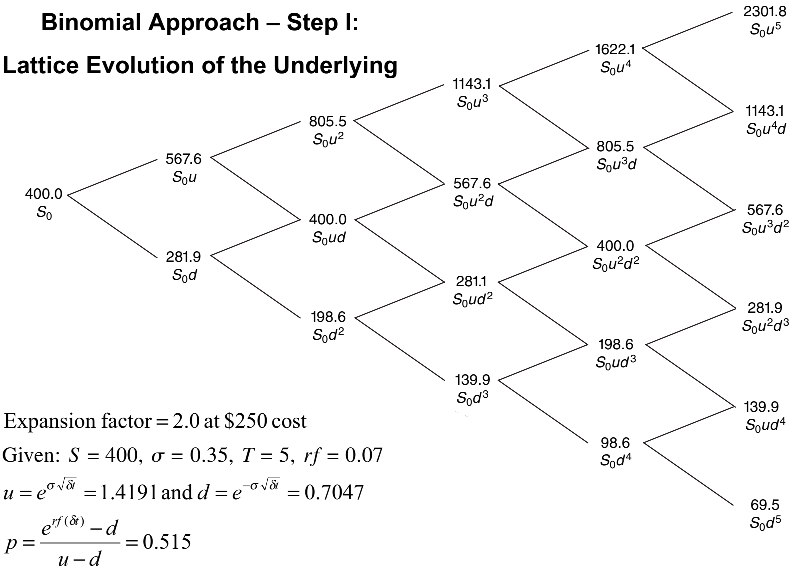


FIGURE 7.3 Expansion Option (Underlying Asset Lattice)

of the underlying asset for this expansion option and that of the abandonment option.

The second step is to calculate the option valuation lattice as shown in Figure 7.4, using the values calculated in Figure 7.3's lattice evolution of the underlying.

In Figure 7.4, we see that the sample circled terminal node (denoted D) reveals a value of $4,353.7, which can be obtained through the value maximization of expansion versus continuation. At the end of five years, the firm has the option to acquire the competition and expand its existing operations or not. Obviously, management will choose the strategy that maximizes profitability. The value of acquiring and expanding its operations is equivalent to doubling its existing capacity of $2,301.8 at the same node shown in Figure 7.3. Hence, the value of acquiring and expanding the firm's operations is double this existing capacity less any acquisition costs, or *2($2,301.8) − $250 = $4,353.7 million.*

The value of continuing with existing business operations can be found in Figure 7.3's lattice evolution of the underlying, at the same node (S_0u^5), which is $2,301.8 million. The profit-maximizing decision is to acquire the firm for $250 million, and hence, we have the value $4,353.7 million on that node (denoted D). Similarly, for the terminal node E in Figure 7.4, we see that the value of continuing existing operations at that time is $69.5 million as seen in Figure 7.3. In comparison, by expanding its operations through acquisition,

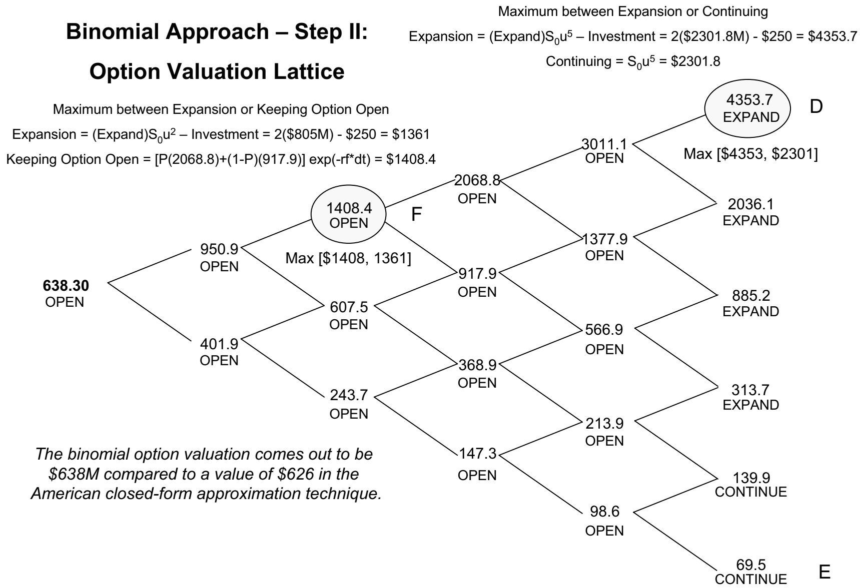


FIGURE 7.4 Expansion Option (Valuation Lattice)

the value is only *2($69.5) − $250 = −$111 million*. Hence, the decision at that node is to continue with existing operations without expanding, and the profit-maximizing value on that node is $69.5 million. This is intuitive because the underlying asset value of pursuing existing business operations is such that if it is very high based on current market conditions (node D), then it is wise to double the firm's operations through acquisition of the competitor. Otherwise, if circumstances force the value of the firm's operations down to such a low level as specified by node E, then it is more optimal to continue with the existing business and not worry about expanding because the project will be a loser at that point.

Moving on to the intermediate nodes, we see that node F is calculated as $1,408.4 million. At this particular node, the firm again has two options, to expand its operations at that point or to keep the option to expand open for the future in the hopes that when the market is up, the firm has the ability to execute the option and acquire its competitor. The value of expanding at that node is *2($805) − $250 = $1,361 million* (rounded). The value of continuing is simply the discounted weighted average of potential future option values using the risk-neutral probability. Because the risk adjustment is performed on the probabilities of future option cash flows, the discounting can be done using the risk-free rate. That is, for the value of keeping the option alive and open, we have $[(P)(\$2{,}068.8) + (1 - P)(\$917.9)]exp[(-rf)(\delta t)] =$

\$1,408.4 million, which is higher than the expansion value. This assumes a 7 percent risk-free rate *rf,* a time-step δt of 1, and a *P* of 0.515. Using this backward-induction technique, the lattice is calculated back to the starting point to obtain the value of \$638.30 million. As the value obtained through a discounted cash flow is \$400 million for current existing operations, the value of acquiring the competitor today is *2(\$400) − \$250 = \$550 million,* the value of twice its current operations less the acquisition costs.

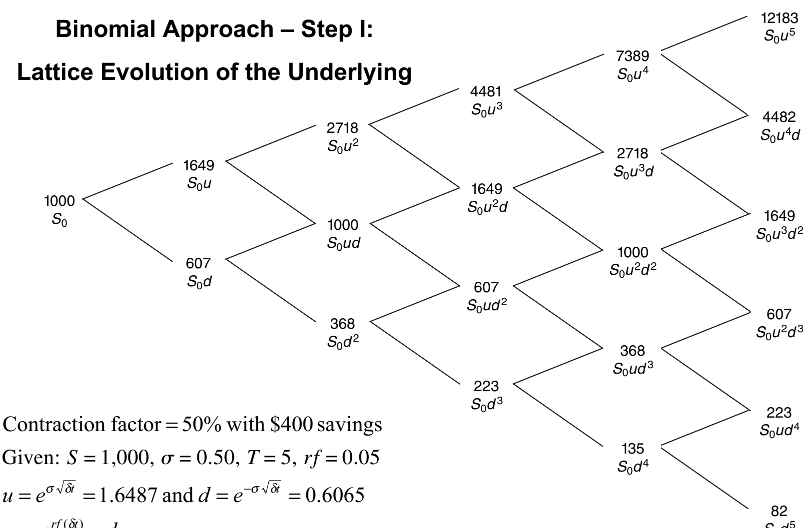
By not executing the acquisition today but still having an option for management given great market and economic outlook to acquire the competitor then, the firm is worth more than its static value of \$550 million. The \$550 million is the static NPV without flexibility, the \$88.30 million is the real options value, and the combined value of \$638.30 million is the total strategic value or ENPV (expanded NPV) or NPV$^+$O (NPV with real options flexibility), the correct total value of this firm. The real options value is worth an additional 16 percent of existing business operations. If a real options approach is not used, the firm will be undervalued because it has a strategic option to expand its current operations but not an obligation to do so and will most likely not do so unless market conditions deem it optimal. The firm has in essence hedged itself against any potential downside if it were to acquire the competitor immediately without regard for what may potentially happen in the future. Having an option and sometimes keeping this option open are valuable given a highly uncertain business environment. Clearly, to mirror actual business conditions, the cost of acquisition can change over time, and the expansion factor (doubling its operations) can also change as business conditions change. All these variables can be accounted for in the lattice.[3] Chapters 10 and 11 have more advanced expansion option examples solved using the Super Lattice Solver software.

OPTION TO CONTRACT

You work for a large aeronautical manufacturing firm that is unsure of the technological efficacy and market demand of its new fleet of long-range supersonic jets. The firm decides to hedge itself through the use of strategic options, specifically an option to contract 50 percent of its manufacturing facilities at any time within the next five years. Suppose the firm has a current operating structure whose static valuation of future profitability using a discounted cash flow model (that is, the present value of the expected future cash flows discounted at an appropriate market risk-adjusted discount rate) is found to be \$1 billion. Using Monte Carlo simulation, you calculate the implied volatility of the logarithmic returns on the projected future cash flows to be 50 percent. The risk-free rate on a riskless asset for the next five years is found to be yielding 5 percent. Suppose the firm has the option to contract 50 percent of its current operations at any time over the next five years, thereby creating an additional \$400 million in savings after this contraction. This is done through a legal contractual agreement with one of its vendors, who has agreed to take up the excess capacity and space of the firm, and at the same time, the firm can scale back its existing work force to obtain this level of savings.

A closed-form approximation of an American option can be used as a gross approximation, because the option to contract the firm's operations can be exercised at any time up to the expiration date and can be confirmed with a binomial lattice calculation. Figures 7.5 and 7.6 show the results of an analysis using a binomial approach. Using the Barone-Adesi-Whaley closed-form equation, you calculate the value of the American option to contract as \$102.23 million.[4] However, using the binomial approach, you calculate the value of the contraction option as \$105.61 million using 5 time-steps and \$102.98 million using 1,000 time-steps. Again, the results from the binomial lattices should be used as they are more accurate than the closed-form approximation models.

All the required calculations and steps in Figure 7.5 are based on the up factor, down factor, and risk-neutral probability analysis previously alluded to in Chapter 6. For instance, the up factor is calculated to be 1.6487, and the down factor is 0.6065 as shown in Figure 7.5. Hence, starting with the underlying value of \$1,000, we multiply this value by the up and down factors to obtain \$1,649 and \$607, respectively. Readers can verify for themselves the rest of the lattice calculations in Figure 7.5.



Contraction factor = 50% with \$400 savings

Given: $S = 1{,}000$, $\sigma = 0.50$, $T = 5$, $rf = 0.05$

$u = e^{\sigma\sqrt{\delta t}} = 1.6487$ and $d = e^{-\sigma\sqrt{\delta t}} = 0.6065$

$$p = \frac{e^{rf(\delta t)} - d}{u - d} = 0.427$$

FIGURE 7.5 Contraction Option (Underlying Asset Lattice)

The second step is to calculate the option valuation lattice as shown in Figure 7.6, using the values calculated in Figure 7.5's lattice evolution of the underlying asset.

In Figure 7.6, we see that the sample terminal node (denoted G) reveals a value of \$12,183, which can be obtained through the value maximization of contraction versus continuation. At the end of five years, the firm has the option to contract its existing operations or not, thereby letting the option expire. Obviously, management will choose the strategy that maximizes profitability. The value of contracting 50 percent of its operations is equivalent to half of its existing operations plus the \$400 million in savings. Hence, the value of contracting the firm's operations is *0.5(\$12,183) + \$400 = \$6,491 million*. The value of continuing with existing business operations can be found in Figure 7.5's lattice evolution of the underlying at the same node (S_0u^5), which is \$12,183 million. The profit-maximizing decision is to continue with the firm's current level of operations at \$12,183 million on that node (denoted G). Similarly, for the terminal node H in Figure 7.6, we see that the value of continuing existing operations at that time is \$82 million as seen in Figure 7.5. In comparison, by contracting its operations by 50 percent, the value is *0.5(\$82) + \$400 = \$441*. Hence, the decision at that node

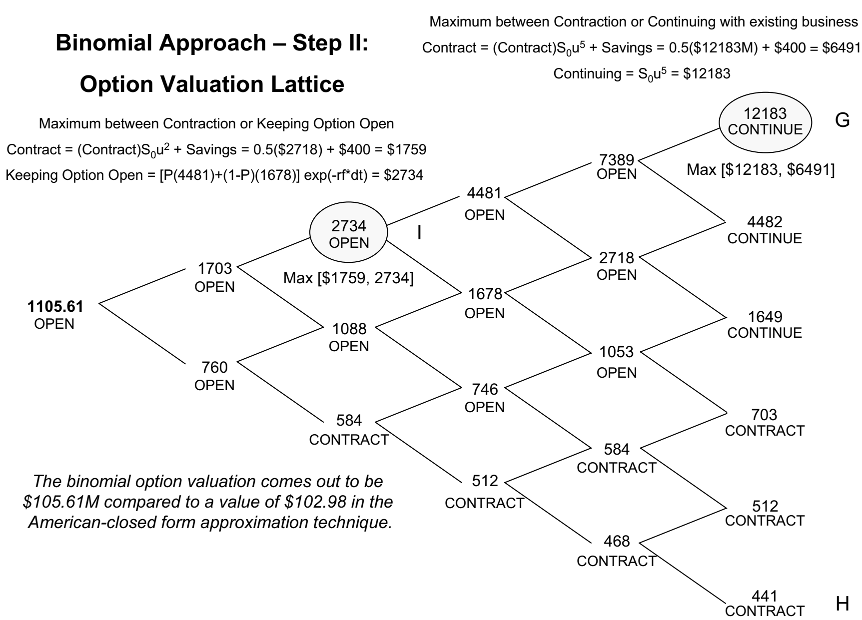


FIGURE 7.6 Contraction Option (Valuation Lattice)

is to contract operations by 50 percent and the profit-maximizing value on that node is $441 million. This is intuitive, because if the underlying asset value of pursuing existing business operations is such that it is very high based on current good operating conditions (node G), then it is wise to continue its current levels of operation. Otherwise, if circumstances force the value of the firm's operations down to such a low level as specified by node H, then it is optimal to contract the existing business by 50 percent (e.g., demand falls, economic downturn, technological failures, and so forth).

Moving on to the intermediate nodes, we see that node I is calculated as $2,734 million. At this particular node, the firm again has two options, to contract its operations at that point or not to contract, thereby keeping the option to contract available and open for the future in the hopes that when the market is down, the firm has the ability to execute the option and contract its existing operations. The value of contracting at that node is *0.5($2,718) + $400 = $1,759 million.* The value of continuing is simply the discounted weighted average of potential future option values using the risk-neutral probability. As the risk adjustment is performed on the probabilities of future option cash flows, the discounting can be done using the risk-free rate. That is, for the value of keeping the option alive and open, we have $[(P)(\$4{,}481) + (1 - P)(\$1{,}678)]exp[(-rf)(\delta t)] = \$2{,}734$ *million,* which is higher than the contraction value. This assumes a 5 percent risk-free rate rf, a time-step δt of 1, and a risk-neutral probability P of 0.427. Using this backward induction technique, the lattice is back-calculated to the starting point to obtain the value of $1,105.61 million. Because the value obtained through a discounted cash flow is $1,000 million for current existing operations, the option value of being able to contract 50 percent of its operations is $105.61 million. The $1,000 million is the static NPV without flexibility, the $105.61 million is the real options value, and the combined value of $1,105.61 million is the total strategic value or ENPV (expanded NPV). The real options value is worth an additional 10.56 percent of existing business operations. If a real options approach is not used, the manufacturing initiative will be undervalued. This is the maximum value the firm should be willing to spend, on average, to obtain this option (e.g., fees paid to the counterparty).

To modify the business case and make it more in line with actual business conditions, different option types can be accounted for at once (Option to Choose) or in phases (Compound Options). For instance, not only has the firm the ability to contract its operations in a down market, it also has the ability to expand its existing business in an up market, or to completely abandon its operations should the future outlook be bleak. These strategic options can exist simultaneously in time or come into being in sequence over a much longer period. With the use of binomial lattices, any and all of these conditions can be modeled and accounted for. No matter how customized the real options analysis may get, the fundamental building blocks of binomial lattice modeling hold true, and these simple cases provide the reader a set of powerful tools to start building upon, when tackling difficult real options problems.

OPTION TO CHOOSE

Suppose a large manufacturing firm decides to hedge itself through the use of strategic options. Specifically it has the option to choose among three strategies: expanding its current manufacturing operations, contracting its manufacturing operations, or completely abandoning its business unit at any time within the next five years. Suppose the firm has a current operating structure whose static valuation of future profitability using a discounted cash flow model (that is, the present value of the future cash flows discounted at an appropriate market risk-adjusted discount rate) is found to be \$100 million. Using Monte Carlo simulation, you calculate the implied volatility of the logarithmic returns on the projected future cash flows to be 15 percent. The risk-free rate on a riskless asset for the next five years is found to be yielding 5 percent annualized returns. Suppose the firm has the option to contract 10 percent of its current operations at any time over the next five years, thereby creating an additional \$25 million in savings after this contraction. The expansion option will increase the firm's operations by 30 percent with a \$20 million implementation cost. Finally, by abandoning its operations, the firm can sell its intellectual property for \$100 million.

A binomial lattice calculation can be used here. Figures 7.7 and 7.8 show the results of the analysis using a binomial approach. The real options value is calculated as \$19.03 million using five lattice steps. An example of the first lattice, the lattice of the underlying asset, is shown in Figure 7.7. Notice that for an option to choose like this example, no closed-form approximations are available. The best that an analyst can do is to use the binomial lattice.

All the required calculations and steps in Figure 7.7 are based on the up factor, down factor, and risk-neutral probability. For instance, the up factor is calculated to be 1.1618, and the down factor is 0.8607 as shown in Figure 7.7. Hence, starting with the underlying value of \$100.0, we multiply this value by the up and down factors to obtain \$116.2 and \$86.1, respectively. The reader can verify the rest of the lattice calculations in Figure 7.7.

The second step is to calculate the option valuation lattice as shown in Figure 7.8, using the values calculated in Figure 7.7's lattice evolution of the underlying asset.

In Figure 7.8, we see that the sample terminal node (denoted J) reveals a value of \$255.2, which can be obtained through the value maximization of expansion, contraction, abandonment, and continuation. At the end of five years, the firm has the option to choose how it wishes to continue its existing operations through these options. Obviously, management will choose the strategy that maximizes profitability. The value of abandoning the firm's busi-

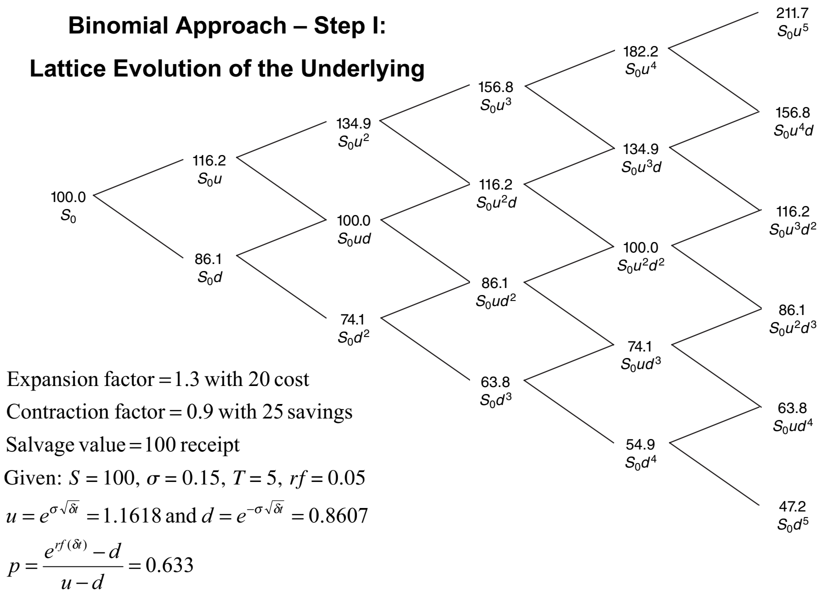


FIGURE 7.7 Option to Choose (Underlying Asset Lattice)

ness unit is \$100 million. The value of expansion is *1.3(\$211.7) − \$20 = \$255.2 million.* The value of contracting 10 percent of its operations is equivalent to 90 percent of its existing operations plus the \$25 million in savings. Hence, the value of contracting the firm's operations is *0.9(\$211.7) + \$25 = \$215.5 million.* The value of continuing with existing business operations can be found in Figure 7.7's lattice evolution of the underlying at the same node (S_0u^5), which is \$211.7 million. The profit-maximizing decision is to expand the firm's current level of operations at \$255.2 million on that node (denoted J).

Similarly, for the terminal node K in Figure 7.8, we see that the value of contracting existing operations at that time is the maximum value of \$102.5 million as seen in Figure 7.8; that is, by contracting the firm's operations by 10 percent, the value is *0.9(\$86.1) + \$25 = \$102.5 million.* In comparison, continuing operations is valued at \$86.1 million, the abandonment strategy is valued at \$100.0 million, and the expansion strategy is valued at *1.3(\$86.1) − \$20 = \$91.9 million.*

This is intuitive because if the underlying asset value of pursuing existing business operations is such that it is very high based on current market demand (node J), then it is wise to expand the firm's current levels of operation. Otherwise, if circumstances force the value of the firm's operations down to such a low level as specified by node K, then it is more optimal to contract the

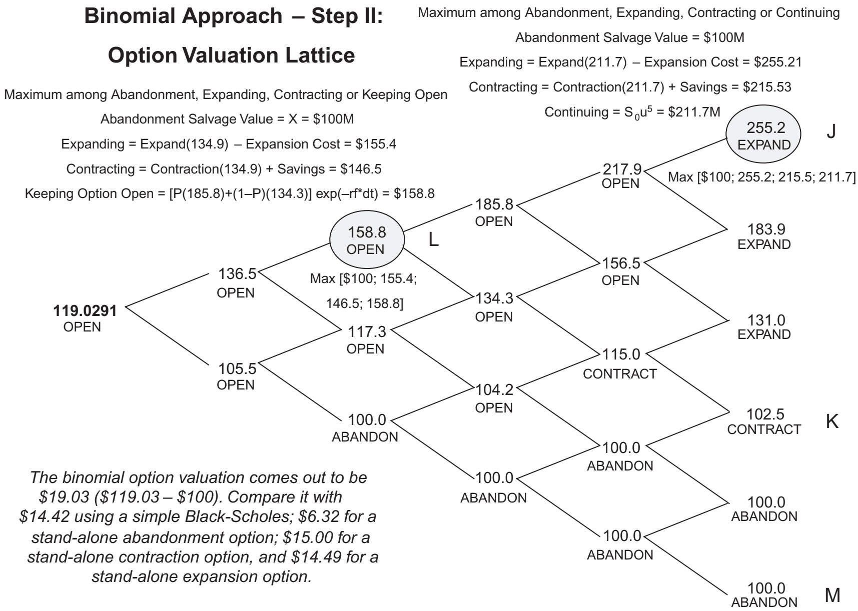


FIGURE 7.8 Option to Choose (Valuation Lattice)

existing business by 10 percent. At any time below level K, for instance, at node M, it is better to abandon the business unit all together.

Moving on to the intermediate nodes, we see that node L is calculated as $158.8 million. At this particular node, the firm again has four options: to expand, contract, abandon its operations, or not execute anything, thus keeping these options open for the future. The value of contracting at that node is *0.9($134.9) + $25 = $146.5 million* (rounded). The value of abandoning the business unit is $100.0 million. The value of expanding is *1.3($134.9) − $20 = $155.4 million.* The value of continuing is simply the discounted weighted average of potential future option values using the risk-neutral probability. As the risk adjustment is performed on the probabilities of future option cash flows, the discounting can be done using the risk-free rate. That is, for the value of keeping the option alive and open, we have $[(P)(\$185.8) + (1 - P)(\$134.3)]exp[(-rf)(\delta t)] = \$158.8$ *million,* which is the maximum value. This assumes a 5 percent risk-free rate *rf*, a time-step δt of 1, and a risk-neutral probability *P* of 0.633. Using this backward induction technique, the lattice is calculated back to the starting point to obtain the value of $119.03 million. As the present value of the underlying is $100 million, the real options value is $19.03 million. In comparison, if we use the Black-Scholes model on the problem, we obtain an incorrect value of $14.42 million.

If the project is analyzed separately, we get differing and misleading results as in the following:

Abandonment option only	\$6.32 million
Contraction option only	\$15.00 million
Expansion option only	\$14.49 million
Sum of all individual options	\$35.81 million

Clearly, valuing a combination of real options by performing them individually and then summing them yields wildly different and incorrect results. We need to account for the interaction of option types within the same project as we have done above. The reason why the sum of individual options does not equal the interaction of the same options is due to the mutually exclusive and independent nature of these specific options. That is, the firm can never both expand and contract on the same node at the same time, or to expand and abandon on the same node at the same time, and so forth. This mutually exclusive behavior is captured using the chooser option. If performed separately on a particular node in the lattice, the expansion option analysis may indicate that it is optimal to expand, while the contraction option analysis may indicate that it is optimal to contract, and so forth, thereby creating a higher total value. However, in a chooser option, the interaction among the three options precludes this from happening, and the option is not overvalued because in the example, multiple option execution cannot occupy the same state. However, in more advanced real options problems, this multiple interaction in a single state is highly desirable.

The same analysis can be further complicated by changing some parameters over time (changing the cost of implementation at some growth rate correlated to the rate of inflation, changing the salvage amount that can be obtained over time, and so forth), all of which can be easily accounted for in the binomial lattices. See Chapter 10 for more details on modeling path dependent, path independent, mutually exclusive, nonmutually exclusive, and complex nested options.

SIMULTANEOUS COMPOUND OPTIONS

In a simultaneous compound option analysis, the value of the option depends on the value of another option. For instance, a pharmaceutical company currently going through a particular FDA drug approval process has to go through human trials. The success of the FDA approval depends heavily on the success of human testing, both occurring at the same time. Suppose that the former costs \$900 million and the latter \$500 million. Further suppose that both phases occur simultaneously and take three years to complete.

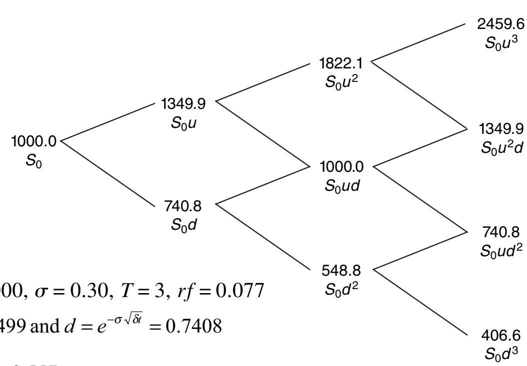
Using Monte Carlo simulation, you calculate the implied volatility of the logarithmic returns on the projected future cash flows to be 30 percent. The risk-free rate on a riskless asset for the next three years is found to be yielding 7.7 percent. The drug development effort's static valuation of future profitability using a discounted cash flow model (that is, the present value of the expected future cash flows discounted at an appropriate market risk-adjusted discount rate) is found to be \$1 billion. Figures 7.9, 7.10, and 7.11 show the calculation involved in obtaining the compound option value. Figure 7.9 shows the usual first lattice of the underlying asset, Figure 7.10 shows an intermediate equity lattice of the first option, and Figure 7.11 shows the option valuation lattice of the compound option, whose valuation lattice is based on the first option as its underlying asset.

All the required calculations and steps in Figure 7.9 are based on the up factor, down factor, and risk-neutral probability analysis previously alluded to in Chapter 6. For instance, the up factor is calculated to be 1.3499, and the down factor is 0.7408 as shown in Figure 7.9. Hence, starting with the underlying value of \$1,000, we multiply this value with the up and down factors to obtain \$1,349.9 and \$740.8, respectively. The rest of the lattice is filled in using the same approach.

The second step involves the calculation of the intermediate or equity lattice as seen in Figure 7.10. We see that the sample terminal node (denoted N) reveals a value of \$1,559.6, which can be obtained through the value maximization of executing the option or not, thereby letting the option expire worthless. The value of the option is *\$2,459.6 − \$900 = \$1,559.6 million.*

Binomial Approach – Step I:

Lattice Evolution of the Underlying



Given: $S = 1{,}000$, $\sigma = 0.30$, $T = 3$, $rf = 0.077$

$u = e^{\sigma\sqrt{\delta t}} = 1.3499$ and $d = e^{-\sigma\sqrt{\delta t}} = 0.7408$

$$p = \frac{e^{rf(\delta t)} - d}{u - d} = 0.557$$

FIGURE 7.9 Simultaneous Compound Option (Underlying Asset Lattice)

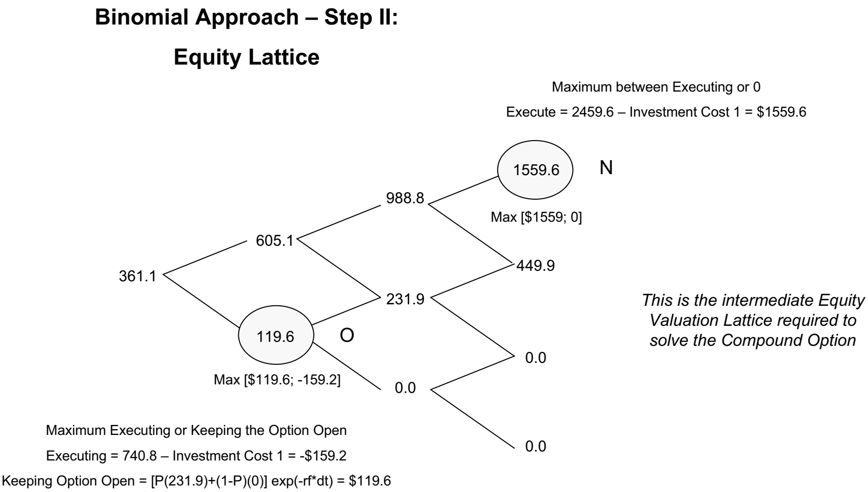


FIGURE 7.10 Simultaneous Compound Option (Intermediate Equity Lattice)

The profit-maximizing value is determined using *MAX*[*1,559.6; 0*], which yields $1,559.6 million.

Moving on to the intermediate nodes, we see that node O is calculated as $119.6 million. At this particular node, the value of executing the option is *$740.8 − $900 = −$159.2 million.* Keep in mind that the value $740.8 comes from the lattice of the underlying at the same node as seen previously in Figure 7.9. The value of continuing is simply the discounted weighted average of potential future option values using the risk-neutral probability. As the risk adjustment is performed on the probabilities of future option cash flows, the discounting can be done using the risk-free rate. That is, for the value of keeping the option alive and open, we have $[(P)(\$231.9) + (1 - P)(\$0)]exp[(-rf)(\delta t)] = \$119.6$ *million,* which is the maximum of the two values. This calculation assumes a 7.7 percent risk-free rate *rf,* a time-step δt of 1, and a risk-neutral probability *P* of 0.557. Using this backward induction technique, this first equity lattice is back-calculated to the starting point to obtain the value of $361.1 million.

The third step is to calculate the option valuation lattice as shown in Figure 7.11. For instance, at the terminal node P, we see the value of the option as $1,059.6, which is nothing but the maximization between zero and the option value. The option value at that node is calculated as *$1,559.6 − $500 = $1,059.6 million.* Notice that the value $1,559.6 comes directly from the equity lattice in Figure 7.10 and not from the underlying asset lattice in Figure 7.9. This is because the underlying asset of a compound option is another option. At node Q, similarly, we see that the value of the option is $0, which is obtained through *MAX*[*−$500; 0*]. Using backward induction, the

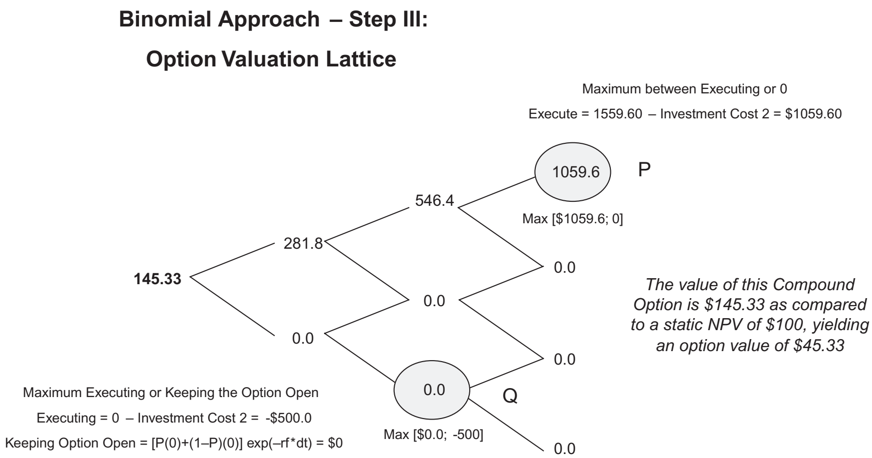


FIGURE 7.11 Simultaneous Compound Option (Valuation Lattice)

value of the compound option is calculated as $145.33 million (rounded). Notice how this compares to a static decision value of *$1,000 − $900 = $100 million* for the first investment. We obtain $165.10 by applying 1,000 steps in the software, $165.10 using a closed-form compound option model, and $165.11 using a modified American call option model, thereby verifying the results and approach. Notice that node P is the same as MAX [*$2459.6 – $1400; 0*]. That is, a simultaneous compound option (regardless of how many different implementations) yields the same value as a simple call option where the implementation cost is the sum of all the different phases' costs.

CHANGING STRIKES

A modification to the option types we have thus far been discussing is the idea of changing strikes—that is, implementation costs for projects may change over time. Putting off a project for a particular period may mean a higher cost. Figures 7.12 and 7.13 show the applications of this concept. Keep in mind that changing strikes can be applied to any previous option types as well; in other words, one can mix and match different option types. Suppose the implementation of a project in the first year costs $80 million but increases to $90 million in the second year due to expected increases in raw materials and input costs. Using Monte Carlo simulation, the implied volatility of the logarithmic returns on the projected future cash flows is calculated to be 50 percent. The risk-free rate on a riskless asset for the next two years is found to be yielding 7.0 percent. The static valuation of future profitability using a discounted cash flow model (that is, the present value of the expected future

Binomial Approach – Step I:

Lattice Evolution of the Underlying

Cost at year 1 = \$80; cost at year 2 = \$90

Given: $S = 100,\ \sigma = 0.50,\ T = 2,\ rf = 0.07$

$u = e^{\sigma\sqrt{\delta t}} = 1.6487$ and $d = e^{-\sigma\sqrt{\delta t}} = 0.6065$

$$p = \frac{e^{rf(\delta t)} - d}{u - d} = 0.447$$

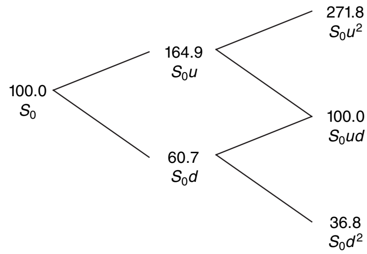


FIGURE 7.12 Changing Strike Option (Underlying Asset Lattice)

cash flows discounted at an appropriate market risk-adjusted discount rate) is found to be \$100 million. The underlying asset lattice evolution can be seen in Figure 7.12.

Similar to the approach used for calculating an American-type call option, Figure 7.13 shows the stepwise calculations on an option with changing strike prices. Notice that the value of the call option on changing strikes is \$37.53 million. Compare this to a naive static discounted cash flow net present value of \$20 million for the first year and \$10 million for the second year.

Binomial Approach – Step II:

Option Valuation Lattice

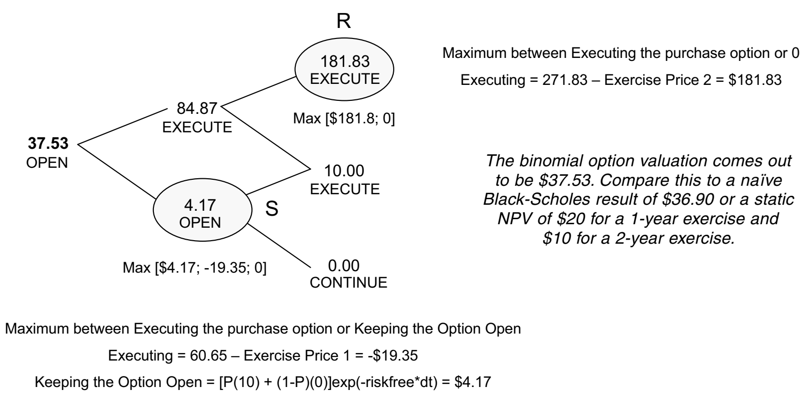


FIGURE 7.13 Changing Strike Option (Valuation Lattice)

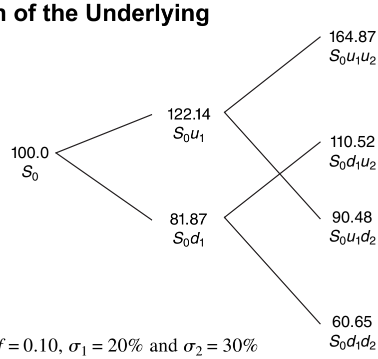
Obviously for simplicity in illustration, only two periods are used. In actual business conditions, multiple strike costs can be accounted for over many time periods and modeled on binomial lattices with more steps. Based on the time-step size (δt), the different costs associated with different time periods can be mapped into the lattice easily. In addition, changing cost options can also be used in conjunction with all other types of real options models, such as the expansion option, compound option, volatility option, and so forth. See Chapters 9 and 10 for details on applying different implementation costs (as well as other parameters changing over time) using the Super Lattice Solver software. Note that it is extremely difficult mathematically to allow multiple changing parameters in a closed-form model.

CHANGING VOLATILITY

Instead of changing strike costs over time, in certain cases, volatility on cash flow returns may differ over time. This can be seen in Figures 7.14 and 7.15. In Figure 7.14, we see the example for a two-year option where volatility is 20 percent in the first year and 30 percent in the second year. In this circumstance, the up and down factors are different over the two time periods. Thus, the binomial lattice will no longer be recombining. As a matter of fact, the underlying asset lattice branches cross over each other as shown in Figure 7.14. The upper bifurcation of the first lower branch (from \$81.87

Binomial Approach – Step I:

Lattice Evolution of the Underlying



Given: $S = 100$, $X = 110$, $T = 2$, $rf = 0.10$, $\sigma_1 = 20\%$ and $\sigma_2 = 30\%$

$$u_1 = e^{\sigma\sqrt{\delta t}} = 1.2214 \text{ and } d_1 = e^{-\sigma\sqrt{\delta t}} = 0.8187$$

$$u_2 = e^{\sigma\sqrt{\delta t}} = 1.3499 \text{ and } d_2 = e^{-\sigma\sqrt{\delta t}} = 0.7408$$

$$p_1 = \frac{e^{rf(\delta t)} - d}{u - d} = 0.7113 \text{ and } p_2 = \frac{e^{rf(\delta t)} - d}{u - d} = 0.5983$$

FIGURE 7.14 Changing Volatility Option (Underlying Asset Lattice) to \$110.52) crosses the lower bifurcation of the upper first branch (from \$122.14 to \$90.48). This complex crossover will be compounded for multiple time-steps.

Figure 7.15 shows the option valuation lattice. Similar calculations are performed for an option with changing volatilities as for other option types. For instance, node T has a value of \$54.87, which is the maximum of zero and *\$164.87 − \$110 = \$54.87*. For node U, the value of \$0.28 million comes from the maximization of executing the option *\$81.87 − \$110 = −\$28.13 million* and keeping the option open with $[(P)(\$0.52) + (1 - P)(\$0)]exp[(-rf)(\delta t)] = \$0.28$ *million,* which is the maximum value. This calculation assumes a 10 percent risk-free rate *rf,* a time-step δt of 1, and a risk-neutral probability *P* of 0.5983. Using this backward induction technique, this valuation lattice is back-calculated to the starting point to obtain the value of \$19.19 million, as compared to the static net present value of −\$10 million (benefits of \$100 million with a cost of \$110 million).

More complicated analyses can be obtained through this changing volatility condition. For example, where there are multiple stochastic underlying variables driving the value of the option, each variable may have its own unique volatility, but the variables are correlated with each other. Examples include the price and quantity sold where there is a negative correlation between these two variables (the downward-sloping demand curve). The Real Options Valuation's Super Lattice Solver software CD-ROM handles some of these more difficult calculations. Note that due to the nonrecombining lattice requirement when volatility changes over time, software applications are

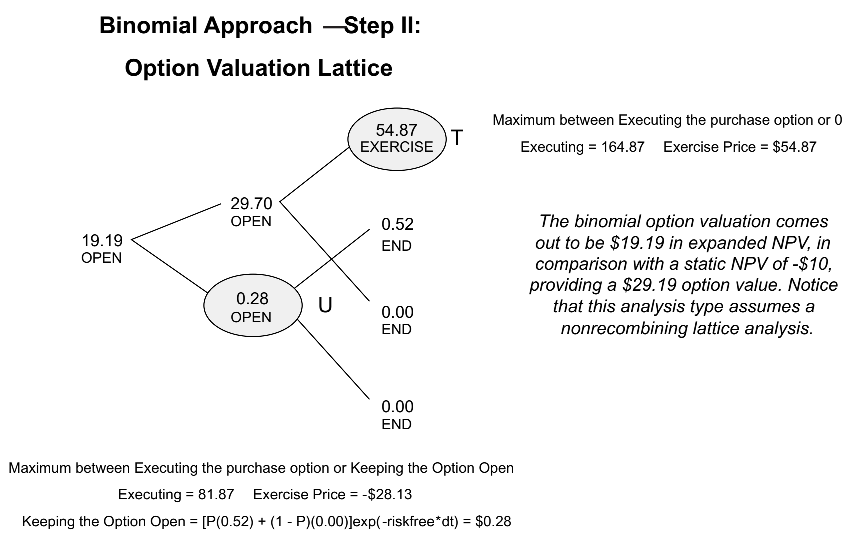


FIGURE 7.15 Changing Volatility Option (Valuation Lattice)

required as the computations become intractable quickly when attempted manually.

SEQUENTIAL COMPOUND OPTION

A sequential compound option exists when a project has multiple phases and latter phases depend on the success of previous phases. Figures 7.16 to 7.19 show the calculation of a sequential compound option. Suppose a project has two phases, where the first phase has a one-year expiration that costs $500 million. The second phase's expiration is three years and costs $700 million. Using Monte Carlo simulation, the implied volatility of the logarithmic returns on the projected expected present value of the returns of future cash flows is calculated to be 20 percent. The risk-free rate on a riskless asset for the next three years is found to be yielding 7.7 percent. The static valuation of future profitability using a discounted cash flow model (that is, the present value of the future cash flows discounted at an appropriate market risk-adjusted discount rate) is found to be $1,000 million. The underlying asset lattice is seen in Figure 7.16.

The calculation of this initial underlying asset lattice is similar to previous option types by first calculating the up and down factors and evolving the present value of the future cash flow for the next three years.

Figure 7.17 shows the second step in calculating the equity lattice of the second option. The analysis requires the calculation of the longer-term option first and then the shorter-term option because the value of a compound

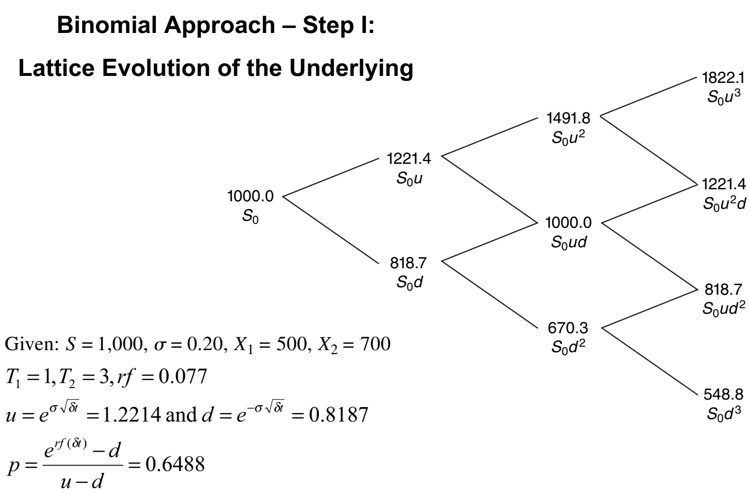


FIGURE 7.16 Sequential Compound Option (Underlying Asset Lattice)

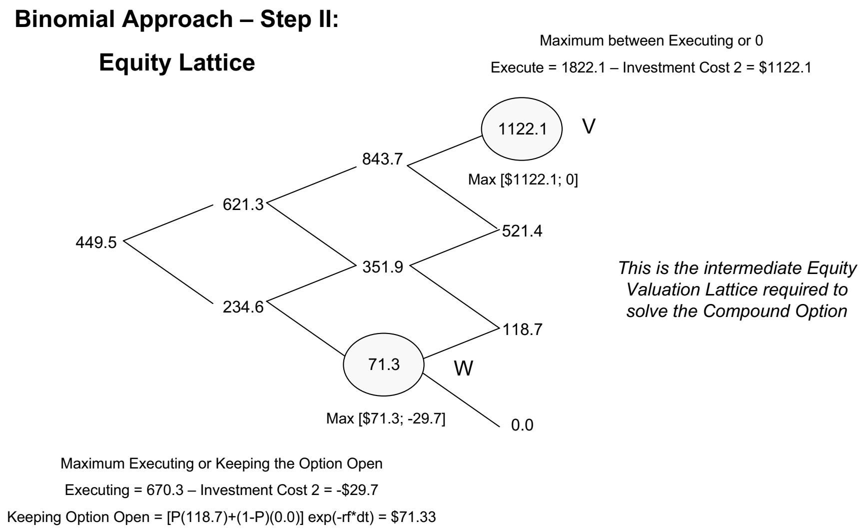


FIGURE 7.17 Sequential Compound Option (Equity Lattice)

option is based on another option. At node V, the value is $1,122.1 million because it is the maximum between zero and executing the option through *$1,822.1 − $700 = $1,122.1 million.* The intermediate node W is $71.3 million, the maximum between executing the option *$670.3 − $700 = −$29.7 million* and keeping the option open with $[(P)(\$118.7) + (1 - P)(\$0.0)]$ $exp[(-rf)(\delta t)] = \$71.3$ *million,* which is the maximum value. This calculation assumes a 7.7 percent risk-free rate *rf,* a time-step δt of 1, and a risk-neutral probability *P* of 0.6488. Using this backward induction technique, this first equity lattice is back-calculated to the starting point to obtain the value of $449.5 million.

Figure 7.18 shows the valuation of the first, shorter-term option. The analysis on this lattice depends on the lattice of the second, longer-term option as shown in Figure 7.17. For instance, node X has a value of $121.3 million, which is the maximum between zero and executing the option *$621.27 − $500 = $121.27 million.* Notice that $621.27 is the value of the second, longer-term equity lattice as shown in Figure 7.17 and $500 is the implementation cost on the first option.

Node Y on the other hand uses a backward induction calculation, where the value $72.86 million is obtained through the maximization between executing the option *$449.5 − $500 = −$50.5 million* and keeping the option open with $[(P)(\$121.3) + (1 - P)(\$0.0)]exp[(-rf)(\delta t)] = \$72.86$ *million,* which is the maximum value. The maximum value comes from keeping the option open. This calculation assumes a 7.7 percent risk-free rate *rf,* a time-step

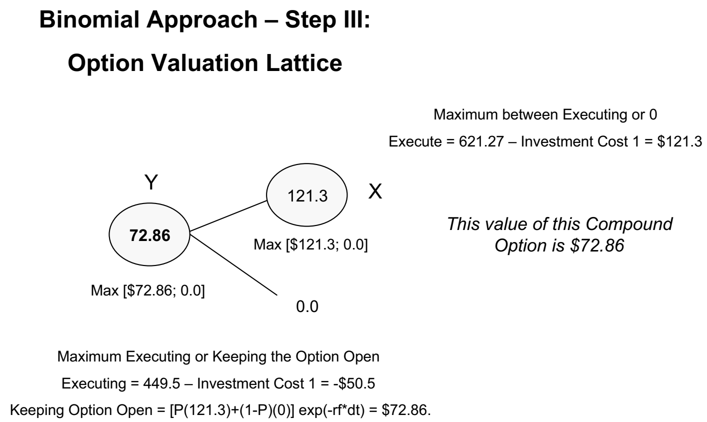


FIGURE 7.18 Sequential Compound Option (Valuation Lattice)

δt of 1, and a risk-neutral probability P of 0.6488. Again notice that \$500 million is the implementation cost of the first option.

Figure 7.19 shows the combined option analysis from Figures 7.17 and 7.18, complete with decision points on when to invest in the first and second rounds versus keeping the option to invest open for the future.

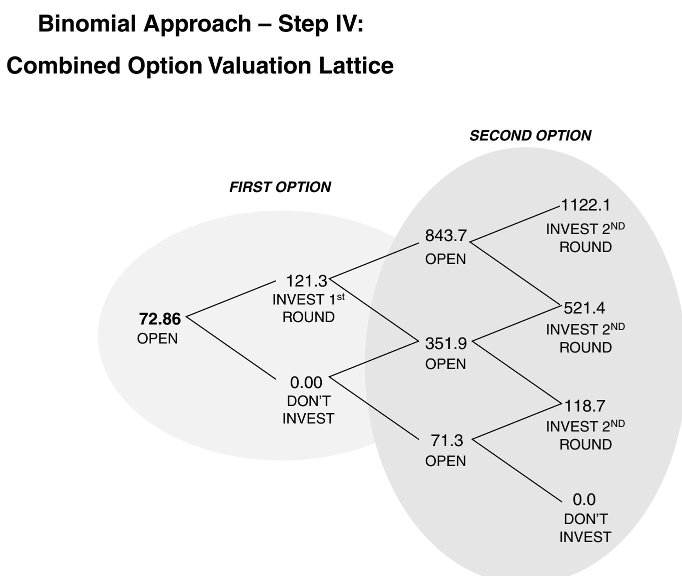


FIGURE 7.19 Sequential Compound Option (Combined Lattice)

See Chapters 9 to 11 for illustrations and examples of how to combine sequential-type options with other more complex options in solving real-life problems using the Multiple Asset Super Lattice Solver software.

EXTENSION TO THE BINOMIAL MODELS

As discussed in the previous examples, multiple tweaks can be performed using the binomial lattices. For instance, Figure 7.20 illustrates a simple chooser option with the same parameters as in Figure 7.7 but with a twist. For instance, the expansion factor increases at a 10 percent rate per year, while the cost of expanding decreases at a 3 percent deflation per year. Similarly, the savings projected from contracting will reduce at a 10 percent rate. However, the salvage value of abandoning increases at a 5 percent rate. Custom changes like these can be easily accommodated in a binomial lattice but are very difficult, if not impossible, to solve in closed-form solutions, because every time a slight modification is made to a closed-form model, stochastic calculus is a necessary evil in solving the problem, as compared to a simple change in the maximization routines inherent in the binomial lattices.

Taking this approach a little further, the reader can very easily create a custom option to accommodate almost any situation, to more closely reflect actual business cases. For instance, the growth rates can be inflation rates or

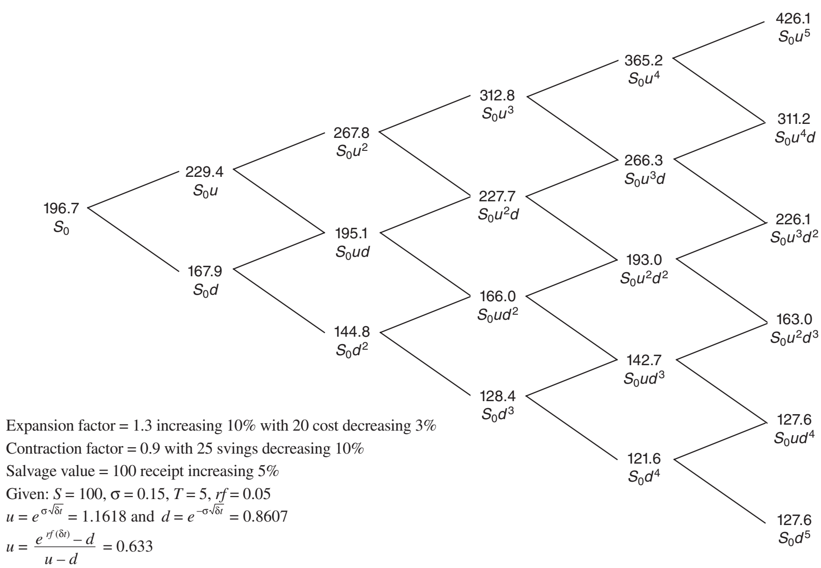


FIGURE 7.20 Extension to the Binomial Models

changes in the cost of execution, savings, or salvage values over time. In addition, the expansion factor or contraction factor can also be changed. This is more appropriate as it is less credible to say that executing the same project at any time within a specified period will cost exactly the same no matter what the circumstances are. With these simple building blocks discussed in this chapter, readers are well on their way to developing more sophisticated and customized real options models. Chapter 8 briefly discusses some additional real options models and problems, while Chapters 9 to 11 illustrate how these more advanced problems can be easily tackled using the Super Lattice Solver trial software included in the CD-ROM.

SUMMARY

Closed-form solutions are exact, quick, and easy to implement with the assistance of some programming skills but are highly difficult to explain. They are also very specific in nature, with limited modeling flexibility. Binomial lattices, in contrast, are easy to implement and easy to explain. They are also highly flexible but require significant computing power and time-steps to obtain good approximations. In the limit, binomial lattices tend to approach closed-form solutions; hence, it is always recommended that both approaches be used to verify the results, whenever appropriate. The results from closed-form solutions may be used as benchmarks in conjunction with the binomial lattice structure when presenting to management a complete real options solution. Even a Black-Scholes model can be used as a means of credibility testing. That is, if the real options results have similar magnitude as the Black-Scholes, the analysis becomes more credible.

CHAPTER 7 QUESTIONS

1. Using the example in Figures 7.1 and 7.2 on the abandonment option, recalculate the value of the option assuming that the salvage value increases from the initial $100 (at time 0) by 10 percent at every period starting from time 1.
2. The expansion option example in Figures 7.3 and 7.4 assumes that the competitor has the same level of growth and uncertainty as the firm being valued. Describe what has to be done differently if the competitor is assumed to be growing at a different rate and facing a different set of risks and uncertainties. Rerun the analysis assuming that the competitor's volatility is 45 percent instead of 35 percent.
3. Figures 7.7 and 7.8 illustrate the chooser option, that is, the option to choose among expanding, contracting, and abandoning current opera-

tions. Rerun these three options separately using the Super Lattice Solver software in the enclosed CD-ROM and verify that the summary provided in Figure 7.8 is correct. Why is it that the sum of the individual option values does not equal the chooser option value?

4. In the compound option example illustrated in Figures 7.9 through 7.11, the first phase cost is $900 and the second phase cost is $500. However, in a simultaneous compound option, these two phases occur concurrently. Rerun the example by changing the first phase cost to $500 and the second phase cost to $900. Should the results be comparable? Why or why not?
5. Based on the example in Appendix 7G, create a European call option model using Monte Carlo simulation in Excel. For a simulated standard-normal random distribution, use the function "=NORMSINV(RAND( ))". Assume a one-year expiration, 40 percent annualized volatility, $100 asset and strike costs, 5 percent risk-free rate, and no dividend payments. Verify your results using Black-Scholes and a binomial lattice.
6. Solve an American call option using the risk-neutral probability approach, and then solve the same option using the market-replicating portfolio approach based on the example in Appendix 7C. For the market-replicating portfolio approach, assume continuous discounting at the risk-free rate. Verify that theory holds such that both approaches obtain identical call option values. Which approach is simpler to apply? For both approaches, assume the following parameters: asset value = $100, strike cost = $100, maturity = 3 years, volatility = 10 percent, risk-free rate = 5 percent, dividends = 0 percent, and binomial lattice steps = 3.

APPENDIX 7A

Volatility Estimates

There are several ways to estimate the volatility used in the option models. The most common and valid approaches are:

- ***Logarithmic Cash Flow Returns Approach or Logarithmic Stock Price Returns Approach:*** This method is used mainly for computing the volatility on liquid and tradable assets such as stocks in financial options; however, it is sometimes used for other traded assets such as price of oil and price of electricity. The drawback is that DCF models with only a few cash flows will generally *overstate* the volatility and this method cannot be used when negative cash flows occur. The benefits include its computational ease, transparency, and modeling flexibility of the method. In addition, no simulation is required to obtain a volatility estimate.
- ***Logarithmic Present Value Returns Approach:*** This approach is used mainly when computing the volatility on assets with cash flows. A typical application is in real options. The drawback of this method is that simulation is required to obtain a single volatility and is not applicable for highly traded liquid assets such as stock prices. The benefit includes the ability to accommodate certain negative cash flows and applies more rigorous analysis than the logarithmic cash flow returns approach, providing a more accurate and conservative estimate of volatility when assets are analyzed.
- ***Generalized Autoregressive Conditional Heteroskedasticity (GARCH) Models:*** These models are used mainly for computing the volatility on liquid and tradable assets such as stocks in financial options and are sometimes used for other traded assets such as price of oil and price of electricity. The drawback is that a lot of data is required, advanced econometric modeling expertise is required, and this approach is highly susceptible to user manipulation. The benefit is that rigorous statistical analysis is performed to find the best-fitting volatility curve, providing different volatility estimates over time.

- *Management Assumptions and Guesses:* This approach is used for both financial options and real options. The drawback is that the volatility estimates are very unreliable and are only subjective best guesses. The benefit of this approach is its simplicity—this method is very easy to explain to management the concept of volatility, both in execution and interpretation.
- *Market Proxy Using Comparables or Indices:* This approach is used mainly for comparing liquid and nonliquid assets, as long as comparable market-, sector-, or industry-specific data are available. The drawback is that it is sometimes hard to find the right comparable firms and the results may be subject to gross manipulation by subjectively including or excluding certain firms. The benefit is its ease of use.

LOGARITHMIC CASH FLOW RETURNS OR LOGARITHIMIC STOCK PRICE RETURNS APPROACH

The Logarithmic Cash Flow Returns or Logarithmic Stock Price Returns Approach calculates the volatility using the individual future cash flow estimates, comparable cash flow estimates, or historical prices, generating their corresponding logarithmic relative returns, as illustrated in Table 7A.1. Starting with a series of forecast future cash flows or historical prices, convert them into relative returns. Then take the natural logarithms of these relative returns. The standard deviation of these natural logarithm returns is the *periodic volatility* of the cash flow series. The resulting periodic volatility from the sample dataset in Table 7A.1 is 25.58%. This value then must be annualized.

No matter what the approach used, the periodic volatility estimate used in a real options or financial options analysis has to be an *annualized* volatility. Depending on the periodicity of the raw cash flow or stock price data used,

TABLE 7A.1 Natural Logarithmic Cash Flow Returns Approach

Time Period	Cash Flows	Cash Flow Relative Returns	Natural Logarithm of Cash Flow Returns (X)
0	\$100	—	—
1	\$125	\$125/\$100 = 1.25	ln(\$125/\$100) = 0.2231
2	\$ 95	\$ 95/\$125 = 0.76	ln (\$95/\$125) = –0.2744
3	\$105	\$ 105/\$95 = 1.11	ln (\$105/\$95) = 0.1001
4	\$155	\$155/\$105 = 1.48	ln(\$155/\$105) = 0.3895
5	\$146	\$146/\$155 = 0.94	ln(\$146/\$155) = –0.0598

the volatility calculated should be converted into annualized values using $\sigma\sqrt{P}$, where P is the number of periods in a year and σ is the periodic volatility. For instance, if the calculated volatility using monthly cash flow data is 10%, the annualized volatility is $10\%\sqrt{12} = 35\%$. Similarly, P is 365 (or about 250 if accounting for trading days and not calendar days) for daily data, 4 for quarterly data, 2 for semiannual data, and 1 for annual data.

Notice that the number of returns in Table 7A.1 is one less than the total number of periods. That is, for time periods 0 to 5, we have six cash flows but only five cash flow relative returns. This approach is valid and correct when estimating the volatilities of liquid and highly traded assets—historical stock prices, historical prices of oil and electricity—and is less valid for computing volatilities in a real options world, where the underlying asset generates cash flows. This is because to obtain valid results, many data points are required, and in modeling real options, the cash flows generated using a DCF model may only be for 5 to 10 periods. In contrast, a large number of historical stock prices or oil prices can be downloaded and analyzed. With smaller data sets, this approach typically overestimates the volatility.

The volatility estimate is then calculated as

$$Volatility = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2} = 25.58\%$$

where n is the number of Xs, and $\bar{x}$ is the average X value.

To further illustrate the use of this approach, Figure 7A.1 shows the stock prices for Microsoft downloaded from Yahoo! Finance, a publicly available free resource.* You can follow along the example by loading the example file: *Start* | *Programs* | *Real Options Valuation* | *Real Options Super Lattice Solver* | *Sample Files* | *Volatility Estimates* and select the worksheet tab *Log Cash Flow Approach*. The data in columns A to G in Figure 7A.1 are downloaded from Yahoo! The formula in cell I3 is simply *LN(G3/G4)* to compute the natural logarithmic value of the relative returns week over week, and is copied down the entire column. The formula in cell J3 is *STDEV(I3:I54)* SQRT(52)* which computes the annualized (by multiplying the square root of the number of weeks in a year) volatility (by taking the standard deviation of the entire 52 weeks of the year 2004 data). The formula in cell J3 is then copied down the entire column to compute a moving window of annualized volatilities. The volatility used in this example is the average of a 52-week

*Go to http://finance.yahoo.com and enter a stock symbol (e.g., MSFT). Click on Quotes: Historical Prices and select Weekly and select the period of interest. You can then download the data to a spreadsheet for analysis.

	Downloaded Weekly Historical Stock Prices of Microsoft							Volatility Computations	
2	Date	Open	High	Low	Close	Volume	Adj. Close*	LN Relative Returns	Moving Average Volatilities
3	27-Dec-04	27.01	27.10	26.63	26.72	52386840	26.64	-0.0108	17.87%
4	20-Dec-04	27.01	27.17	26.73	27.01	77413174	26.93	0.0019	17.84%
5	13-Dec-04	27.10	27.40	26.80	26.96	100620300	26.88	-0.0045	17.85%
6	06-Dec-04	27.10	27.44	26.91	27.06	83312720	27.00	-0.0055	18.00%
7	29-Nov-04	26.64	27.44	26.61	27.23	83103200	27.15	0.0235	18.13%
8	22-Nov-04	26.75	26.82	26.10	26.60	61034599	26.52	-0.0090	18.03%
9	15-Nov-04	27.34	27.50	26.84	26.86	75575960	26.78	-0.0011	18.10%
10	08-Nov-04	29.18	30.20	29.13	29.97	109385736	26.81	0.0225	18.20%
11	01-Nov-04	28.16	29.56	27.95	29.51	85044019	26.22	0.0468	18.28%
12	25-Oct-04	27.67	28.54	27.55	27.97	70791679	25.02	0.0084	17.71%
13	18-Oct-04	28.07	28.89	27.58	27.74	74671318	24.81	-0.0092	17.80%
14	11-Oct-04	28.20	28.27	27.80	27.95	48396360	25.04	0.0000	19.68%
15	04-Oct-04	28.44	28.59	27.97	27.95	52998320	25.04	-0.0091	19.65%
16	27-Sep-04	27.17	28.32	27.04	28.25	61785760	25.27	0.0346	19.68%
17	20-Sep-04	27.44	27.74	27.07	27.29	59162520	24.41	-0.0082	19.62%
18	13-Sep-04	27.53	27.57	26.74	27.51	51599880	24.61	0.0008	20.52%
19	07-Sep-04	27.29	27.51	27.14	27.49	51935175	24.59	0.0139	21.30%
20	30-Aug-04	27.30	27.68	26.85	27.11	45125980	24.25	-0.0127	21.25%
21	23-Aug-04	27.27	27.67	27.09	27.46	40526880	24.56	0.0123	22.29%
22	16-Aug-04	27.03	27.50	26.89	27.20	52571740	24.26	0.0066	22.29%
23	09-Aug-04	27.26	27.75	26.85	27.02	51244080	24.10	-0.0041	22.42%
24	02-Aug-04	28.27	28.55	27.05	27.14	56739100	24.20	-0.0488	22.42%
25	26-Jul-04	28.36	28.81	28.13	28.49	65555220	25.41	0.0163	21.97%
26	19-Jul-04	27.62	29.89	27.60	28.03	114579322	25.00	0.0198	22.11%
27	12-Jul-04	27.67	28.36	27.25	27.48	57970740	24.51	-0.0130	22.02%
28	06-Jul-04	28.32	28.33	27.55	27.06	61197249	24.05	-0.0250	22.04%
29	28-Jun-04	28.60	29.04	28.17	28.57	66214339	25.40	0.0000	22.07%
30	21-Jun-04	28.22	28.66	27.81	28.57	82202478	25.40	0.0079	22.30%
31	14-Jun-04	26.55	28.50	26.53	28.35	97727648	25.20	0.0574	22.48%
32	07-Jun-04	26.02	26.79	25.97	26.77	55540250	23.87	0.0311	22.71%
33	01-Jun-04	26.13	26.28	25.85	25.95	49284475	23.14	-0.0107	22.86%
34	24-May-04	26.05	26.35	25.60	26.23	51927460	23.39	0.0129	23.15%
35	17-May-04	25.47	26.27	25.42	25.89	56652040	23.09	0.0013	23.21%
36	10-May-04	25.63	26.19	25.43	25.86	58864200	23.06	0.0030	23.87%
37	03-May-04	26.19	26.60	25.75	25.78	60847680	22.99	-0.0134	24.07%
38	26-Apr-04	27.45	27.55	25.96	26.13	77381899	23.30	-0.0527	24.05%
39	19-Apr-04	25.08	27.72	25.06	27.54	102244677	24.56	0.0903	23.67%
40	12-Apr-04	25.48	25.77	25.10	25.16	56472679	22.44	-0.0124	21.92%
41	05-Apr-04	25.81	25.98	25.35	25.48	52838950	22.72	-0.0144	22.50%
42	29-Mar-04	25.25	25.90	24.85	25.85	69704180	23.05	0.0322	22.76%
43	22-Mar-04	24.48	25.51	24.01	25.03	92829802	22.32	0.0158	22.59%
44	15-Mar-04	25.50	25.46	24.55	24.63	80008619	21.97	-0.0296	23.72%
45	08-Mar-04	25.51	26.35	25.07	25.38	76061740	22.63	-0.0377	24.48%

One-Year Annualized Volatility Analysis	
Average	21.09%
Median	22.50%

FIGURE 7A.1 Computing Microsoft's One-Year Annualized Volatility

moving window, which covers two years of data. That is, cell L8's formula is *AVERAGE(J3:J54)*, where cell J54 has the following formula: *STDEV (I54:I105)*SQRT(52)*, and of course row 105 is January 2003. This means that the 52-week moving window captures the average volatility over a two-year period and smoothes the volatility such that infrequent but extreme spikes will not dominate the volatility computation. Of course, a median volatility should also be computed. If the median is far off from the average, the distribution of volatilities is skewed and the median should be used, otherwise, the average should be used. Finally, these 52 volatilities can be fed into Monte Carlo simulation—using the enclosed Risk Simulator software's custom nonparametric simulation (see Chapter 9 for details).

Clearly there are advantages and shortcomings to this simple approach. This method is very easy to implement, and Monte Carlo simulation is not required to obtain a single-point volatility estimate. This approach is mathematically valid and is widely used in estimating volatility of financial assets. However, for real options analysis, there are several caveats that deserve closer attention. When cash flows are negative over certain time periods, the relative returns will have negative values, and the natural logarithm of a negative value does not exist. Hence, the volatility measure does not fully capture the possible cash flow downside and may produce erroneous results. In addition, autocorrelated cash flows (estimated using time-series forecasting techniques) or cash flows following a static growth rate will yield erroneous volatility estimates. Great care should be taken in such instances. This flaw is neutralized in larger datasets that only carry positive values such as historical stock prices or price of oil or electricity.

This approach is valid and correct as computed in Figure 7A.1 for liquid and traded assets with a lot of historical data. The reason why this approach is not valid for computing the volatility of cash flows in a DCF for the purposes of real options analysis is the lack of data. For instance, the annualized cash flows 100, 200, 300, 400, 500 would yield a volatility of 20.80 percent, as compared to the annualized cash flows 100, 200, 400, 800, 1600, which would yield a volatility of 0 percent, versus the cash flows 100, 200, 100, 200, 100, 200, which yield 75.93 percent. All these cash flow streams seem fairly deterministic and yet provide very different volatilities. In addition, the third set of negatively autocorrelated cash flows should actually be less volatile (due to its predictive cyclical nature and reversion back to a base level) but its volatility is computed to be the highest. The second cash flow seems more risky than the first set due to larger fluctuations but has a volatility of 0 percent. Therefore, be careful when applying this method to small datasets.

When applied to stock prices and historical data that are nonnegative, this approach is easy and valid. However, if used on real options assets, the DCF cash flows may very well take on negative values, returning an error in your computation (i.e., log of a negative value does not exist). However, you can take certain approaches to avoid this error. The first is to move up your DCF model, from free cash flows to net income, to operating income (EBITDA), and even all the way up to revenues and prices, where all the values are positive. If doing it this way, then care must be taken such that all other options and projects are modeled this way for comparability's sake. Also, this approach is justified in situations where the volatility, risk, and uncertainty stem from a certain variable above the line is used. For instance, the only critical success factor for an oil and gas company is the price of oil (price) and the production rate (quantity), where both are multiplied to obtain revenues. In addition, if all other items in the DCF are proportional ratios (e.g., operating expenses are 25 percent of revenues or EBITDA values are 10 percent of revenues, and so forth), then we are only interested in the volatility of revenues. In fact, if the proportions remain constant, the volatilities computed are identical (e.g., revenues of \$100, \$200, \$300, \$400, \$500 versus a 10 percent proportional EBITDA of \$10, \$20, \$30, \$40, \$50, yields identical 20.80 percent volatilities). Finally, taking the oil and gas example a step further, computing the volatility of revenues, assuming no other market risks exist below this revenue line in the DCF, is justified because this firm may have global operations with different tax conditions and financial leverages (different ways of funding projects). The volatility should only apply to market risks and not private risks (how good a negotiator the CFO is on getting foreign loans, or how shrewd your CPAs are in creating offshore tax shelters).

Now that you understand the mechanics of computing volatilities this way, we need to explain why we did what we did! Merely understanding the mechanics is insufficient in justifying the approach or explaining the rationale why we analyzed it the way we did. Hence, let us look at the steps undertaken and explain the rationale behind them.

Step 1: Collect the relevant data and determine the periodicity and time frame. You can use forecast financial data (cash flows from a DCF model), comparable data (comparable market data such as sector indexes and industry averages), or historical data (stock prices or price of oil and electricity). Consider the periodicity and time frame of the data. In using forecast and comparable data, your choices are limited to what is available or what models have been built, and are typically annual, quarterly, or monthly data, usually for a limited amount of time. When using historical data, your choices are more varied. Typically, daily data has too much random fluctuation and white noise that may erroneously impact the volatility computations. Monthly, quarterly, and annual historical data are spread too far out and all the fluctuations inherent in the time-series data may be smoothed out. The optimal periodicity is weekly data, if available. Any intraday and intraweek fluctuations are smoothed out but weekly fluctuations are still inherent in the dataset. Finally, the time frame of the historical data is also important. Periods of extreme events (e.g., dot-com bubble, global recession, depression, terrorist attacks) need to be carefully considered; that is, are these actual events that will recur and hence are not outliers but part of the undiversifiable systematic risk of doing business? In Figure 7A.1's example, a two-year cycle was used. Clearly, if the option has a three-year maturity, then a three-year cycle should be considered, with the exception that data is not available, or if certain extreme events mitigate our using the data back that far.

Step 2: Compute relative returns. Relative returns are used in geometric averages while absolute returns are used in arithmetic averages. To illustrate, suppose you purchase an asset or stock for \$100. You hold it for one period and it doubles to \$200, which means you made 100 percent absolute returns. You get greedy and keep it for one more period when you should have sold it and obtained the capital gains. The next period, the asset goes back down to \$100, which means you lost half the value or –50 percent absolute returns. Your stockbroker calls you up and tells you that you made an average of 25 percent returns in the two periods (the arithmetic average of 100 percent and –50 percent is 25 percent)! You started with \$100 and ended up with \$100. You clearly did not make a 25 percent return. Thus, an arithmetic average will overinflate the average when fluctuations occur. Fluctuations do occur in the stock market or for your real options project, otherwise your volatility is very low and there's no option value, and hence, no point in doing an options analysis. A geometric average is a better way to compute the return. The computation is seen below, and you can clearly see that as part of the geometric average calculation, relative returns are computed. That is, if \$100 goes to \$200, the relative return is 2.0 and the absolute return is 100 percent; or when \$100 goes down to \$90, the relative return is 0.9 (anything less than 1.0 is a loss) or –10 percent absolute returns. Thus, to avoid over-inflating the computations, we use relative returns in Step 2.

$$\text{Geometric Average} = \sqrt[\text{PERIODS}]{\left(\frac{\text{Period 1 End Value}}{\text{Period 1 Start Value}}\right)\left(\frac{\text{Period 2 End Value}}{\text{Period 2 Start Value}}\right)\cdots\left(\frac{\text{Period n End Value}}{\text{Period n Start Value}}\right)}$$

$$= \sqrt[2]{\left(\frac{200}{100}\right)\left(\frac{100}{200}\right)} = 1.0$$

Hence, the calculated geometric average of 1.0 implies a 0 percent average return (simply take the geometric average minus 1.0), which more accurately represents the situation.

Step 3: Compute natural logarithm of the relative returns. The natural log is used for two reasons. The first is to be comparable to the exponential Brownian Motion stochastic process; that is, recall that a Brownian Motion is written as:

$$\frac{\delta S}{S} = e^{\mu(\delta t)+\sigma\varepsilon\sqrt{\delta t}}$$

To compute the volatility (σ) used in an equivalent computation (regardless of whether it is used in simulation, lattices, or closed-form models because these three approaches require the Brownian Motion as a fundamental assumption), a natural log is used. The exponential of a natural log cancels each other out in the previous equation. The second reason is that in computing the geometric average, relative returns were used, then multiplied and taken to the root of the number of periods. By taking a natural log of a root (n), we reduce the root (n) in the geometric average equation. This is why natural logs are used in Step 3.

Step 4: Compute the sample standard deviation to obtain the periodic volatility. A sample standard deviation is used instead of a population standard deviation because your dataset might be small. For larger datasets, the sample standard deviation converges to the population standard deviation, so it is always safer to use the sample standard deviation. Of course, the following sample standard deviation is simply the average (sum of all and then divided by some variation of n) of the deviations of each point of a dataset from its mean ($x - \bar{x}$), adjusted for a degree of freedom for small datasets, where a higher standard deviation implies a wider distributional width and, thus, carries a higher risk. The variation of each point around the mean is squared to capture its absolute distances (otherwise for a symmetrical distribution, the variations to the left of the mean might equal the variations to the right of the mean, creating a zero sum), and the entire result is taken to the square root, to bring the value back to its original unit. Finally, the denominator ($n - 1$) adjusts for a degree of freedom in small sample sizes. To illustrate, suppose there are three people in a room and we ask all three of them to randomly choose a number of their choice, as long as the average is \$100. The first person might choose any value, and so could the second person. However, when it comes to the third person, he or she can only choose a single unique value such that the average is exactly \$100. Thus, in a room of 3 people (n), only 2 people ($n - 1$) are truly free to choose. So, for smaller sample sizes, taking the $n - 1$ correction makes the computations more conservative. This is why we use sample standard deviations in Step 4.

$$Volatility = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2}$$

Step 5: Compute the annualized volatility. The volatility used in options analysis is annualized for several reasons. The first reason is that all other inputs are annualized inputs (e.g., annualized risk-free rate, annualized dividends, and maturity in years). The second reason is that a cash flow or stock price stream of \$10 to \$20 to \$30 that occurs in three different months versus three different days has very different volatilities. Clearly, if it takes days to double or triple your asset value, that asset is a lot more volatile than if it takes months. All these have to be common-sized in time, that is, annualized. Finally, the Brownian Motion stochastic equation has the values $\sigma\sqrt{\delta t}$; that is, suppose we have a one-year option modeled using a 12-step lattice, then δt is 1/12. If we use monthly data, compute the monthly volatility, and use this figure as the input, then this monthly volatility will again be partitioned into 12 pieces per $\sigma\sqrt{\delta t}$. Therefore, we need to first annualize the volatility to an annual volatility (multiplied by the square root of 12), input this annual volatility into the model, and let the model partition the volatility (multiplied by the square root of $1/12$) into its periodic volatility. This is why we annualize volatilities in Step 5.

LOGARITHMIC PRESENT VALUE RETURNS APPROACH

The Logarithmic Present Value Returns Approach to estimating volatility collapses all future cash flow estimates into two present value sums, one for the first time period and another for the present time (Table 7A.2). The

TABLE 7A.2 Natural Logarithmic Present Value Returns

Time Period	Cash Flows	Present Value at Time 0	Present Value at Time 1
0	\$100	$\frac{\$100}{(1+0.1)^0} = \$100.00$	—
1	\$125	$\frac{\$125}{(1+0.1)^1} = \$113.64$	$\frac{\$125}{(1+0.1)^0} = \$125.00$
2	\$ 95	$\frac{\$95}{(1+0.1)^2} = \$78.51$	$\frac{\$95}{(1+0.1)^1} = \$86.36$
3	\$105	$\frac{\$105}{(1+0.1)^3} = \$78.89$	$\frac{\$105}{(1+0.1)^2} = \$86.78$
4	\$155	$\frac{\$155}{(1+0.1)^4} = \$105.87$	$\frac{\$155}{(1+0.1)^3} = \$116.45$
5	\$146	$\frac{\$146}{(1+0.1)^5} = \$90.65$	$\frac{\$146}{(1+0.1)^4} = \$99.72$
SUM		\$567.56	\$514.31

calculations assume a constant discount rate. The cash flows are discounted all the way to Time 0 and again to Time 1, with the cash flows in Time 0 ignored (sunk cost). Then the values are summed, and the following logarithmic ratio is calculated:

$$X = \ln\left(\frac{\sum_{i=1}^{n} PVCF_i}{\sum_{i=0}^{n} PVCF_i}\right)$$

where $PVCF_i$ is the present value of future cash flows at different time periods i.

This approach is more appropriate for use in real options where actual assets and projects' cash flows are computed and their corresponding volatility is estimated. This approach is applicable for project and asset cash flows, and can accommodate less data points; however, it requires the use of Monte Carlo simulation to obtain a volatility estimate. This approach reduces the measurement risks of autocorrelated cash flows and negative cash flows. However, the bottom line is still that this approach is the best for estimating volatilities in most real options problems.

In the foregoing example, X is simply *ln($514.31/$567.56) = – 0.0985*. Using this intermediate X value, perform a Monte Carlo simulation on the discounted cash flow model (thereby simulating the individual cash flows) and obtain the resulting forecast distribution of X. As seen previously, the sample standard deviation of the forecast distribution of X is the volatility estimate used in the real options analysis. *It is important to note that only the numerator is simulated while the denominator remains unchanged.*

The downside to estimating volatility this way is that the approach requires Monte Carlo simulation, but the calculated volatility measure is a single-digit estimate, as compared to the Logarithmic Cash Flow Returns or Stock Price Returns Approach, which yields a distribution of volatilities, that in turn yields a distribution of calculated real options values.

The main objection to using this method is its dependence on the variability of the discount rate used. For instance, we can expand the X equation as follows:

$$X = \ln\left(\frac{\sum_{i=1}^{n} PVCF_i}{\sum_{i=0}^{n} PVCF_i}\right) = \ln\left(\frac{\dfrac{CF_1}{(1+D)^0} + \dfrac{CF_2}{(1+D)^1} + \dfrac{CF_3}{(1+D)^2} + \cdots + \dfrac{CF_N}{(1+D)^{N-1}}}{\dfrac{CF_0}{(1+D)^0} + \dfrac{CF_1}{(1+D)^1} + \dfrac{CF_2}{(1+D)^2} + \cdots + \dfrac{CF_N}{(1+D)^N}}\right)$$

where D represents the constant discount rate used. Here we see that the cash flow series CF for the numerator is offset by one period, and the discount factors are also offset by one period. Therefore, by performing a Monte Carlo simulation on the cash flows alone versus performing a Monte Carlo simulation on both cash flow variables as well as the discount rate will yield very different X values. The main critique of this approach is that in a real options analysis, the variability in the present value of cash flows is the key driver of option value and not the variability of discount rates used in the analysis. Modifications to this method include duplicating the cash flows and simulating only the numerator cash flows, thereby providing different numerator values but a static denominator value for each simulated trial, while keeping the discount rate constant. In fact, when running this approach, it might be advisable to set the discount rate as a static risk-free rate, simulate the DCF inputs, and obtain the volatility as an output, then reset the discount rate back to its original value.

Figure 7A.2 illustrates an example of how this approach can be implemented easily in Excel. To follow along, open the example file: *Volatility Computations* and select the worksheet tab *Log Present Value Approach*. The example shows a sample DCF model where the cash flows (row 46) and implementation costs (row 48) are computed separately. This is done for several reasons. The first is to separate the market risks (revenues and associated operating expenses) from the private risks (cost of implementation)—of course only if it makes sense to separate them, as there might be situations where the implementation cost is subject to market risk as well. Here we assume that implementation cost is subject to only private risks and will be discounted at a risk-free or the cost of money that is close to risk-free rate of return, to discount it for time value of money. The market-risk cash flows are discounted at a market risk-adjusted rate of return (which can also be seen as discounting at 5 percent risk-free rate to account for time value of money, and discounted again at the market risk premium of 10 percent for risk, or simply discounted one time at 15 percent). As discussed in Chapter 2, if you do not separate the market and private risks, you end up discounting the private risks heavily and making the DCF a lot more profitable than it actually is (i.e., if the costs that should be discounted at 5 percent are discounted at 15 percent, the NPV will be inflated). By separately discounting these cash

	A	C	D	E	F	G	H	I
2			**Log Present Value Approach**					
7								
8		Input Parameters			Results			
9		Discount Rate (Cash Flow)	15.00%		Present Value (Cash Flow)		**$328.24**	
10		Discount Rate (Impl. Cost)	5.00%		Present Value (Impl. Cost)		**$189.58**	
11		Tax Rate	10.00%		Net Present Value		**$138.67**	
12								
17			**2002**	**2003**	**2004**	**2005**	**2006**	
18		Revenue	$100.00	$200.00	$300.00	$400.00	$500.00	
22		Cost of Revenue	$40.00	$80.00	$120.00	$160.00	$200.00	
26		Gross Profit	$60.00	$120.00	$180.00	$240.00	$300.00	
27		Operating Expenses	$22.00	$44.00	$66.00	$88.00	$110.00	
31		Depreciation Expense	$5.00	$5.00	$5.00	$5.00	$5.00	
35		Interest Expense	$3.00	$3.00	$3.00	$3.00	$3.00	
39		Income Before Taxes	$30.00	$68.00	$106.00	$144.00	$182.00	
40		Taxes	$3.00	$6.80	$10.60	$14.40	$18.20	
41		Income After Taxes	$27.00	$61.20	$95.40	$129.60	$163.80	
42		Non-Cash Expenses	$12.00	$12.00	$12.00	$12.00	$12.00	
46		Cash Flow	$39.00	$73.20	$107.40	$141.60	$175.80	
47								
48		Implementation Cost	$25.00	$25.00	$50.00	$50.00	$75.00	
49								
50		**Volatility Estimates (Logarithmic PV Approach)**						
51		PV (0)	$39.00	$63.65	$81.21	$93.10	$100.51	
52		PV (1)	N/A	$73.20	$93.39	$107.07	$115.59	
53		Static PV (0)	$39.00	$63.65	$81.21	$93.10	$100.51	
54		Variable X	0.0307					
55		Volatility	Simulate!					

FIGURE 7A.2 Logarithmic Present Value Approach flows, the present value of cash flows and implementation costs can be computed (cells H9 and H10). The difference will, of course, be the NPV. The separation here is also key because from the Black-Scholes equation that follows, the call option is computed as the present value of net benefits discounted at some risk-adjusted rate of return or the starting stock price (S) times the standard normal probability distribution (Φ) less the implementation cost or strike price (X) discounted at the *risk-free rate* and adjusted by another standard normal probability distribution (Φ). If volatility (σ) is zero, the uncertainty is zero, and Φ is equal to 100 percent. (The value inside the parenthesis is infinity, meaning that the standard normal distribution value is 100 percent. Alternatively, you can state that with zero uncertainties, you have a 100 percent certainty). By separating the cash flows, you can now use these as inputs into the options model, whether it's using the Black-Scholes or binomial lattices.

$$Call = S\Phi\left(\frac{\ln(S/X)+(r+\sigma^2/2)T}{\sigma\sqrt{T}}\right) - Xe^{-rT}\Phi\left(\frac{\ln(S/X)+(r-\sigma^2/2)T}{\sigma\sqrt{T}}\right)$$

Continuing with the example in Figure 7A.2, the calculations of interest are on rows 51 to 55. Row 51 shows the present values of the cash flows to Year 0 (assume that the base year is 2002), while row 52 shows the present values of the cash flows to Year 1, ignoring the sunk cost of cash flow at Year 0. These two rows are computed in Excel and are linked formulas. You should then copy and paste the values only into row 53 (use Excel's *Edit | Paste Special | Values Only* to do this). Then, compute the intermediate variable X in cell D54 using the following Excel formula: *LN(SUM(E52:H52)/SUM(D53:H53))*. Then, simulate this DCF model by assigning the relevant input assumptions in the model using the Risk Simulator software and set this intermediate variable X as the output forecast. The standard deviation from this X is the periodic volatility. Annualizing the volatility is required, by multiplying this periodic volatility with the square root of the number of periodicities in a year.

Now that you understand the mechanics of computing volatilities this way, we need to explain why we did what we did! Merely understanding the mechanics is insufficient in justifying the approach or explaining the rationale why we analyzed it the way we did. Hence, let us look at the steps undertaken and explain the rationale behind them.

Step 1: Compute the present values at times 0 and 1 and sum them. The theoretical price of a stock is the sum of the present values of all future dividends (for non-dividend-paying stocks, we use market-replicating portfolios and comparables), and the funds to pay these dividends are obtained from the company's net income and free cash flows. The theoretical value of a project or asset is the sum of the present value of all future free cash flows or net income. Hence, the price of a stock is equivalent to the price or value of an asset, the NPV. Thus, the sum of the present values at time 0 is equivalent to the stock price of the asset at time 0, the value today. The sum of the present value of the cash flows at time 1 is equivalent to the stock price at time 1, or a good *proxy* for the stock price in the *future*. We use this as a proxy because in most DCF models, cash flow forecasts are only a few periods. Hence, by running Monte Carlo simulation, we are changing all future possibilities and capturing the uncertainties in the DCF inputs. This future stock price is hence a good proxy of what may happen to the future stream of cash flows. Remember that the sum of the present value of future cash flows at time 1 included in its computations all future cash flows from the DCF, thereby capturing future fluctuations and uncertainties. This is why we perform Step 1 when we compute volatilities using the Log Present Value Returns Approach.

Step 2: Calculate the intermediate variable *X*. This *X* variable is identical to the logarithmic relative returns in the Log Cash Flow Returns Approach. It is simply the natural logarithm of the relative returns of the future stock price (using the sum of present values at time 1 as a proxy) from the current stock price (the sum of present values at time 0). We then set the sum of present values at time 0 as static because it is the base case, and by definition of a base case, the values do not change. The base case can be seen as the NPV of the project's net benefits and is assumed to be the best estimate of the project's net benefit value. It is the future that is uncertain and fluctuates, hence we simulate the DCF model and allow the numerator of the *X* variable to change during the simulation while keeping the denominator static as the base case.

Step 3: Simulate the model and obtain the standard deviation as volatility. This approach requires that the model be simulated. This makes sense because if the model is not simulated it means that there are no uncertainties in the project or asset, and hence, the volatility is equal to zero. You would only simulate when there are uncertainties, hence you obtain a volatility estimate. The rationale for using the sample standard deviation as the volatility is similar to the Logarithmic Cash Flow Returns Approach. If the sums of the present values of the cash flows are fluctuating between positive and negative values during the simulation, you can again move up the DCF model and use items like EBITDA and net revenues as proxy variables for computing volatility.

Another alternative volatility estimate is to combine both approaches if enough data exists. That is, from a DCF with many cash flow estimates, compute the PV Cash Flows for periods 0, 1, 2, 3, and so forth. Then, compute the natural logarithm of the relative returns of these PV Cash Flows. The standard deviation is then annualized to obtain the volatility. This is, of course, the preferred method and does not require the use of Monte Carlo simulation, but the drawback is that a longer cash flow forecast series is required.

GARCH APPROACH

Another approach is the GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model, which can be utilized to estimate the volatility of any time-series data. GARCH models are used mainly in analyzing financial time-series data, in order to ascertain their conditional variances and volatilities. These volatilities are then used to value the options as usual, but the amount of historical data necessary for a good volatility estimate remains significant. Usually, several dozens—and even up to hundreds—of data points are required to obtain good GARCH estimates. In addition, GARCH models are very difficult to run and interpret and require great facility with econometric modeling techniques. GARCH is a term that incorporates a family of models that can take on a variety of forms, known as GARCH(p,q), where p and q are positive integers that define the resulting GARCH model and its forecasts.

For instance, a GARCH (1,1) model takes the form of

$$
\begin{aligned}
y_t &= x_t \gamma + \varepsilon_t \\
\sigma_t^2 &= \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2
\end{aligned}
$$

where the first equation's dependent variable (y_t) is a function of exogenous variables (x_t) with an error term (ε_t). The second equation estimates the variance (squared volatility σ_t^2) at time t, which depends on a historical mean (ω), news about volatility from the previous period, measured as a lag of the squared residual from the mean equation (ε_{t-1}^2), and volatility from the previous period (σ_{t-1}^2). The exact modeling specification of a GARCH model is beyond the scope of this book and is not discussed. Suffice it to say that detailed knowledge of econometric modeling (model specification tests, structural breaks, and error estimation) is required to run a GARCH model, making it less accessible to the general analyst. The other problem with GARCH models is that the model usually does not provide a good statistical fit. That is, it is impossible to predict the stock market, and of course equally if not harder, to predict a stock's volatility over time. Figure 7A.3 shows a GARCH (1,2) on Microsoft's historical stock prices.

Dependent Variable: MSFT
Method: ML - ARCH
Date: 02/25/05 Time: 00:20
Sample(adjusted): 3 52
Included observations: 50 after adjusting endpoints
Convergence achieved after 67 iterations
Bollerslev-Wooldrige robust standard errors & covariance

	Coefficient	Std. Error	z-Statistic	Prob.
C	23.14431	1.301024	17.78930	0.0000
D(MSFT,1)	0.456040	0.062391	7.309364	0.0000
AR(1)	0.967490	0.027575	35.08601	0.0000
		Variance Equation		
C	0.151406	0.028717	5.272435	0.0000
ARCH(1)	0.148308	0.053559	2.769061	0.0056
GARCH(1)	0.735869	0.097780	7.525790	0.0000
GARCH(2)	-0.867066	0.083186	-10.42325	0.0000

R-squared	0.898576	Mean dependent var	24.48620
Adjusted R-squared	0.884424	S.D. dependent var	1.290867
S.E. of regression	0.438849	Akaike info criterion	1.106641
Sum squared resid	8.281300	Schwarz criterion	1.374324
Log likelihood	-20.66602	F-statistic	63.49404
Durbin-Watson stat	1.308287	Prob(F-statistic)	0.000000
Inverted AR Roots	.97		

FIGURE 7A.3 Sample GARCH Results

MANAGEMENT ASSUMPTION APPROACH

A more simple approach is the use of Management Assumptions. This approach allows management to get a rough volatility estimate without performing more protracted analysis. This approach is also great for educating management about what volatility is and how it works. Mathematically and statistically, the width or risk of a variable can be measured through several different statistics, including the range, standard deviation (σ), variance, coefficient of variation, and percentiles. Figure 7A.4 illustrates two different stocks' historical prices. The stock depicted as a dark bold line is clearly less volatile than the stock with the dotted line. The time-series data from these two stocks can be redrawn as a probability distribution as seen in Figure 7A.5. Although the expected value of both stocks is similar, their volatilities and

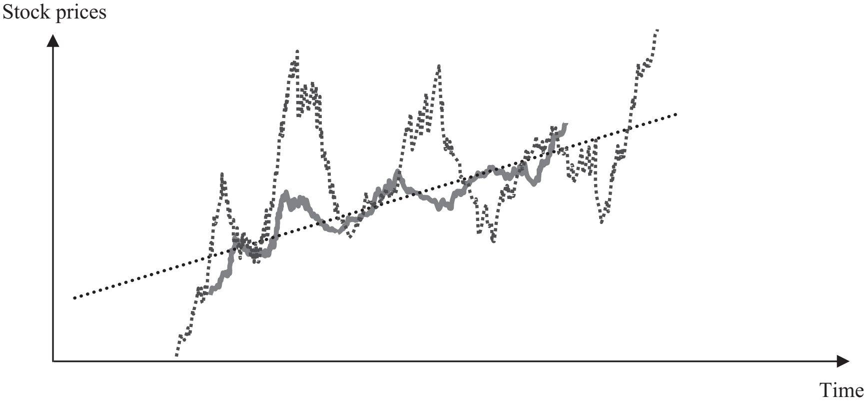


FIGURE 7A.4 Volatility

hence their risks are different. The *x*-axis depicts the stock prices, while the *y*-axis depicts the frequency of a particular stock price occurring, and the area under the curve (between two values) is the probability of occurrence. The second stock (dotted line in Figure 7A.4) has a wider spread (a higher standard deviation σ_2) than the first stock (bold line in Figure 7A.4). The width of Figure 7A.5's *x*-axis is the same width from Figure 7A.4's *y*-axis. One common measure of width is the standard deviation. Hence, standard deviation is a way to measure volatility. The term *volatility* is used and not standard deviation because the volatility computed is not from the raw cash

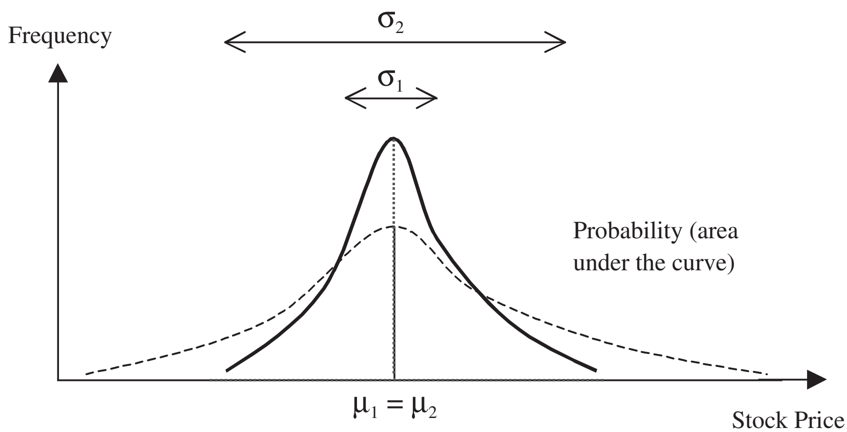


FIGURE 7A.5 Standard Deviation

flows or stock prices themselves, but from the natural logarithm of the relative returns on these cash flows or stock prices. Hence, the term volatility differentiates it from a regular standard deviation.

However, for the purposes of explaining volatility to management, we relax this terminological difference and on a very high level, state that they are one and the same, for discussion purposes. Thus, we can make some management assumptions in estimating volatilities. For instance, starting from an expected NPV (the mean value), you can obtain an alternate NPV value with its probability, and get an approximate volatility. For instance, say that a project's NPV is expected to be \$100M. Management further assumes that the best case scenario exceeds \$150M if everything goes really well, and that there is only a 10 percent probability that this best-case scenario will hit. Figure 7A.6 illustrates this situation. If we assume for simplicity that the underlying asset value will fluctuate within a normal distribution, we can compute the implied volatility using the following equation:

$$Volatility = \frac{Percentile\ Value - Mean}{Inverse\ of\ the\ Percentile \times Mean}$$

For instance, we compute the volatility of this project as:

$$Volatility = \frac{\$150M - \$100M}{Inverse\,(0.90) \times \$100M} = \frac{\$50M}{1.2815 \times \$100M} = 39.02\%$$

where the Inverse of the Percentile can be obtained by using Excel's *NORMS INV(0.9)* function. Similarly, if the worst-case scenario occurring 10 percent of the time will yield an NPV of \$50M, we compute the volatility as:

$$Volatility = \frac{\$50M - \$100M}{Inverse\,(0.10) \times \$100M} = \frac{-\$50M}{-1.2815 \times \$100M} = 39.02\%$$

This methodology implies that the volatility is a symmetrical measure. That is, at an expected NPV of \$100M, a 50 percent increase is equivalent to \$150M while a 50 percent decrease is equivalent to \$50M. And because the normal distribution is assumed as the underlying distribution, this symmetry makes perfect sense. So now, by using this simple approach, if you obtain a volatility estimate of 39.02 percent, you can explain to management by stating that this volatility is equivalent to saying that there is a 10 percent probability the NPV will exceed \$150M. Through this simple analysis, you have converted probability into volatility using the foregoing equation, where the

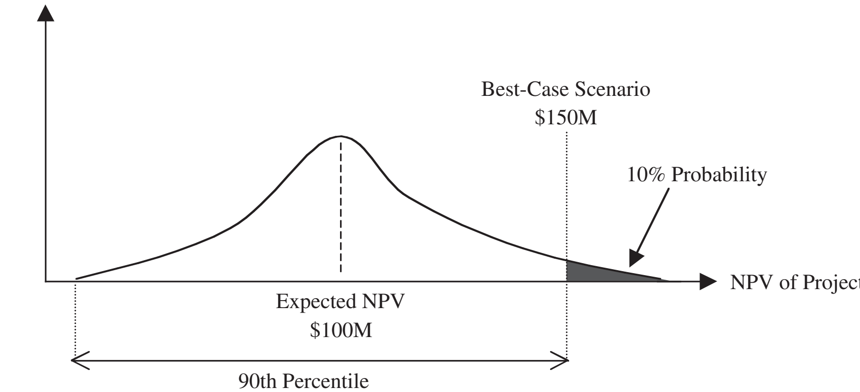


FIGURE 7A.6 Going from Probability to Volatility

latter is a lot easier for management to understand. Conversely, if you model this in Excel, you can convert from volatility back into probability. Figures 7A.7 and 7A.8 illustrate this approach. Open the example file *Volatility Estimates* and select the worksheet tab *Volatility to Probability* to follow along.

Figure 7A.7 allows you to enter the expected NPV and the alternate values (best-case and worst-case) as well as its corresponding percentiles. That is,

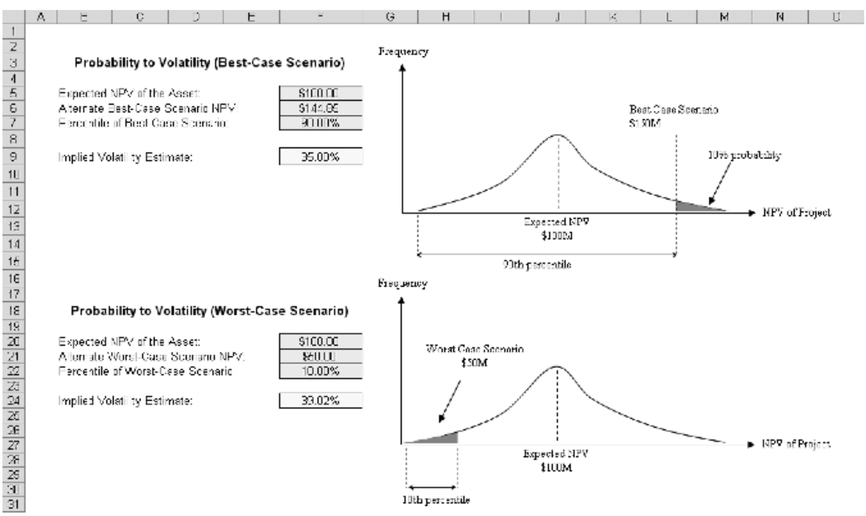


FIGURE 7A.7 Excel Probability to Volatility Model

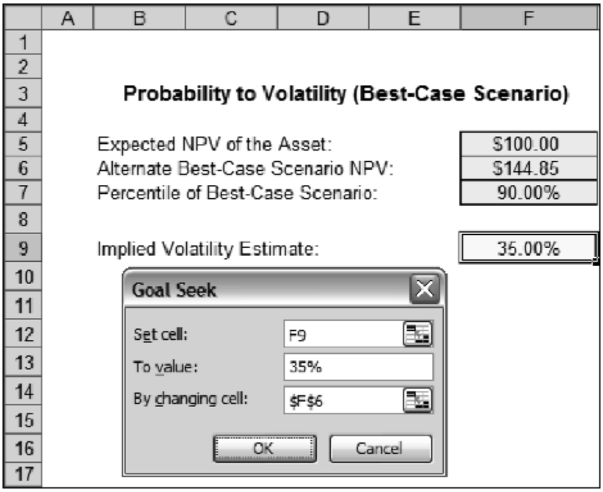


FIGURE 7A.8 Excel Volatility to Probability Model

given some probability and its value, we can impute the volatility. Conversely, Figure 7A.8 shows how you can use Excel's Goal Seek function (click on *Tools* | *Goal Seek* in Excel) to find the probability from a volatility. For instance, if the project's expected NPV is $100M, a 35 percent volatility implies that 90 percent of the time the NPV will be less than $144.85M, and that only 10 percent (best-case scenario) of the time will the true NPV exceed this value.

Now that you understand the mechanics of estimating volatilities this way, again, we need to explain why we did what we did! Merely understanding the mechanics is insufficient in justifying the approach or explaining the rationale why we analyzed it the way we did. Hence, let us look at the assumptions required and explain the rationale behind them.

Assumption 1: We assume that the underlying distribution of the asset fluctuations is normal. We can assume normality because the distribution of the final nodes on a superlattice is normally distributed. In fact, the Brownian Motion equation shown earlier requires a random standard normal distribution (ε). In addition, a lot of distributions will converge to the normal distribution anyway (a Binomial distribution becomes normally distributed when the number of trials increases; a Poisson distribution also becomes normally distributed with a high average rate; a Triangular distribution is a normal distribution with truncated upper and lower values; and so forth) and it is not possible to ascertain the shape and type of the final NPV distribution if the DCF model is simulated with many different types of distributions (e.g., revenues are Lognormally distributed and are negatively correlated to one another over time, while operating expenses are positively correlated to revenues but are assumed to be distributed following a Triangular distribution, while the effects of market competition are simulated using a Poisson distribution with a small rate times the probability of technical success simulated as a Binomial distribution). We cannot determine theoretically what a Lognormal minus a Triangular times Poisson and Binomial, after accounting for their correlations, would be. Instead, we rely on the Central Limit Theorem and assume the final result is normally distributed, especially if a large number of trials are used in the simulations. Finally, we are interested in the logarithmic relative returns' volatility, not the standard deviation of the actual cash flows or stock prices. Stock prices and cash flows are usually Lognormally distributed (stock prices cannot be below zero) but the logs of the relative returns are always normally distributed. In fact, this can be seen in Figures 7A.9 and 7A.10, where the historical stock prices of Microsoft from March 1986 to December 2004 are tabulated.

Assumption 2: We assume that the standard deviation is the same as the volatility. Again, referring to Figure 7A.10, using the expected returns chart, the average is computed at 0.58 percent and the 90th percentile is 8.60 percent, and the implied volatility is found to be 37 percent. Using the data downloaded, we compute the empirical volatility for this entire period to be 36 percent. So, the computation is close enough such that we can use this approach for management discussions. This is why the normality assumption and using a regular standard deviation as a proxy are sufficient.

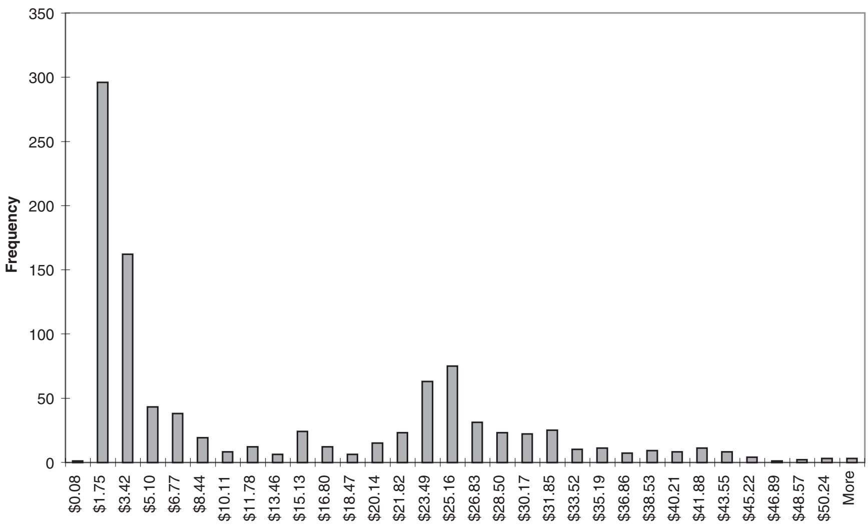


FIGURE 7A.9 Probability Distribution of Microsoft's Stock Prices (Since 1986)

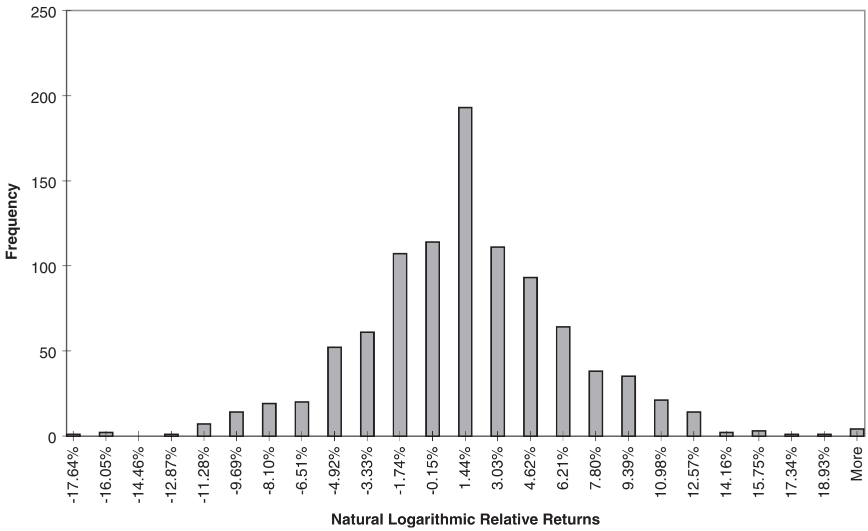


FIGURE 7A.10 Probability Distribution of Microsoft's Log Relative Returns

Assumption 3: We used a standard-normal calculation to impute the volatility. As we are assuming that the underlying distribution is normal, we can compute the volatility by using the standard-normal distribution. The standard normal distribution Z-score is such that:

$$Z = \frac{x - \mu}{\sigma} \text{ this means that } \sigma = \frac{x - \mu}{Z}$$

and because we normalize the volatility as a percentage (σ^*), we divide this by the mean to obtain:

$$\sigma^* = \frac{x - \mu}{Z\mu}$$

In layman's terms, we have:

$$Volatility = \frac{Percentile\ Value - Mean}{Inverse\ of\ the\ Percentile \times Mean}$$

Again, the inverse of the percentile is obtained using Excel's function: *NORMSINV*.

MARKET PROXY APPROACH

An often used (not to mention abused and misused) method in estimating volatility applies to publicly available market data. That is, for a particular project under review, a set of market comparable firms' publicly traded stock prices are used. These firms should have *functions*, *markets*, *risks*, and *geographical locations* similar to those of the project under review. Then, using closing stock prices, the standard deviation of natural logarithms of relative returns is calculated. The methodology is identical to that used in the logarithm of cash flow returns approach previously alluded to. The problem with this method is the assumption that the risks inherent in comparable firms are identical to the risks inherent in the specific project under review. The issue is that a firm's equity prices are subject to investor overreaction and psychology in the stock market, as well as countless other exogenous variables that are irrelevant when estimating the risks of the project. In addition, the market valuation of a large public firm depends on multiple interacting and diversified projects. Finally, firms are levered, but specific projects are usually unlevered. Hence, the volatility used in a real options analysis (σ_{RO}) should be adjusted to discount this leverage effect by dividing the volatility in equity prices (σ_{EQUITY}) by $(1 + D/E)$, where D/E is the debt-to-equity ratio of the public firm. That is, we have

$$\sigma_{RO} = \frac{\sigma_{EQUITY}}{1 + \frac{D}{E}}$$

This approach can be used if there are market comparables such as sector indexes or industry indexes. It is incorrect to state that a project's risk as measured by the volatility estimate is identical to the entire industry, sector, or the market. There are a lot of interactions in the market such as diversification, overreaction, and marketability issues that a single project inside a firm is not exposed to. Great care must be taken in choosing the right comparables as the major drawback of this approach is that it is sometimes hard to find the right comparable firms and the results may be subject to gross manipulation by subjectively including or excluding certain firms. The benefit is its ease of use—industry averages are used and it requires little to no computation.

VOLATILITY VERSUS PROBABILITY OF TECHNICAL SUCCESS

The discussion of volatility will be incomplete if we do not discuss what does and does not go into a volatility estimate. Volatility drives the value of an option, and the value of an option is the value of strategic flexibility. Hence, items that should be modeled and simulated in a DCF to obtain the volatility estimate should be those items that will change a person's decision about whether to execute a particular project, such as fluctuations in revenues, price, operating expenses, market size, market share, competition, and so forth. As an example, if prices increase, management may decide to execute an expansion option to spin off another ancillary product or expand to another market. However, in the R&D area, for example, in the pharmaceutical and biotech industries, R&D progresses through stages (Preclinical Phase I, Phase II, Phase III, Biologics FDA approval, go to market strategy, and so forth) where each stage has a particular probability of technical success or PTS (based on historical experience, industry averages, scientific analysis, best-guesses, or scenario analysis).

These PTS probabilities are *not* strategic options! That is, if a phase fails, you're out of the game, regardless of all the options that exist currently or in the future. These PTS values should be modeled in the DCF and can be simulated as well (use a Bernoulli distribution and set the PTS as the probability parameter in the Risk Simulator software). These PTSs can be assumed to be statistically independent of each other and can be multiplied with one another, or a correlation can be set among them, or a modeling relationship can be created—set all future cash flows to a particular phase's PTS just like an on-off switch, where all future cash flows at and after a particular phase either occur or they don't if the phase fails. Simulate the model and use the resulting average NPV as the expected value of the project, after accounting for these PTS events. Then, simulate the model again without these PTSs changing, and capture the volatility as discussed in this appendix. The volatility therefore captures the uncertainties in the market events that determine what options will be executed. For instance, PTS events are discrete event simulations that are *not* options—you are stuck with whatever happens. However, if a phase is successful, management may still have the strategic option to stop and abandon a phase, or decide to continue onto another phase (sequential compound option) even if the market outlook is bad but the R&D phase is successful. The strategic option here is the stage-gate development, not the PTS. Therefore, both PTS and stage-gate options have to be computed, but do not double count their values—incorrectly simulating PTS to obtain the volatility will artificially inflate the volatility and the option value, which makes no sense because a highly risky project with wide swings in PTS should not have higher project values.

APPENDIX 7B

Black-Scholes in Action

This appendix discusses the fundamentals of the Black-Scholes model. Although the Black-Scholes model is not a good approach to use in its entirety, it is often useful as a gross approximation method as well as a benchmark. Hence, understanding the fundamentals of the Black-Scholes model is important.

The Black-Scholes model is summarized as follows, with a detailed explanation of the procedures by which to obtain each of the variables.

$$Call = S_t\Phi(d_1) - Xe^{-rf(T)}\Phi(d_2)$$

$$\text{where } d_1 = \frac{\ln\left(\dfrac{S_0}{X}\right) + \left(rf + \dfrac{1}{2}\sigma^2\right)(T)}{\sigma\sqrt{T}}$$

$$\text{and } d_2 = d_1 - \sigma\sqrt{T}$$

Φ is the cumulative standard normal distribution function;
S is the value of the underlying asset or the stock price;
X is the strike price or the cost of executing the option;
rf is the nominal risk-free rate;
σ is the annualized volatility; and
T is the time to expiration or the economic life of the strategic option.

In order to fully understand and use the model, we need to understand the assumptions under which the model was constructed. These are essentially the caveats that go into using real options in valuing any asset. These assumptions are violated quite often, but the model should still hold up to scrutiny. The main assumption is that the underlying asset's price structure follows a Geometric Brownian Motion with static drift and volatility parameters and that this motion follows a Markov-Weiner stochastic process. The general derivation of a Markov-Weiner stochastic process takes the form of $dS = \mu Sdt + \sigma SdZ$, where $dZ = \varepsilon\sqrt{dt}$ and dZ is a Weiner process, μ is the drift rate, and σ is the volatility measure. The other assumptions are fairly standard, including a fair and timely efficient market with no riskless arbitrage opportunities, no transaction costs, and no taxes. Price changes are also assumed to be continuous and instantaneous.

The variables in the Black-Scholes model have the following relationships to the resulting call value, assuming a European call:

- Underlying asset value +
- Expiration cost −
- Time to expiration +
- Volatility +
- Risk-free rate +

APPENDIX 7C

Binomial Path-Dependent and Market-Replicating Portfolios

Another method for solving a real options problem includes the use of binomial lattice structures coupled with market-replicating portfolios. In order to correctly value market-replicating portfolios, we must be able to create a cash-equivalent replicating portfolio from a particular risky security and risk-free asset. This cash-equivalent replicating portfolio will have the same exact payoff series as the project in each state where the price of the cash equivalent replicating portfolio will be the value of the project itself. This is because we introduce a Martingale-based q measure, which is in essence a risk-adjusted or risk-neutral parameter. It is therefore not necessary to use probability estimates of the states of nature. The risk-adjusted discount rate is not computed, and nothing is known about the risk tolerances of the firm. All the required information is implicitly included in the relative prices of the risk-free asset and risky asset. The assumption is that as long as prices are in true equilibrium, the market information tells us all that we need to know. The other assumption is that the portfolio is arbitrage-free, such that the Arbitrage Pricing Theory holds true at any point in time. However, the Arbitrage Pricing Theory does not require the actual portfolio to be observable, and the portfolio set does not have to be intertemporally stationary.

Compare this complicated method using market-replicating portfolio with a much simpler to use *risk-neutral probability approach*. In theory, both approaches obtain the same results, but the latter approach is much simpler to apply. Thus, in this book, we focus on the risk-neutral probabilities approach to solve sample real options problems. However, for completeness, the following text illustrates a simple market-replicating approach to solving a real options problem.

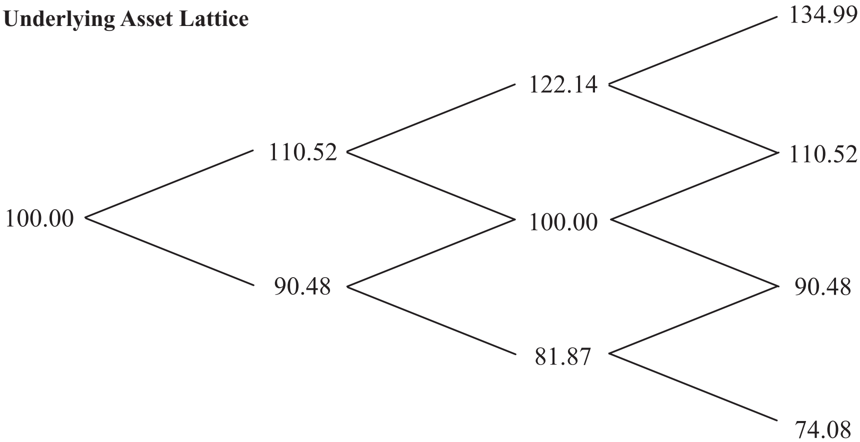


FIGURE 7C.1 Underlying Asset Lattice in Risk-Neutral Probability Approach

In this example, we solve a simple American call option using the risk-neutral probability approach (Figures 7C.1 and 7C.2), and then solve the same option using the market-replicating portfolio approach to compare the results. We will verify that theory holds such that both approaches obtain identical call option values.

For both approaches, assume the following parameters: asset value = \$100, strike or implementation cost = \$100, maturity = 3 years, volatility = 10%, risk-free rate = 5%, dividends = 0%, and steps = 3.

Risk-Neutral Probability Approach

Stepping Time = 1.00
Up Jump Size (up) = 1.1052
Down Jump Size (down) = 0.9048
Risk-Neutral Probability = 0.7309

Figures 7C.1 and 7C.2 show the computations using the risk-neutral probability approach.

Market-Replicating Portfolio Approach

In solving the market-replicating approach, we require the use of the following formulae:

- Hedge ratio (h):

$$h_{i-1} = \frac{C_{up} - C_{down}}{S_{up} - S_{down}}$$

- Debt load (D):

$$D_{i-1} = S_i(h_{i-1}) - C_i$$

- Call value (C) at node i:

$$C_i = S_i(h_i) - D_i e^{-rf(\delta t)}$$

- Risk-adjusted probability (q):

$$q_i = \frac{S_{i-1} - S_{down}}{S_{up} - S_{down}}$$

obtained assuming $S_{i-1} = q_i S_{up} + (1 - q_i) S_{down}$. This means that $S_{i-1} = q_i S_{up} + S_{down} - q_i S_{down}$ and $q_i[S_{up} - S_{down}] = S_{i-1} - S_{down}$, so we get

$$q_i = \frac{S_{i-1} - S_{down}}{S_{up} - S_{down}}$$

In addition, a new naming convention is required, as seen in Figure 7C.3.

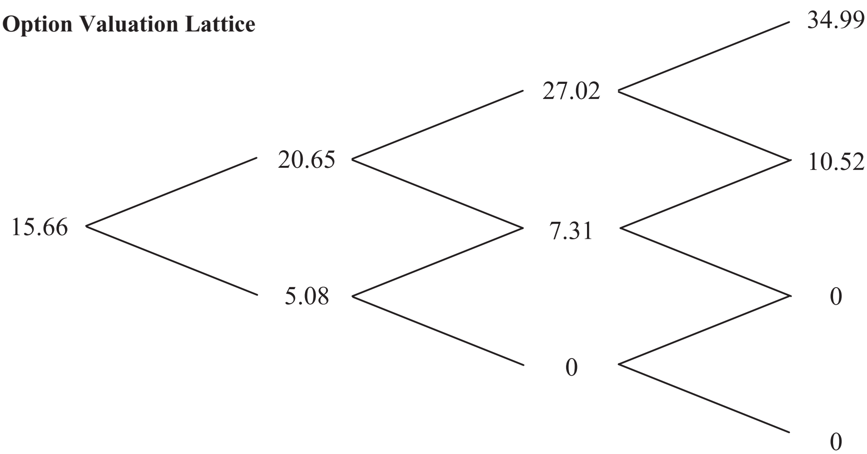


FIGURE 7C.2 Valuation Lattice in Risk-Neutral Probability Approach

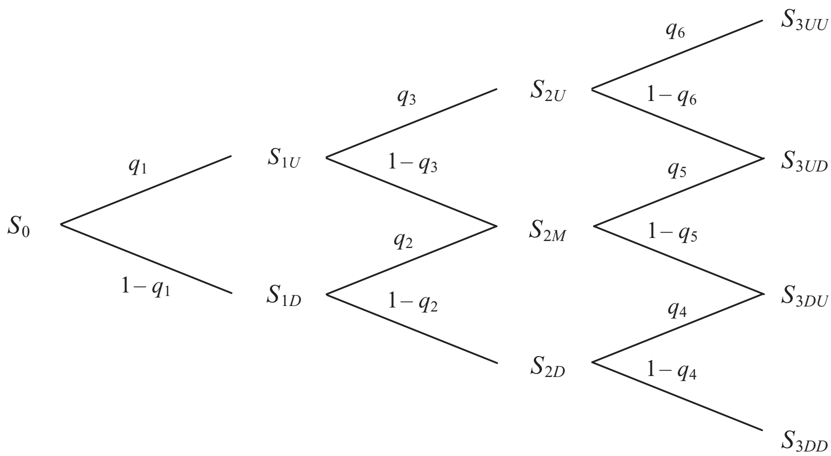


FIGURE 7C.3 New Naming Convention in the Market-Replicating Approach

Step 1: Get the call values at the terminal nodes. Using call value (C) at node i: $C_i = max\ [S_{3ij} - cost,\ 0]$ we get:

$$C_{3UU} = 34.99$$
$$C_{3UD} = 10.52$$
$$C_{3DU} = 0$$
$$C_{3DD} = 0$$

Step 2: Get the hedge ratios for the terminal branches. Using the hedge ratio *(h)*:

$$h_{i-1} = \frac{C_{up} - C_{down}}{S_{up} - S_{down}}$$

we get:

$$h_{2U} = 1.0000$$
$$h_{2M} = 0.5250$$
$$h_{2D} = 0.0000$$

Step 3: Get the debt load for the terminal branches. Using the debt load (D): $D_{i-1} = S_i(h_{i-1}) - C_i$ we get:

$$D_{2U} = 100.0000$$
$$D_{2M} = 47.5030$$
$$D_{2L} = 0.0000$$

Step 4: Repeat to obtain the call values one node back, $t = 2$.

$$C_{2U} = 27.0171$$
$$C_{2M} = 7.3137$$
$$C_{2D} = 0.0000$$

Step 5: Repeat and obtain the hedge ratios for the one branch back, $t = 1$.

$$h_{1U} = 0.8899 \qquad h_{1D} = 0.4034$$

Step 6: Repeat and obtain the debt load for one branch back, $t = 1$.

$$D_{1U} = 81.6753$$
$$D_{1D} = 33.0263$$

Step 7: Get the call values one node back.

$$C_{1U} = 20.6598$$
$$C_{1D} = 5.0840$$

Step 8: Get the hedge ratios for two branches back, $t = 0$.

$h_0 = 0.7772$ (rounded)

Step 9: Get the debt load for two branches back, $t = 0$.

$D_0 = 65.2363$ (rounded)

Step 10: Get the call value at $t = 0$, the option value of this analysis.

$C_0 = 15.66$

The resulting 15.66 is identical to the results from the risk-neutral probability approach (Figure 7C.2) but this market-replicating method is more tedious and very difficult to apply and very difficult to explain. Therefore, the risk-neutral probability approach is preferred for real options analysis. Finally, the risk-neutral probability approach is more flexible and changing its inputs or adding certain exotic conditions are very easy but can sometimes be mathematically intractable when applying the market replicating approach.

APPENDIX 7D

Single-State Static Binomial Example

DIFFERENTIAL EQUATIONS

This appendix illustrates yet another approach to solving real options problems, that of basic differential equations. In this example, suppose that a certain firm has an option to change its current mummification embalming technology from the traditional Dull Old Method (D) to a new and revolutionary approach using the latest liquid nitrogen freezing equipment in Cryogenic Technology (C). Obviously, in order to do so, it would cost the firm some restructuring cost of approximately \$9,000 to convert the existing lab into a freezing chamber and an additional \$1,000 scrapping cost to dismantle the old equipment. Hence, the total cost of implementation is assumed to be \$10,000 and, for simplicity, assumed to be fixed no matter when the implementation takes place, either at present or sometime in the future. The benefit of the new cryogenics technology is that the mummification cost will be fixed at \$500 each. This incremental fixed cost is highly desirable to senior management as it assists in cost-cutting strategies and provides a really good way to forecast future profitability.

Based on the current technology using the same Dull Old Method, it costs on average \$2,000 in incremental marginal cost. However, this cost fluctuates depending on market demand. For instance, if the market is good (G), where the demand for mummification increases, the firm will have to hire additional help and have employees work overtime, costing on average \$3,000 marginal cost per unit. In a down or bad (B) market, when demand is significantly low, the firm can lay off individuals, put key employees on a rotating part-time schedule, and cut overhead costs significantly, resulting in only a \$400 incremental marginal cost. The question is, will the new cryogenics be financially feasible assuming there is a 50 percent chance of a good upswing market for mummification and a 10 percent cost of capital (r)? If it is feasible, then should the implementation be done now or later? The cost structure is presented graphically in Figure 7D.1 (all values in \$).

If the firm moves and starts at time 0, profits or the benefits from cost savings will be

$$\pi_0 = DMC_0 - CMC_0 = \$2{,}000 - \$500 = \$1{,}500$$

Restructure Cost = \$9,000 (*RC*)
Scrapping Cost = \$1,000 (*SC*)
Total Cost = *RC* + *SC* = \$10,000 (*TC*)

Time = 0

$DMC_0 = 2{,}000$ cost
$CMC_0 = 500$ fixed cost
$\pi_0 = 1{,}500$

Time ≥ 1

Good outcome

$CMC_n^G = 500$ fixed cost
$DMC_n^G = 3{,}000$ cost
$\pi_n^G = 2{,}500$

Bad outcome

$CMC_n^B = 500$ fixed cost
$DMC_n^B = 400$ cost
$\pi_n^B = -100$

where we define

DMC_0	Dull Old Method's marginal cost at time 0
CMC_0	Cryogenics method's marginal cost at time 0
π_0	Profits through savings, at time 0
CMC_n^G	Cryogenics method's marginal cost at time *n* with good market conditions
DMC_n^G	Dull method's marginal cost at time *n* with good market conditions
π_n^G	Profits through savings, at time *n* with good market conditions
CMC_n^B	Cryogenics method's marginal cost at time *n* with bad market conditions
DMC_n^B	Dull method's marginal cost at time *n* with bad market conditions
π_n^B	Profits through savings, at time *n* with bad market conditions

FIGURE 7D.1 Cost Structure

This is the current period (time 0) cost savings only. Because the implementation has already begun, the future periods will also derive cost savings such that for time where $n \geqslant 1$, we have under the good market conditions

$$\pi_n^G = DMC_n^G - CMC_n^G = \$3{,}000 - \$500 = \$2{,}500$$

Under the bad market conditions, we have

$$\pi_n^B = DMC_n^B - CMC_n^B = \$400 - \$500 = -\$100$$

Assuming we know from historical data and experience that there is a 50 percent chance of a good versus a bad market, we can take the expected value of the profits $E(\pi)$ or cost savings of these two market conditions:

$$E(\pi_1) = p\pi_1^G + (1-p)\pi_1^B = 0.5(\$2{,}500) + (0.5)(-\$100) = \$1{,}200$$

Because this expected value of \$1,200 occurs for every period in the future with the same fixed value with zero growth, the future cash flow stream can be summarized as perpetuities, and the present value of executing the implementation now $E(\pi_0)$ will be

$$E(\pi_0) = \sum_{n=0}^{\infty} E(\pi_n)/(1+r)^n \cong \pi_0 + E(\pi_1)/r = \$1{,}500 + \frac{\$1{,}200}{0.1} = \$13{,}500$$

Hence, the net present value of the project is simply the value generated through the cost savings of the cryogenics technology less the implementation cost:

$$NPV = \pi_0 - TC_0 = \$13{,}500 - \$10{,}000 = \$3{,}500$$

If the firm decides to switch at a future time when $k \geq 1$, given a good market (ω_G):

$$\Pi_k^G = \sum_{n=k}^{\infty} E(\pi_n^G/\omega_G)(1+r)^{k-n} \cong \sum_{n=k}^{\infty} \pi_1^G (1+r)^{k-n} \cong \pi_1^G \left[\frac{r+1}{r}\right]$$

$$\Pi_k^G = \$2{,}500\left[\frac{1.1}{0.1}\right] = \$27{,}500$$

Similarly, given that the market is unfavorable (ω_U), at time $k \geq 1$:

$$\Pi_k^B = \sum_{n=k}^{\infty} E(\pi_n^B/\omega_u)(1+r)^{k-n} \cong \pi_1^B \left[\frac{r+1}{r}\right]$$

$$\Pi_k^B = -\$100\left[\frac{1.1}{0.1}\right] = -\$1{,}100$$

For instance, assume $k = 1$,

$$\Pi_0 = \pi_0 + [p^G(\pi_1^G) + (1 - p^G)(\pi_1^B)]/(1 + r)$$

$$\begin{aligned}\Pi_0 &= \$1{,}500 + [0.5(\$27{,}500) + (1 - 0.5)(-\$1{,}100)]/(1 + 0.1) \\ &= \$13{,}500\end{aligned}$$

$$NPV = \Pi_0 - TC_0 = \$13{,}500 - \$10{,}000 = \$3{,}500$$

However, this is incorrect because we need to consider the analysis in terms of strategic optionalities. As we have the opportunity, the right to execute and not the obligation to do so, the firm would execute the option if it is financially feasible and not execute otherwise. Hence, the options are feasible only when the good market outcome occurs in the future and not executed in the bad market condition. Therefore, the actual net present value should be

$$\frac{\Pi_k^B p_B}{(1 + r)} + \frac{\Pi_k^G p_G}{(1 + r)} = 0 + 0.5\left[\frac{\$27{,}500 - \$10{,}000}{1.1}\right] = \$7{,}954$$

Hence, we can create a generic valuation structure for the option value as above. To add a level of complexity, the total cost should be discounted at a risk-free rate (r_f), as we segregate the market risk (Π_G) and private risk (TC), and the structure could be represented as

$$\Pi_{CALL} = \max\left\{[\Pi_0 - TC], \frac{p^G(\Pi_1^G)}{1 + r} - \frac{TC}{1 + r_f}\right\}$$

$$= \max\left\{\left[\pi_0 + \frac{E(\pi_1)}{r} - TC\right]^+, \left[\frac{p^G\pi_1^G\left(\frac{r + 1}{r}\right)}{1 + r} - \frac{TC}{1 + r_f}\right]^+\right\}$$

This simply is to calculate the maximum value of either starting now, which is represented by $[\Pi_0 - TC]$ or starting later, which is represented as

$$\frac{p^G(\pi_1^G)}{1 + r} - \frac{TC}{1 + r_f}$$

Because the future starting point has been collapsed into a single static state, any starting points in the future can be approximated by the valuation of a single period in the future.

OPTIMAL TRIGGER VALUES

A related analysis is that of optimal trigger values. Looking at the formulation for the call valuation price structure, if there is a change in total cost, that is, the initial capital outlay, something interesting occurs. The total cost in starting now is not discounted because the outlay occurs immediately. However, if the outlay occurs in the future, the total cost will have to be discounted at the risk-free rate. Therefore, the higher the initial cost outlay, the discounting effect of starting in the future decreases the effective cost in today's dollar, hence making it more efficient to wait and defer the cost until a later time. If the cost is lower and the firm becomes more operationally efficient, it is beneficial to begin now as the value of starting now is greater than waiting. The total cost break-even point can be obtained by solving the call valuation equation above for total cost and can be represented as

$$TC^* = \left[1 - \frac{1}{(1+r_f)^n}\right]^{-1}\left[\pi_0 + \frac{E[\pi_1]}{r} - \frac{p^G\pi_1^G\left(\frac{r+1}{r}\right)}{(1+r)^n}\right]$$

If total cost of implementation exceeds TC^* above, it is optimal to wait, and if total cost does not exceed TC^*, it is beneficial to execute the option now. Remember that the optimal trigger value depends on the operational efficiency of the firm as well, because it is a dynamic equation given that the optimal trigger value depends on how much money can be saved with implementation of the Cryogenic modifications. Refer to Chapters 10 and 11 for details on optimal timing and optimal trigger values computed using binomial lattices.

Uncertainty Effects on Profit or Cost Savings π

Suppose we keep the first moment and change the second moment, that is, change the spread and, hence, the risk or uncertainty of the profit or cost savings while leaving the expected payoffs the same. It would make sense that waiting is better. Let's see how this works. Recall that the original case had $\pi_n^G = \$2{,}500$, $\pi_n^B = -\$100$, with a 50 percent chance of going either way, creating an expected value of 0.5(\$2,500 − \$100) = \$1,200. Now, suppose we change the values to $\pi_n^{G*} = \$3{,}000$ and $\pi_n^{B*} = -\$600$, with a 50 percent chance of going either way, creating a similar expected value of 0.5(\$3,000 – \$600) = \$1,200 as in the original case. However, notice that the risk has increased in the second case as the variability of payoffs has increased. So, we can easily recalculate the value of

$$\Pi_k^G = \pi_1^G\left[\frac{1+r}{r}\right] = \$3{,}000\left[\frac{1.1}{0.1}\right] = \$33{,}000,$$

(for all $k \geqslant 1$) which is higher than the original \$27,500.

The conclusion is that the higher the uncertainty, the higher is the value of waiting. This is because the firm has no information on the market demand fluctuations. The higher the market volatility, the better off the firm will be by waiting until this market uncertainty has been resolved and it knows what market demand looks like before proceeding with the capital investment.

APPENDIX 7E

Sensitivity Analysis with Delta, Gamma, Rho, Theta, Vega, and Xi

Using the corollary outputs generated by options theory, we can use the results—namely, Delta, Gamma, Rho, Theta, Vega, and Xi—as a form of sensitivity analysis. By definition, sensitivity analysis, or stress testing, looks at the outcome of the change in the option price given a change in one unit of the underlying variables. In our case, these sensitivities reflect the instantaneous changes of the value of the option given a unit change in a particular variable, ceteris paribus. In other words, we can form a sensitivity table by simply looking at the corresponding values in Delta, Gamma, Rho, Theta, Vega, and Xi. Delta provides the change in value of the option given a unit change in the present value of the underlying asset's cash flow series. Gamma provides the rate of change in delta given a unit change in the underlying asset's cash flow series. Rho provides us with the change in the value of the option given that we change the interest rate one unit, Theta looks at the change per unit of time, Vega looks at the change per unit of volatility, and Xi looks at the change per unit of cost. In other words, one can provide a fairly comprehensive view of the way the value of the option changes given changes in these variables, thereby providing a test of the sensitivity of the option's value. A worse-case, nominal case, and best-case scenario can then be constructed. The sensitivity table not only provides a good test of the robustness of our results but also provides great insight into the value drivers in the firm, that is, which variables have the most impact on the firm's bottom line. The following provides the derivations of these sensitivity measures for a European option without dividend payments. In actual real options analysis, it might be easier to compute the sensitivities based on a percentage change to each input rather than instantaneous changes (refer to Chapter 9's tornado and sensitivity analysis in Risk Simulator).

CALL DELTA

Starting from $C = S_t N(d_1) - Xe^{-rT}N(d_2)$, where

$$d_1 = \frac{\ln\left(\frac{S_t}{X}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T)}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T}$$

we can get the call Delta, defined as the change in call value for a change in the underlying asset value, that is, the partial derivative

$$\frac{\partial C_t}{\partial S_t}$$

at an instantaneous time t. Differentiating, we obtain:

$$Delta = \Delta = \frac{\partial C_t}{\partial S_t} = N(d_1) + S_t \frac{\partial N(d_1)}{\partial S_t} - Xe^{-rT} \frac{\partial N(d_2)}{\partial S_t}$$

$$\frac{\partial C_t}{\partial S_t} = N(d_1) + S_t \frac{e^{-\frac{1}{2}d_1^2}}{\sqrt{2\pi}} \frac{\partial d_1}{\partial S_t} - Xe^{-rT} \frac{e^{-\frac{1}{2}d_2^2}}{\sqrt{2\pi}} \frac{\partial d_2}{\partial S_t}$$

$$\frac{\partial C_t}{\partial S_t} = N(d_1) + S_t \frac{e^{-\frac{1}{2}d_1^2}}{\sqrt{2\pi}} \frac{1/S_t}{\sigma\sqrt{T}} - Xe^{-rT} \frac{e^{-\frac{1}{2}(d_1 - \sigma\sqrt{T})^2}}{\sqrt{2\pi}} \frac{1/S_t}{\sigma\sqrt{T}}$$

$$\frac{\partial C_t}{\partial S_t} = N(d_1) + \frac{e^{-\frac{1}{2}d_1^2}}{\sigma\sqrt{2\pi T}} \left[1 - Xe^{-rT} \frac{e^{-\frac{1}{2}\sigma^2 T + d_1\sigma\sqrt{T}}}{S_t}\right]$$

$$\frac{\partial C_t}{\partial S_t} = N(d_1) + \frac{e^{-\frac{1}{2}d_1^2}}{\sigma\sqrt{2\pi T}} \left[1 - \frac{Xe^{-rT}}{S_t} e^{-\frac{1}{2}\sigma^2 T} e^{\ln(S_t/X) + (r + \sigma^2/2)T}\right]$$

$$\frac{\partial C_t}{\partial S_t} = N(d_1) + \frac{e^{-\frac{1}{2}d_1^2}}{\sigma\sqrt{2\pi T}} \left[1 - \frac{Xe^{-rT}}{S_t} e^{-\frac{1}{2}\sigma^2 T} \frac{S_t}{X} e^{rT} e^{\frac{1}{2}\sigma^2 T}\right]$$

$$Delta = \Delta = \frac{\partial C_t}{\partial S_t} = N(d_1)$$

CALL GAMMA

$$Gamma = \Gamma = \frac{\partial \Delta}{\partial S_t}$$

$$\frac{\partial \Delta}{\partial S_t} = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_1^2} \frac{\partial d_1}{\partial S_t}$$

$$\frac{\partial \Delta}{\partial S_t} = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_1^2} \frac{1}{S_t \sigma \sqrt{T}}$$

$$Gamma = \Gamma = \frac{\partial \Delta}{\partial S_t} = \frac{e^{-\frac{1}{2}d_1^2}}{S_t \sigma \sqrt{2\pi T}}$$

CALL RHO

$$Rho = \mathrm{P} = \frac{\partial C_t}{\partial r} = S_t \frac{\partial N(d_1)}{\partial r} + XTe^{-rT}N(d_2) - Xe^{-rT}\frac{\partial N(d_2)}{\partial r}$$

$$\frac{\partial C_t}{\partial r} = S_t \frac{e^{-\frac{1}{2}d_1^2}}{\sqrt{2\pi}} \frac{\partial d_1}{\partial r} + XTe^{-rT}N(d_2) - Xe^{-rT} \frac{e^{-\frac{1}{2}d_2^2}}{\sqrt{2\pi}} \frac{\partial d_2}{\partial r}$$

$$\frac{\partial C_t}{\partial r} = \frac{e^{-\frac{1}{2}d_1^2}}{\sqrt{2\pi}} \left[S_t \frac{\partial d_1}{\partial r} - Xe^{-rT} e^{\frac{1}{2}(d_1^2 - d_2^2)} \frac{\partial d_1}{\partial r} \right] + XTe^{-rT}N(d_2)$$

$$\frac{\partial C_t}{\partial r} = \frac{e^{-\frac{1}{2}d_1^2}}{\sqrt{2\pi}} \frac{\partial d_1}{\partial r} \left[S_t - Xe^{-rT} e^{-\frac{1}{2}\sigma^2 T} e^{\ln(S/X) + (r + \sigma^2/2)T} \right] + XTe^{-rT}N(d_2)$$

$$\frac{\partial C_t}{\partial r} = \frac{e^{-\frac{1}{2}d_1^2}}{\sqrt{2\pi}} \frac{\partial d_1}{\partial r} \left[S_t - \frac{XS_t}{X} \right] + XTe^{-rT}N(d_2)$$

$$Rho = \mathrm{P} = \frac{\partial C_t}{\partial r} = XTe^{-rT}N(d_2)$$

CALL THETA

Start with $\dfrac{\partial C_t}{\partial T} = S_t \dfrac{\partial N(d_1)}{\partial T} - X \dfrac{\partial}{\partial T}\left[e^{-rT}N(d_2) \right]$

$$\frac{\partial C_t}{\partial T} = S_t \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_1^2} \frac{\partial d_1}{\partial T} + rXe^{-rT}N(d_2) - Xe^{-rT}\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_2^2} \frac{\partial d_2}{\partial T}$$

As $\frac{\partial d_1}{\partial T} = \frac{-\ln\frac{S_t}{X}}{2\sigma T^{3/2}} + \frac{1}{2\sigma\sqrt{T}}\left[r + \frac{\sigma^2}{2}\right]$ we have $\frac{\partial d_2}{\partial T} = \frac{\partial d_1}{\partial T} - \frac{\sigma}{2\sqrt{T}}$ and

$$\frac{\partial C_t}{\partial T} = S_t \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_1^2} \frac{\partial d_1}{\partial T} + rXe^{-rT}N(d_2) - Xe^{-rT}\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_2^2} \left[\frac{\partial d_1}{\partial T} - \frac{\sigma}{2\sqrt{T}}\right]$$

$$\frac{\partial C_t}{\partial T} = \frac{e^{-\frac{1}{2}d_1^2}}{\sqrt{2\pi}}\left[S_t \frac{\partial d_1}{\partial T} - Xe^{-rT}e^{-\frac{1}{2}(d_1^2 - d_2^2)}\left(\frac{\partial d_1}{\partial T} - \frac{\sigma}{2\sqrt{T}}\right)\right] + rXe^{-rT}N(d_2)$$

$$\frac{\partial C_t}{\partial T} = \frac{e^{-\frac{1}{2}d_1^2}}{\sqrt{2\pi}}\left[S_t \frac{\partial d_1}{\partial T} - Xe^{-rT}\, e^{-\frac{1}{2}(\sigma^2 T)}\, e^{\sigma d_1\sqrt{T}}\left(\frac{\partial d_1}{\partial T} - \frac{\sigma}{2\sqrt{T}}\right)\right] + rXe^{-rT}N(d_2)$$

$$\frac{\partial C_t}{\partial T} = \frac{e^{-\frac{1}{2}d_1^2}}{\sqrt{2\pi}}\left[S_t \frac{\partial d_1}{\partial T} - Xe^{-rT\, -\frac{1}{2}\,(\sigma^2 T) + \ln(S/X) + (r+\sigma^2/2)T}\left(\frac{\partial d_1}{\partial t} - \frac{\sigma}{2\sqrt{T}}\right)\right] + rXe^{-rT}N(d_2)$$

$$\frac{\partial C_t}{\partial T} = \frac{e^{-\frac{1}{2}d_1^2}}{\sqrt{2\pi}}\left[S_t \frac{\partial d_1}{\partial T} - S_t\frac{\partial d_1}{\partial T} + \frac{S_t\sigma}{2\sqrt{T}}\right] + rXe^{-rT}N(d_2)$$

$$Theta = \Theta = \frac{-\partial C_t}{\partial T} = \frac{-S\sigma e^{-\frac{1}{2}d_1^2}}{2\sqrt{2\pi T}} - rXe^{-rT}N(d_2)$$

CALL VEGA

$$Vega = V = \frac{\partial C_t}{\partial \sigma} = \frac{\partial}{\partial \sigma}[S_t N(d_1) - Xe^{-rT}N(d_2)]$$

$$\frac{\partial C_t}{\partial \sigma} = \frac{S_t}{\sqrt{2\pi}} e^{-\frac{1}{2}d_1^2} \frac{\partial d_1}{\partial \sigma} - Xe^{-rT}\, e^{-\frac{1}{2}d_2^2} \frac{\partial d_2}{\partial \sigma}$$

$$\frac{\partial C_t}{\partial \sigma} = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_1^2}\left[S_t \frac{\partial d_1}{\partial \sigma} - Xe^{-rT}\, e^{-(d_1^2 - d_2^2)} \frac{\partial d_2}{\partial \sigma}\right]$$

$$\frac{\partial C_t}{\partial \sigma} = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_1^2}\left[S_t \frac{\partial d_1}{\partial \sigma} - Xe^{-rT}\, e^{-\frac{1}{2}\sigma^2 T + d_1\sigma\sqrt{T}} \frac{\partial d_2}{\partial \sigma}\right]$$

$$\frac{\partial C_t}{\partial \sigma} = \frac{1}{\sqrt{2\pi}}\ e^{-\frac{1}{2}d_1^2}\left[S_t\frac{\partial d_1}{\partial \sigma} - Xe^{-rT}\,e^{-\frac{1}{2}\sigma^2 T}\ e^{\ln(S/X)+(r+\sigma^2/2)T}\frac{\partial d_2}{\partial \sigma}\right]$$

$$\frac{\partial C_t}{\partial \sigma} = \frac{1}{\sqrt{2\pi}}\ e^{-\frac{1}{2}d_1^2}\left[S_t\frac{\partial d_1}{\partial \sigma} - S_t\frac{\partial d_2}{\partial \sigma}\right]$$

$$\frac{\partial C_t}{\partial \sigma} = \frac{1}{\sqrt{2\pi}}\ e^{-\frac{1}{2}d_1^2}\,S_t\left[\frac{\partial d_1}{\partial \sigma} - \frac{\partial d_1}{\partial \sigma} + \sqrt{T}\right]$$

$$Vega = V = \frac{\partial C_t}{\partial \sigma} = \frac{S_t\sqrt{T}e^{\frac{-d_1^2}{2}}}{\sqrt{2\pi}}$$

CALL XI

$$Xi = \Xi = \frac{\partial C_t}{\partial X} = -N(d_2)\,e^{-rT} + S_t\frac{\partial N(d_1)}{\partial X} - Xe^{-rT}\,\frac{\partial N(d_2)}{\partial X}$$

$$\frac{\partial C_t}{\partial X_t} = -N(d_2)e^{-rT} + S_t\frac{e^{-\frac{1}{2}d_1^2}}{\sqrt{2\pi}}\,\frac{\partial d_1}{\partial X} - Xe^{-rT}\,\frac{e^{-\frac{1}{2}d_2^2}}{\sqrt{2\pi}}\,\frac{\partial d_2}{\partial X}$$

$$\frac{\partial C_t}{\partial X_t} = -N(d_2)e^{-rT} + S_t\frac{e^{-\frac{1}{2}d_1^2}}{\sqrt{2\pi}}\,\frac{1/S_t}{\sigma\sqrt{T}} - Xe^{-rT}\,\frac{e^{-\frac{1}{2}(d_1-\sigma\sqrt{T})^2}}{\sqrt{2\pi}}\,\frac{1/S_t}{\sigma\sqrt{T}}$$

$$\frac{\partial C_t}{\partial X_t} = -N(d_2)e^{-rT} + \frac{e^{-\frac{1}{2}d_1^2}}{\sigma\sqrt{2\pi T}}\left[1 - Xe^{-rT}\,\frac{e^{-\frac{1}{2}\sigma^2 T + d_1\sigma\sqrt{T}}}{S_t}\right]$$

$$\frac{\partial C_t}{\partial X_t} = -N(d_2)e^{-rT} + \frac{e^{-\frac{1}{2}d_1^2}}{\sigma\sqrt{2\pi T}}\left[1 - \frac{Xe^{-rT}}{S_t}\,e^{-\frac{1}{2}\sigma^2 T}\ e^{\ln(S_t/X)+(r+\sigma^2/2)T}\right]$$

$$\frac{\partial C_t}{\partial X_t} = -N(d_2)e^{-rT} + \frac{e^{-\frac{1}{2}d_1^2}}{\sigma\sqrt{2\pi T}}\left[1 - \frac{Xe^{-rT}}{S_t}\,e^{-\frac{1}{2}\sigma^2 T}\,\frac{S_t}{X}\,e^{rT}\,e^{\frac{1}{2}\sigma^2 T}\right]$$

$$Xi = \Xi = \frac{\partial C_t}{\partial X_t} = -N(d_2)e^{-rT}$$

APPENDIX 7F

Reality Checks

THEORETICAL RANGES FOR OPTIONS

One of the tests to verify whether the results calculated using the real options analytics are plausible is to revert back to financial options pricing theory. By construction, the value of a call option can be no lower than zero when the option is left to expire, that is, we have the call option value, $C \geq max[S - Xe^{-rT}, 0]$, and it can be no higher than the value of the asset, which we have defined as S, such that $C \leq S$. If the calculated results fall outside this range, we can reasonably say that the analysis is flawed, potentially due to unreasonable assumptions on creating the forecast cash flows. However, if the results fall comfortably within the range, we cannot be certain it is correct, only reasonably sure the analysis is correct assuming all the input variables are also reasonable. The main thrust of using this option range spread is to test the width of this spread, that is, the tighter the spread, the higher the confidence that the results are reasonable. Also, one could perform a sensitivity analysis by changing the input variables and assumptions to see if the spread changes, that is, if the spread widens or shifts.

SMIRR AND SNPV CONSISTENCY

Another plausibility test includes the use of a sequential modified internal rate of return (SMIRR) method and a sequential net present value (SNPV) method. If all goes well in the forecast of free cash flow and the discounted cash flow analysis holds up, then the MIRR[1] and NPV of the cash flow stream should theoretically be smooth. That is, the entire stream of cash flow should have MIRR and NPV similar to that of the cash flow stream less the first year's free cash flow, eliminating the first year's free cash flow as a reduction of the original net present value and setting the first year's cash flows to zeros. This method is repeated for all subsequent years. The reinvestment rate and the discount rate could be set at different levels for the computation of the MIRR and NPV. This interest-rate-jackknifing approach looks at the consistency and smoothness of the predicted cash flows over time. However, this approach is cumbersome and is seldom used.

MINIMAX APPROACH

If relevant probabilities are provided by the firm's management on specific outcomes of the cash flow over time, a *regret analysis* can be performed as a means of calculating the relevant value of the asset. This regret analysis takes the form of a Minimax approach in Bayesian probability theory in the context of decision sciences. Essentially, it measures the relevant outcome of a forward-looking cash flow series given the appropriate probabilities, calculates the expected monetary value of the scenario, and identifies the scenario at which one minimizes the maximum amount of regret—hence the name Minimax. However, even if relevant probabilities are provided, they should not be used because these forecasted values add an additional element of uncertainty and because management can hardly be expected to provide a solid, dependable, and reliable set of economic forecasts, let alone the respective probabilities associated with each forecast's outcome. The analysis can be coupled within a Game Theory framework, where the best strategic outcome under the Nash equilibrium will always be observed. The specifics of Game Theory are beyond the scope of this book.

IMPLIED VOLATILITY TEST

Using the developed real options model and approach, we could set the volatility measure as the dependent variable to calculate. This implied volatility can then be measured against the historical volatility of the firm's cash flow situation or benchmarked against the volatility of cash flows of corresponding comparable companies under similar risks, functions, and products. The implied volatility can then be tested using a parametric t-test or a nonparametric Wilcoxon sign-rank test[2] to see if it is statistically identical to the mean and median of the set of comparable firms' volatilities.

An alternative is to use the Newton-Raphson search criteria for implied volatility measures through a series of guesses.

$$\sigma_1^* = \sqrt{\left|\ln\left[\frac{S}{X}\right] + rT\right|\left(\frac{2}{T}\right)}$$

$$\sigma_2^* = \sigma_1^* - \frac{[C(\sigma_1^*) - C(\sigma)]e^{\frac{d_1^2}{2}} 2\sqrt{\pi}}{S\sqrt{T}}$$

where C is the call value

$$\sigma_3^* = \sigma_2^* - \frac{[C(\sigma_2^*) - C(\sigma)]\, e^{\frac{d_1^2}{2}} 2\sqrt{\pi}}{S\sqrt{T}}$$

$$\sigma_{i+1}^* = \sigma_i^* - \frac{[C(\sigma_i^*) - C(\sigma)]\, e^{\frac{d_1^2}{2}} 2\sqrt{\pi}}{S\sqrt{T}}$$

APPENDIX 7G

Applying Monte Carlo Simulation to Solve Real Options

Monte Carlo simulation can be easily adapted for use in a real options paradigm. There are multiple uses of Monte Carlo simulation, including the ability to obtain a volatility estimate as input into the real options models, obtaining a range of possible outcomes in the discounted cash flow analysis, and simulating input parameters that are highly uncertain. Here, the discussion focuses on two distinct applications of Monte Carlo simulation: solving a real options problem versus obtaining a range of real options values. Although these two approaches are discussed separately, they can be used together in an analysis. See Chapter 9 for more technical details of running Monte Carlo simulations using the author's Risk Simulator software.

APPLYING MONTE CARLO SIMULATION TO OBTAIN A REAL OPTIONS RESULT

Monte Carlo simulation can be applied to solve a real options problem, that is, to obtain an option result. Recall that the mainstream approaches in solving real options problems are the binomial approach, closed-form equations, partial-differential equations, and simulation. In the simulation approach, a series of forecast asset values are created using the Geometric Brownian Motion, and the maximization calculation is applied to the end point of the series, and discounted back to time zero, at the risk-free rate. That is, starting with an initial seed value of the underlying asset, simulate out multiple future pathways using a Geometric Brownian Motion, where $\delta S_t = S_{t-1}\left(rf(\delta t) + \sigma\varepsilon\sqrt{\delta t}\right)$. That is, the change in asset value δS_t at time t is the value of the asset in the previous period S_{t-1} multiplied by the Brownian Motion $\left(rf(\delta t) + \sigma\varepsilon\sqrt{\delta t}\right)$. Recall that rf is the risk-free rate, δt is the time-steps,

σ is the volatility, and ε is the simulated value from a standard-normal distribution with mean of zero and a variance of one.

Figure 7G.1 illustrates an example of a simulated pathway used to solve a European option. Note that simulation can be easily used to solve European-type options, but it is fairly difficult to apply simulation to solve American-type options.[1] In this example, the one-year maturity European option is divided into five time-steps in the binomial lattice approach, which yields \$20.75, as compared to \$19.91 using the continuous Black-Scholes equation, and \$19.99 using 1,000 Monte Carlo simulations on 10 steps. In theory, when the number of time-steps in the binomial lattices is large enough, the results approach the closed-form Black-Scholes results. Similarly, if the number of simulation trials are adequately increased, coupled with an increase in the simulation steps, the results stemming from Monte Carlo simulation also approach the Black-Scholes value. See Figure 7G.1

The first step in Monte Carlo simulation is to decide on the number of steps to simulate. In the example, 10 steps were chosen for simplicity. Starting with the initial asset value of \$100 (S_0), the change in value from this initial value to the first period is seen as $\delta S_1 = S_0\left(rf(\delta t) + \sigma\varepsilon\sqrt{\delta t}\right)$. Hence, the value of the asset at the first time-step is equivalent to $S_1 = S_0 + \delta S_1 = S_0 + S_0\left(rf(\delta t) + \sigma\varepsilon\sqrt{\delta t}\right)$. The value of the asset at the second time-step is hence $S_2 = S_1 + \delta S_2 = S_1 + S_1\left(rf(\delta t) + \sigma\varepsilon\sqrt{\delta t}\right)$, and so forth, all the way until the terminal 10th time-step. Notice that because ε changes on each simulation trial, each simulation trial will produce an entirely different asset evolution pathway. At the end of the 10th time-step, the maximization process is then applied. That is, for a simple European option with a \$100 implementation cost, the function is simply $C_{10,i} = Max[S_{10,i} - X, 0]$. This is the call value $C_{10,i}$ at time 10 for the ith simulation trial. This value is then discounted at the risk-free rate to obtain the call value at time zero, that is, $C_{0,i} = C_{10,i}e^{-rf(T)}$. This is a single-value estimate for a single simulated pathway.

Applying Monte Carlo simulation for 1,000 trials and obtaining the mean value of C_0 yields \$19.99. This is termed the path-dependent simulation approach. There is a less precise shortcut to this simulation. That is, collapse all the 10 time-steps into a single time-step, using $S_T = S_0 + \delta S_T = S_0 + S_0(rf(T) + \sigma\varepsilon\sqrt{T})$, where the time T in this case is the one-year maturity. Then the call option value can be estimated using $C_{0,i} = Max[(S_{T,i} - X)e^{-rf(T)}, 0]$. Simulating the results 1,000 times yields the estimated option value of \$18.29. Obviously, the higher the number of simulations and the higher the number of steps in the simulation, the more accurate the results.

Figure 7G.2 illustrates the results generated by performing 1,000 simulation trials. Notice the lognormal distribution of the payoff functions.

Input Parameters

Expiration in Years	1.00
Volatility	45.00%
PV Asset Value	$100
Risk-free Rate	5.00%
Dividend rate	0.00%
Strike Cost	$100

Simulation Calculation

Simulate Value	0.00
Payoff Function	4.76

Intermediate Calculations

Number of Time Steps	5
Time Step Size (dt)	0.2000
Up Jump Size (u)	1.2229
Down Jump Size (d)	0.8177
Risk-Neutral Probability (p)	47.47%

Results

Binomial Approach	**$20.7492**
Black-Scholes Model	**$19.9118**
Path Dependent Simulated	**$19.9929**
Path Independent Simulated	**$18.2891**

Time	Simulate	Steps	Value
0			100.00
1	0.0	0.50	100.50
2	0.0	0.50	101.00
3	0.0	0.50	101.50
4	0.0	0.50	102.00
5	0.0	0.50	102.50
6	0.0	0.50	103.00
7	0.0	0.50	103.50
8	0.0	0.50	104.00
9	0.0	0.50	104.50
10	0.0	0.50	105.00
			4.76

Step 0	Step 1	Step 2	Step 3	Step 4	Step 5
100.00	122.29	149.55	182.89	223.67	273.53
	81.77	100.00	122.29	149.55	182.89
		66.87	81.77	100.00	122.29
			54.68	66.87	81.77
				44.71	54.68
					36.56

20.75	34.35	55.07	84.87	124.66	173.53	continue	continue	continue	continue	continue	Execute
	8.86	16.28	29.20	50.55	82.89		continue	continue	continue	continue	Execute
		2.31	4.92	10.48	22.29			continue	continue	continue	Execute
			0.00	0.00	0.00				continue	continue	End
				0.00	0.00					continue	End
					0.00						End

FIGURE 7G.1 Path-Dependent Simulation Approach to Solving Real Options

Input Parameters

Expiration in Years	1.00
Volatility	45.00%
PV Asset Value	$100
Risk-free Rate	5.00%
Dividend rate	0.00%
Strike Cost	$100

Simulation Calculation

Simulate Value	0.00
Payoff Function	4.76

Intermediate Calculations

Number of Time Steps	5
Time Step Size (dt)	0.2000
Up Jump Size (u)	1.2229
Down Jump Size (d)	0.8177
Risk-Neutral Probability (p)	47.47%

Results

Binomial Approach	**$20.7492**
Black-Scholes Model	**$19.9118**
Path Dependent Simulated	**$19.9929**
Path Independent Simulated	**$18.2891**

Time	Simulate	Steps	Value
0			100.00
1	0.0	0.50	100.50
2	0.0	0.50	101.00
3	0.0	0.50	101.50
4	0.0	0.50	102.00
5	0.0	0.50	102.50
6	0.0	0.50	103.00
7	0.0	0.50	103.50
8	0.0	0.50	104.00
9	0.0	0.50	104.50
10	0.0	0.50	105.00

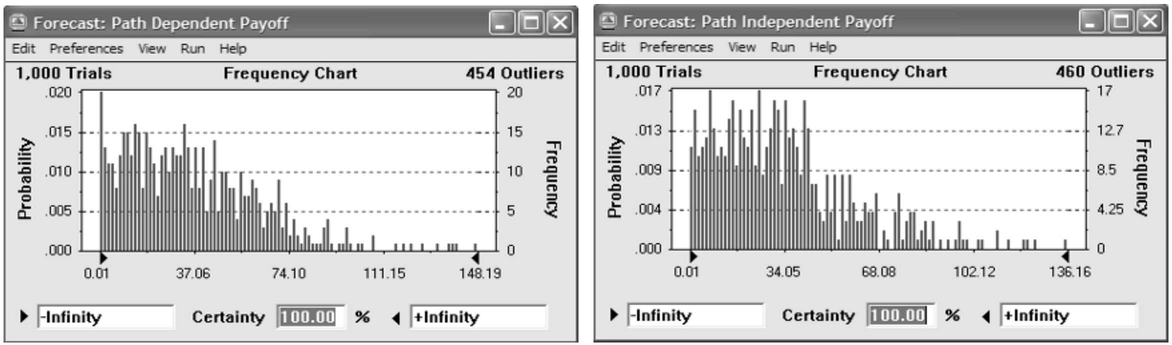


FIGURE 7G.2 Path-Dependent Simulation Forecasting Results

APPLYING MONTE CARLO SIMULATION TO OBTAIN A RANGE OF REAL OPTIONS VALUES

Alternatively, Monte Carlo simulation can be applied to obtain a range of real options values. That is, as seen in Figure 7G.3, risk-free rate and volatility are the two example variables chosen for simulation. Distributional assumptions are assigned to these two variables, and the resulting options values using the Black-Scholes and binomial lattices are selected as forecast cells.

The results of the simulation are essentially a distribution of the real options values as seen in Figure 7G.4.[2] Notice that the ranges of real options values are consistent for both the binomial lattice and the Black-Scholes model. Keep in mind that the simulation application here is used to vary the inputs to a real options model to obtain a range of results, not to model and calculate the real options itself. However, simulation can be applied to both simulate the inputs to obtain the range of real options results and also to solve the real options model through path-dependent modeling. However, a word of caution is in order. Recall that volatility is an input in a real options analysis, which captures the variability in asset value over time, and a binomial lattice is a discrete simulation technique, while a closed-form solution is obtained using continuous simulation models. Simulating real options inputs may end up double-counting a real option's true variability. See Figure 7G.4.

Note that the distribution of the terminal values is lognormal in nature, as all values are non-negative. Another word of caution is important here. Attempting to simulate just the terminal values without using the Brownian Motion approach will most certainly yield incorrect answers in general. The answers may be similar but will never be robust. Thus, simply simulating the terminal value outcomes and valuing them that way is completely flawed.

Input Parameters

Expiration in Years	1.00
Volatility	45.00%
PV Asset Value	$100
Risk-free Rate	5.00%
Dividend rate	0.00%
Strike Cost	$100

Intermediate Calculations

Number of Time Steps	5
Time Step Size (dt)	0.2000
Up Jump Size (u)	1.2229
Down Jump Size (d)	0.8177
Risk-Neutral Probability (p)	47.47%

Results

Binomial Approach	**$20.7492**
Black-Scholes Model	**$19.9118**

Step 0	Step 1	Step 2	Step 3	Step 4	Step 5
100.00	122.29	149.55	182.89	223.67	273.53
	81.77	100.00	122.29	149.55	182.89
		66.87	81.77	100.00	122.29
			54.68	66.87	81.77
				44.71	54.68
					36.56

20.75	34.35	55.07	84.87	124.66	173.53	continue	continue	continue	continue	continue	Execute
	8.86	16.28	29.20	50.55	82.89		continue	continue	continue	continue	Execute
		2.31	4.92	10.48	22.29			continue	continue	continue	Execute
			0.00	0.00	0.00				continue	continue	End
				0.00	0.00					continue	End
					0.00						End

FIGURE 7G.3 Simulating Options Ranges

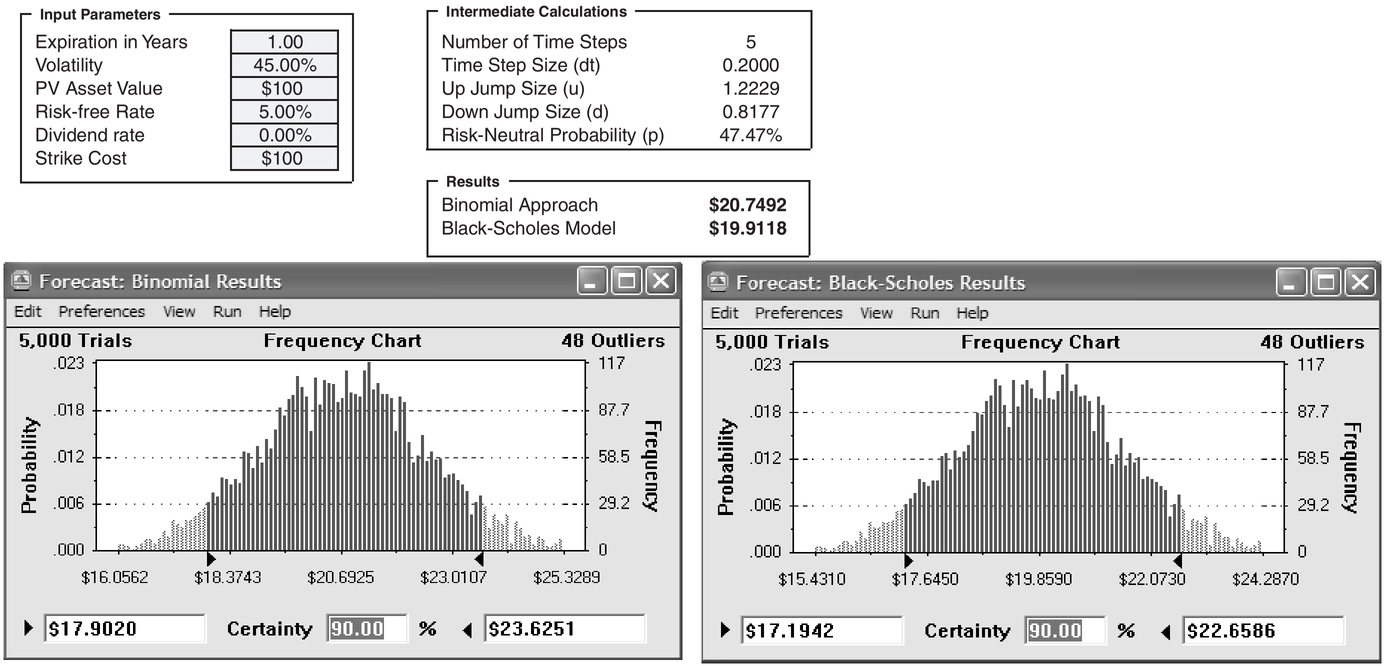


FIGURE 7G.4 Simulating Options Ranges Forecast Results

APPENDIX 7H

Trinomial Lattices

For the sake of completeness, below is an illustration of a trinomial lattice (see Figure 7H.1). Building and solving a trinomial lattice is similar to building and solving a binomial lattice, complete with the up/down jumps and risk-neutral probabilities. However, the following recombining trinomial lattice is more complicated to build. The results stemming from a trinomial lattice are the same as those from a binomial lattice at the limit, but the lattice-building complexity is much higher for trinomials or multinomial lattices. Hence, the examples thus far have been focusing on the binomial lattice, due to its simplicity and applicability. It is difficult enough to create a three-time-step trinomial lattice as shown in Figure 7H.1. Imagine having to keep track of the

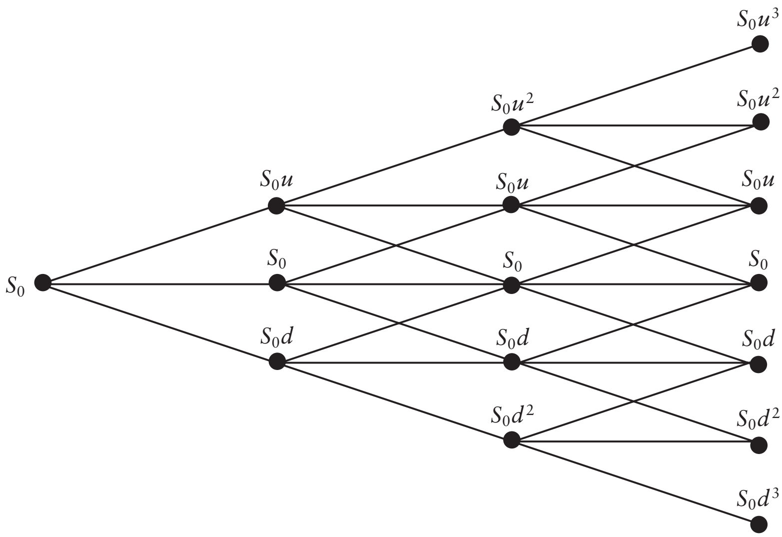


FIGURE 7H.1 Trinomial Lattice

number of nodes, bifurcations, and which branch recombines with which, in a very large lattice. The trinomial lattice's equations are specified below:

$$u = e^{\sigma\sqrt{3\delta t}} \text{ and } d = e^{-\sigma\sqrt{3\delta t}}$$

$$p_L = \frac{1}{6} - \sqrt{\frac{\delta t}{12\sigma^2}}\left[r - q - \frac{\sigma^2}{2}\right]$$

$$p_M = \frac{2}{3}$$

$$p_H = \frac{1}{6} + \sqrt{\frac{\delta t}{12\sigma^2}}\left[r - q - \frac{\sigma^2}{2}\right]$$

See Chapter 10 for more details on applying the Multinomial Super Lattice Solver software to easily and quickly solve complex trinomial lattices and other multinomial lattices (quadranomial and pentanomial lattices) when the underlying assets follow mean-reverting and jump-diffusion tendencies.

APPENDIX 7I

Nonrecombining Lattices

Figure 7I.1 illustrates a five-step nonrecombining lattice for solving an American call option. Each node branches into two pathways that do not meet with other branches along the way (i.e., they do not recombine). The lattice shown here is the first lattice of the underlying asset.

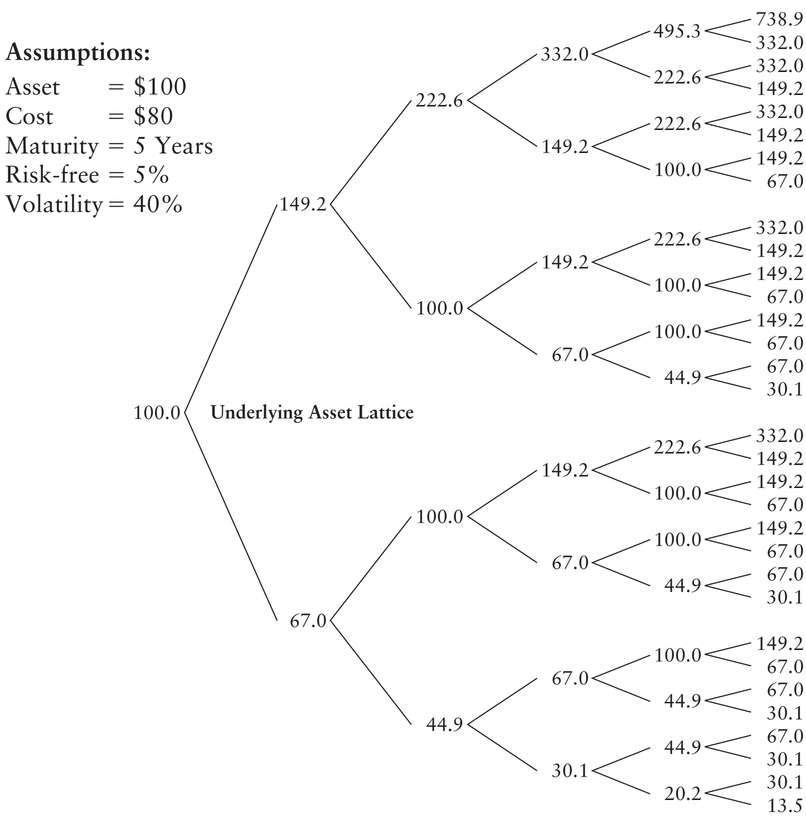


FIGURE 7I.1 Nonrecombining Underlying Asset Lattice

The lattice shown in Figure 7I.2 is the valuation lattice of the American call option, obtained using the backward-induction approach and applying a risk-neutral probability analysis.

The problem can also be solved using a recombining lattice as shown in Figures 7I.3 and 7I.4. Notice the similar values along the nonrecombining and recombining lattices. In the recombining lattice, the amount of computation work is significantly reduced because identical values for a particular time period are collapsed and summarized as unique nodes.

Notice the similar results obtained using the recombining and nonrecombining lattices approach.

However, there is a caveat in comparing the recombining and nonrecombining lattices. For instance, the six terminal nodes on a recombining tree are unique occurrences and a summary of the 32 terminal nodes on the nonrecombining lattice. Therefore, it is incorrect to assume that there is

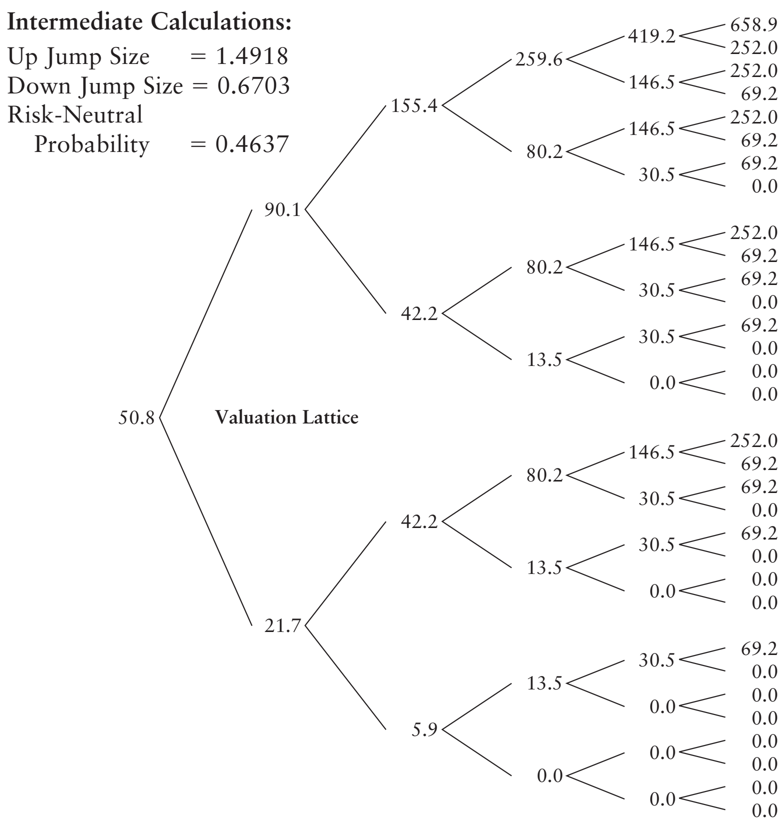


FIGURE 7I.2 Nonrecombining Valuation Lattice

Assumptions:

Asset = \$100
Cost = \$80
Maturity = 5 Years
Risk-free = 5%
Volatility = 40%

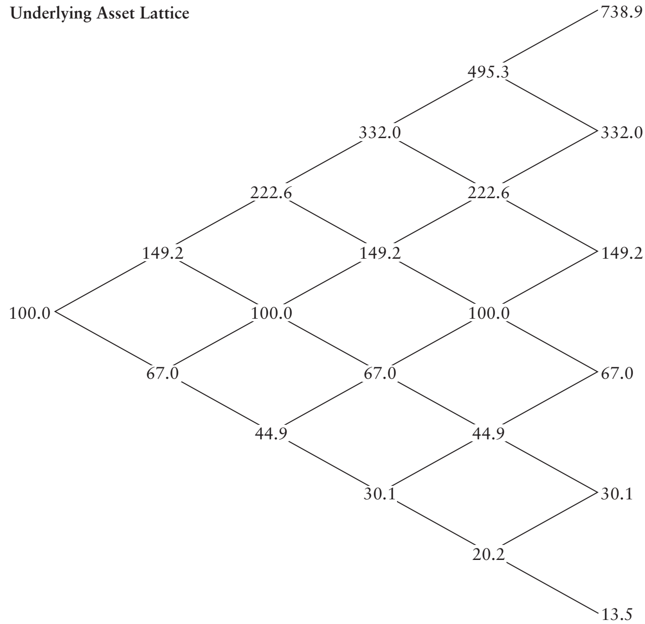


FIGURE 7I.3 Recombining Underlying Asset Lattice

Intermediate Calculations:

Up Jump Size = 1.4918
Down Jump Size = 0.6703
Risk-Neutral Probability = 0.4637

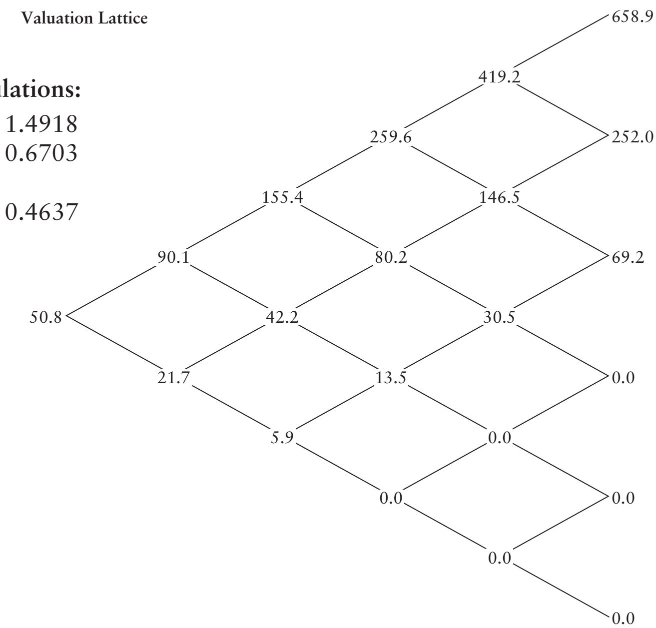


FIGURE 7I.4 Recombining Valuation Lattice a $\frac{1}{6}$ probability of occurrence for the values 738, 332, 149, 67, 30, and 13. See Figure 7I.5.

In reality, the distribution of the terminal nodes looks somewhat normal, with different outcome probabilities as seen in Figure 7I.6. Depending on the input parameters, the distribution of the terminal nodes may change slightly (higher volatility means a higher frequency of occurrence in the extreme values).

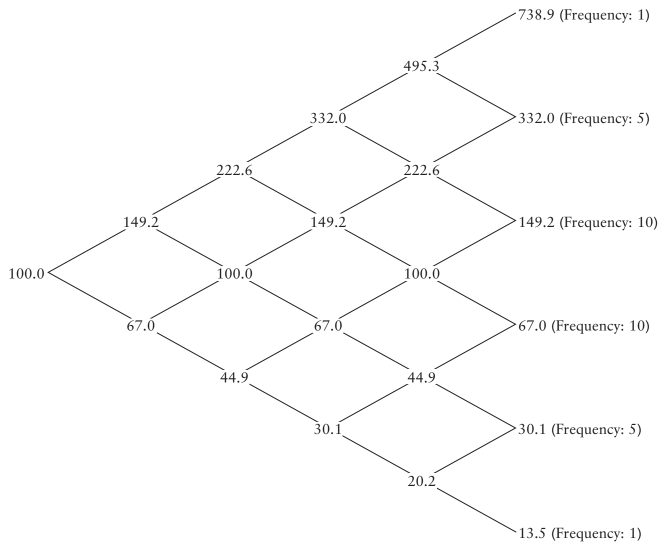


FIGURE 7I.5 Frequency of Occurrence in a Recombining Lattice

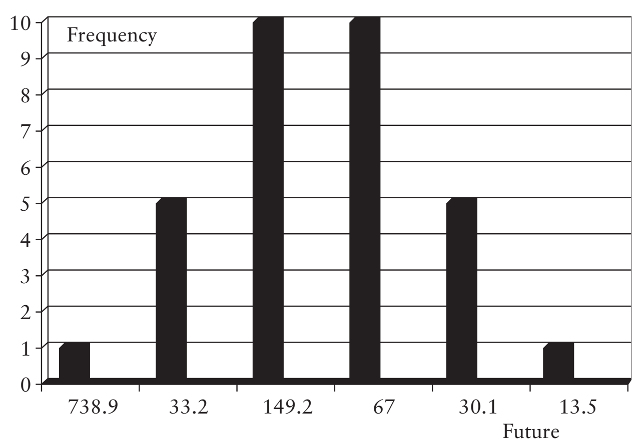


FIGURE 7I.6 Probability Distribution of the End Nodes on a Recombining Lattice

Although recombining lattices are easier to calculate and arrive at identical answers to the nonrecombining lattices, there are conditions when nonrecombining lattices are required for the analysis. These conditions include when there are multiple sources of uncertainty or when volatility changes over time, as in Figure 7I.7.

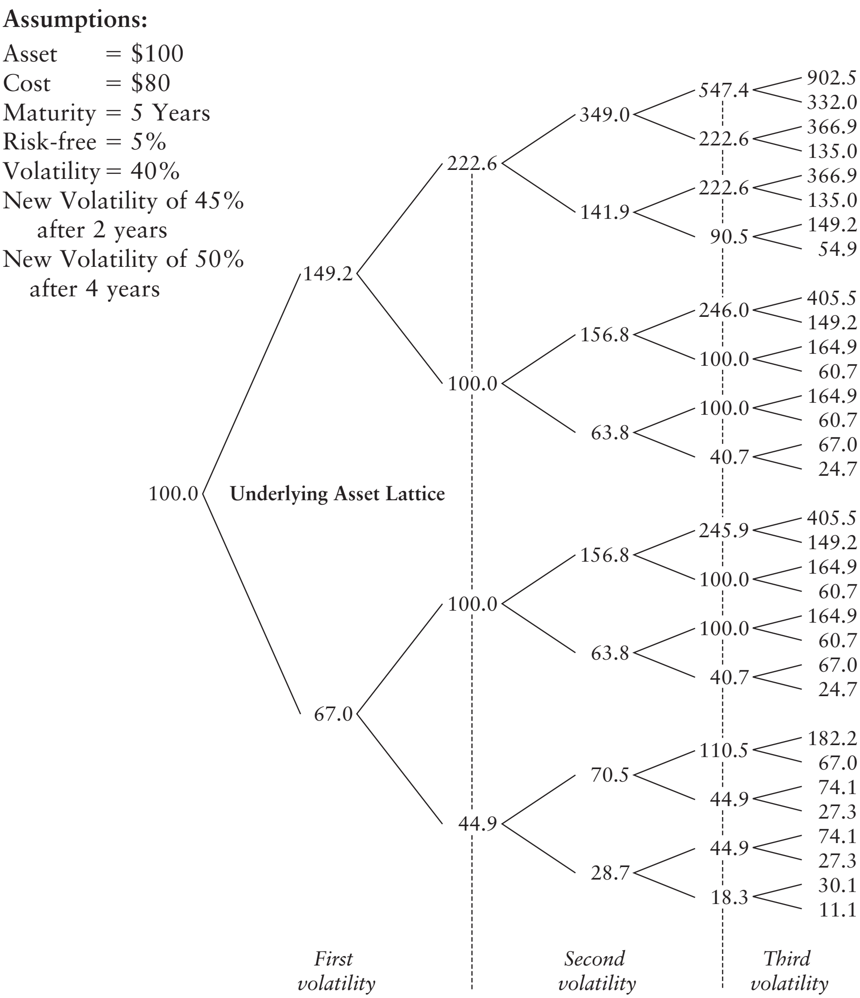


FIGURE 7I.7 Nonrecombining Underlying Asset Lattice for a Changing Volatility Option

Figure 7I.8 shows the valuation lattice on an American call option with changing volatilities using the risk-neutral probability approach.

Although nonrecombining lattices are better suited for solving options with changing volatilities, recombining lattices can also be modified to handle this condition, thereby cutting down on analytical time and effort. The results

Calculations:
up (1) = 1.4918
down (1) = 0.6703
prob (1) = 0.4637
up (2) = 1.5683
down (2) = 0.6376
prob (2) = 0.4445
up (3) = 1.6487
down (3) = 0.6065
prob (3) = 0.4267

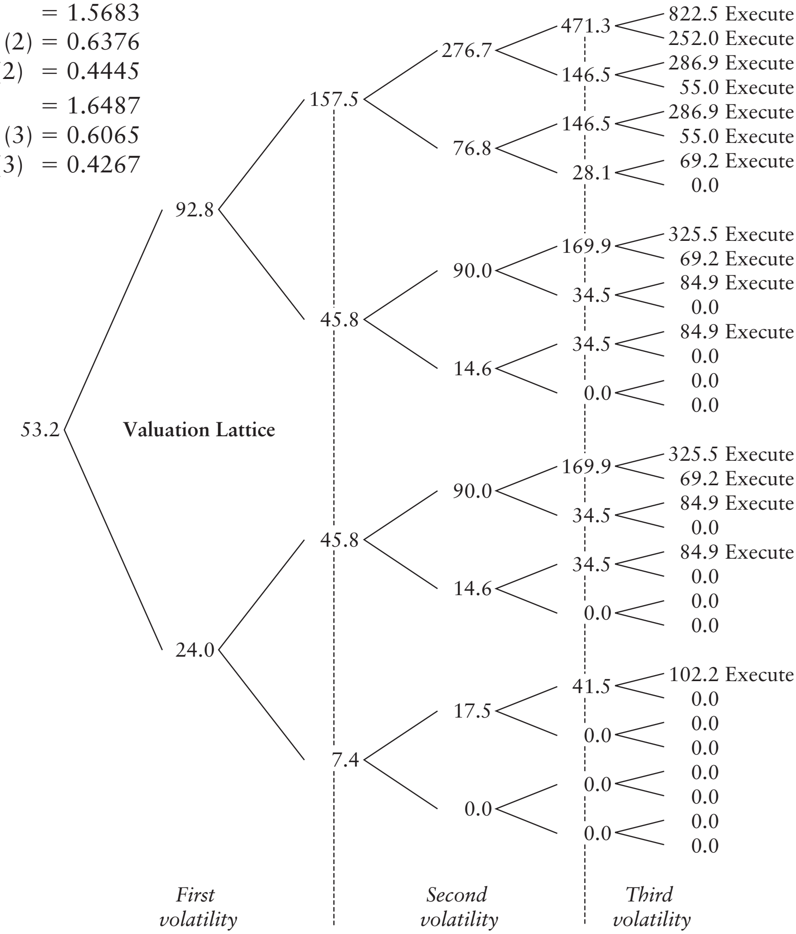


FIGURE 7I.8 Nonrecombining Valuation Lattice for a Changing Volatility Option obtained are identical no matter which approach is used. The modified recombining lattice below makes use of the fact that although volatility changes three times within the five-year maturity period, volatility remains constant within particular time periods. For instance, the 40 percent volatility applies from time 0 to time 2, and the 45 percent volatility holds for time 2 to time 4. Within these time periods, volatility remains constant; hence, the lattice bifurcations are recombining. The entire lattice analysis in Figure 7I.9 can

Assumptions:
Asset = \$100
Cost = \$80
Maturity = 5 Years
Risk-free = 5%
Volatility = 40%
New Volatility of 45% after 2 years
New Volatility of 50% after 4 years

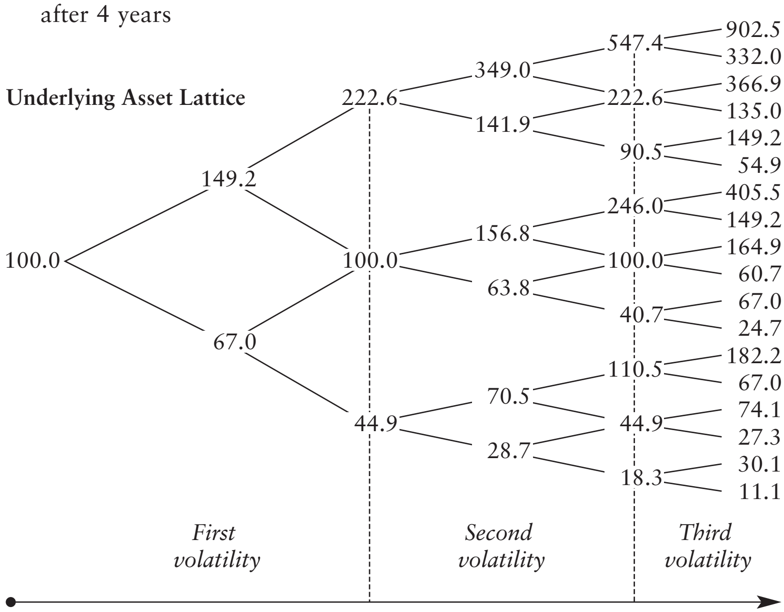


FIGURE 7I.9 Solving the Underlying Asset Lattice Using Multiple Recombining Lattices be segregated into three stages of recombining lattices. At the end of a constant volatility period, each resulting node becomes the starting point of a new recombining lattice.

Figure 7I.10 is the modified recombining valuation lattice approach for the changing volatility option analysis. Notice that the resulting option value of $53.2 is identical to the result obtained using the nonrecombining lattice (Figure 7I.8).

Calculations:

up (1) = 1.4918
down (1) = 0.6703
prob (1) = 0.4637
up (2) = 1.5683
down (2) = 0.6376
prob (2) = 0.4445
up (3) = 1.6487
down (3) = 0.6065
prob (3) = 0.4267

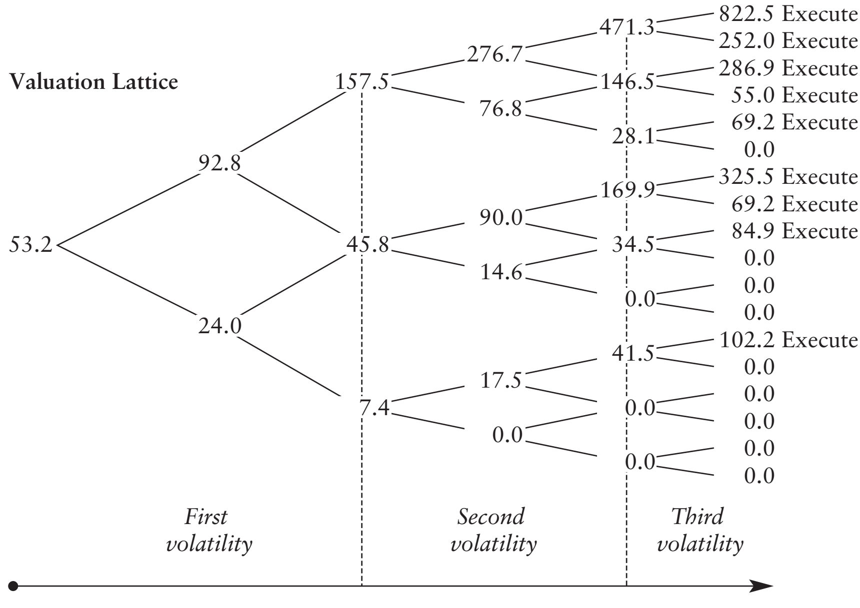


FIGURE 7I.10 Solving the Valuation Lattice Using Multiple Recombining Lattices

To compute the probability of occurrence for a specific node, two simple methods can be used: Pascal's triangle and the binomial probability distribution. First, Pascal's triangle can be used to figure out the frequency of occurrence of a particular node for a particular number of steps. Figure 7I.11 illustrates Pascal's triangle corresponding to a five-step lattice. Compare the results (1, 5, 10, 10, 5, and 1) with the histogram in Figure 7I.6.

Another approach is the use of the binomial probability mass function or simply

$$P(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$P(x)$ is the probability that the number of x events will occur, given the total number of trials n with a specified probability of success p. For instance, if we toss a fair coin four times, the probability that exactly three heads will occur is

$$P(x=3) = \frac{4!}{3!(4-3)!} 0.5^3 (1-0.5)^{4-3} = 25\%$$

Similarly, think of the binomial lattice as a continuous series of coin tosses. This means that by using the five-step lattice illustrated in Figure 7I.11, we can compute the probability of occurrence of say the second node from the top by

$$P(x=1) = \frac{5!}{1!(5-1)!} 0.5^1 (1-0.5)^{5-1} = 15.63\%$$

The result is identical to the second node's frequency of 5 divided by 32 or the sum of all frequencies (5/32 is 15.63 percent).

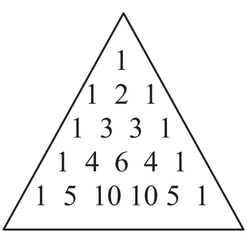


FIGURE 7I.11 Pascal's Triangle

That is, we set $n = 5$ for the fifth step on the lattice, and $x = 1$ for the second node from the top. Note that the first node on the top is $x = 0$, because the binomial distribution is a zero-based distribution. That is, say we toss a fair coin once, we have $n = 1$. Then, we have two outcomes, heads or tails. If we define heads as success, then the two outcomes are such that $x = 0$ and $x = 1$, or no heads (means tails appears) or one head (means tails does not appear). This means that Figure 7I.11's fifth step values of 1, 5, 10, 10, 5, and 1 have the following x values: 0, 1, 2, 3, 4, and 5.

As a final illustration, for a four-step lattice, the following frequencies appear: 1, 4, 6, 4, and 1. It means that the second node with a frequency of 4 has a probability of 4 divided by 16 (sum of all frequencies) or 25 percent. Using the binomial probability mass function, the probability of the second node with a frequency of 4 is

$$P(x = 1) = \frac{4!}{1!(4-1)!} 0.5^1 (1 - 0.5)^{4-1} = 25\%$$

CHAPTER 8

Additional Issues in Real Options

INTRODUCTION

This chapter deals with additional issues in real options. These include the optimal timing of projects, stochastic optimization of options, barrier-type exit and abandonment options, switching options, and multiple compound options. The problems of applying decision trees to real options analysis are also discussed, explaining why decision trees by themselves are problematic when trying to apply and solve real options. The technical appendixes at the end of the chapter detail the different approaches to stochastic optimization as well as present details of multiple exotic-options formulae.

PROJECT RANKING, VALUATION, AND SELECTION

One of the key uses of real options analysis is project ranking and selection, as shown in Figure 8.1. For example, using a traditional net present value metric, management would prioritize the initiatives A-D-B-C, from the most preferred to least. However, considering the strategic management flexibility inherent in each of the initiatives and quantified through real options analysis, the initiative prioritization would now become A-D-C-B. If real options value is not included, the selection criteria may lead to the wrong initiative selection and conclusions.

For instance, suppose that Initiative B is to develop a certain automobile model, while Initiative C is to develop the similar model but with an option for converting it into a gas-electric hybrid. Obviously, the latter costs more than the former. Hence, the NPV for Initiative C is less than that for Initiative B. Therefore, choosing the project that has a higher NPV today is shortsighted. If the option value is included, Initiative C is chosen, the optimal decision,

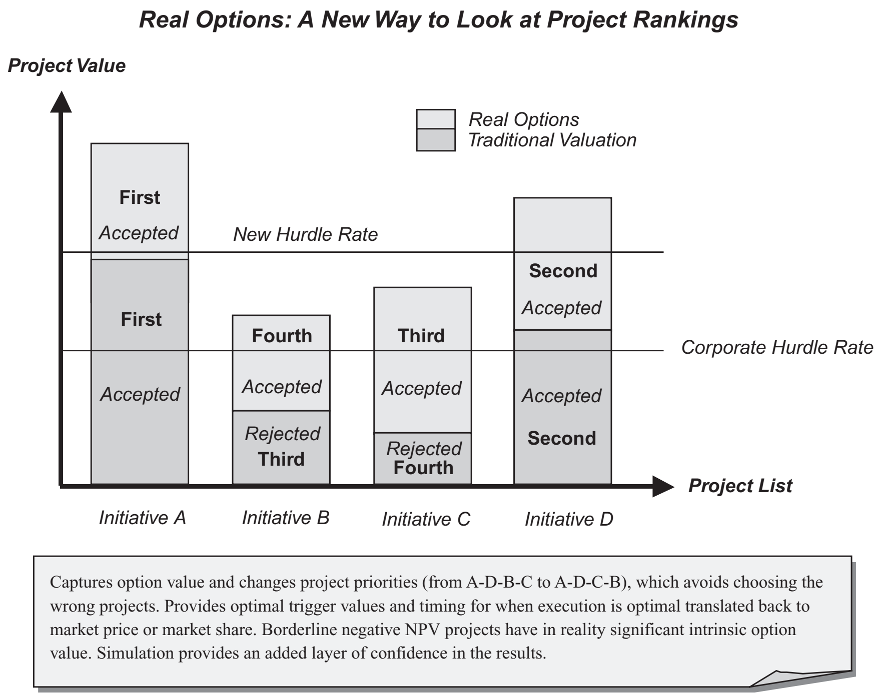


FIGURE 8.1 Project Selection and Prioritization

because given today's uncertain technological environment, hybrid cars may become extremely valuable in the future.

DECISION TREES

Figure 8.2 shows an example of a decision tree. One major misunderstanding that analysts tend to have about real options is that they can be solved using decision trees alone. This is untrue. Instead, decision trees are a great way of depicting strategic pathways that a firm can take, showing graphically a decision road map of management's strategic initiatives and opportunities over time. However, to solve a real options problem, it is better to combine decision tree analytics with real option analytics, and not to replace it completely with decision trees. When used in framing real options, these trees should be more appropriately called strategy trees (used to define optimal strategic pathways) as seen in Chapter 11's cases.

Models used to solve decision tree problems range from a simple expected value to more sophisticated Bayesian probability updating approaches. Neither of these approaches is applicable when trying to solve a real options problem. A decision tree is not the optimal stand-alone methodology when trying to solve real options problems because subjective probabilities are required, as are different discount rates at each node. The difficulties and errors in forecasting the relevant discount rates and probabilities of occurrence are compounded over time, and the resulting calculated values are oftentimes in error. In addition, as shown in Chapter 7, binomial lattices are a much better way to solve real options problems, and because these lattices can also ultimately be converted into decision trees, they are far superior to using decision trees as a stand-alone application for real options. Nonetheless, there is a common ground between decision trees and real options analytics, as seen in Chapter 11's case studies.

Figure 8.2 shows a decision tree but without any valuation performed on it. On each node of the tree, certain projects or initiatives can be attached. The values of these nodes can be determined separately using binomial lattices, closed-form solutions, or any of the other number of ways used to solve real options problems.

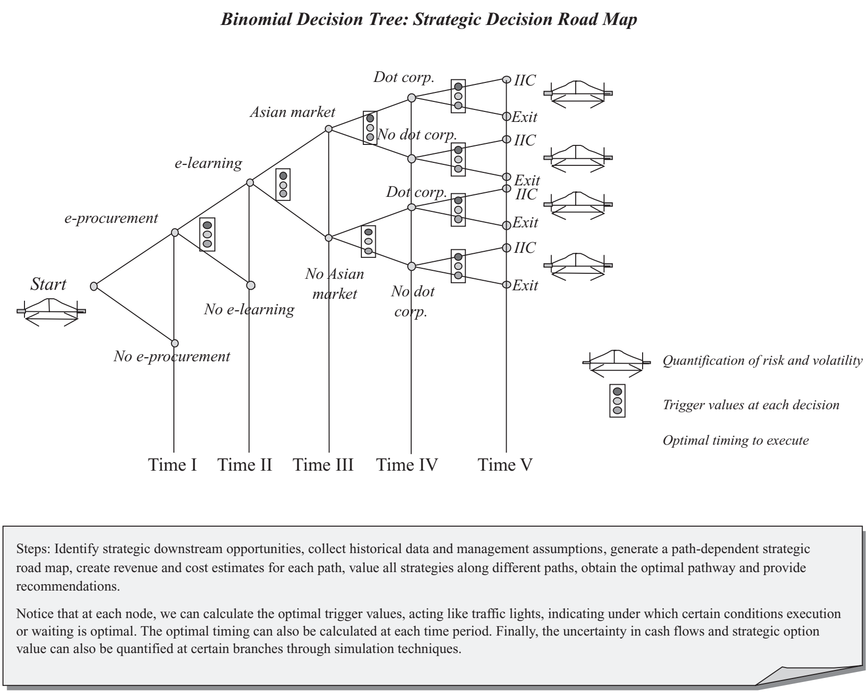


FIGURE 8.2 Decision Tree Analysis

Figure 8.2's hypothetical e-business strategy starts with e-procurement through globalization of an International Internet Coalition (IIC). The decision tree simply shows that there are multiple paths that can lead to this IIC end state. However, at each intermediate state, there is path-dependence. For instance, the firm cannot enter the Asian market without first having the correct infrastructure for setting up e-learning and e-procurement capabilities. The success of the former depends on the success of the latter, which is nothing but a sequential compound option. At each intermediate decision node, there are also abandonment options. In addition, simulation analysis, critical trigger values, and optimal timing can be applied and quantified along each decision node in a real options framework but cannot be done using a simple decision tree analysis. However, presenting strategies in a decision tree provides key insights to management as to what projects are available for execution, and under what conditions.

One of the fatal errors analysts tend to run into includes creating a decision tree and calculating the expected value using risk-neutral probabilities, akin to the risk-neutral probability used in Chapter 7. This is incorrect because risk-neutral probabilities are calculated based on a constant volatility. The risk structures of nodes on a decision tree (for instance, e-learning versus a dot.corp strategy have very different risks and volatilities). In addition, for risk-neutral probabilities, a Martingale process is required. That is, in a binomial lattice, each node has two bifurcations, an up and a down. The up and down jump sizes are identical in magnitude for a recombining lattice. This has to hold before risk-neutral probabilities are valid. Clearly the return magnitudes of different events along the decision tree are different, and risk-neutralization does not work here. Because risk-neutral probabilities cannot be used, the risk-free rate therefore cannot be used here for discounting the cash flows. Also, because risks are different at each strategy node, the market risk-adjusted discount rate, such as a WACC, should also be different at every node. A correct single discount rate is difficult enough to calculate, let alone multiple discount rates on a complex tree, and the errors tend to compound over time, by the time the NPV of the strategy is calculated.

In addition, chance nodes are usually added in decision tree analysis, indicating that a certain event may occur given a specific probability. For instance, chance nodes may indicate a 30 percent chance of a great economy, a 45 percent chance of a nominal one, and a 25 percent chance of a downturn. Then events and payoffs are associated with these chances. Back-calculating these nodes using risk-neutral probabilities will be incorrect because these are chance nodes, not strategic options. Because these three events are complementary—that is, their respective probabilities add up to 100 percent—one of these events *must* occur, and given enough trials, all of these events must occur at one time or another. Real options analysis stipulates that one does not know what will occur, but only what the strategic alternatives are if a certain event occurs. If chance nodes are required in an analysis, the discounted cash flow model can accommodate them to calculate an expected value, which could then be simulated based on the probability and distributional assumptions. These simulated values can then be run in a real options modeling environment. The results can be shown on a strategy tree looking similar to a decision tree as depicted in Chapter 11. However, strategic decision pathways should be shown in the strategy tree environment, and each strategy node or combinations of strategy nodes can be evaluated in the context of real options analysis as described throughout this book. Then the results can be displayed in the strategy tree.

In summary, decision tree analysis alone is incomplete as a stand-alone analysis in complex situations. Both methodologies discussed approach the same problem from different perspectives. However, a common ground could be reached. Taking the advantages of both approaches and melding them into an overall valuation strategy, decision trees should be used to frame the problem, real options analytics should be used to solve any existing strategic optionalities (either by pruning the decision tree into subtrees or solving the entire strategy tree at once), and the results should be presented back on a decision tree. These so-called option strategy trees are useful for determining the optimal decision paths the firm should take (see Chapter 11 for sample applications).

EXIT AND ABANDONMENT OPTIONS

Exit options are abundant in the real business world where projects can be scrapped and salvaged resources can then be redeployed elsewhere. However, certain projects may not be that easily abandoned at certain times because of project stickiness and business psychology, or the fact that management can be stubborn and reluctant to kill a project due to personal reasons.

Figure 8.3 shows a down and out barrier abandonment option. This type of option means that a project will not be terminated immediately once it falls out of profitability. Instead, management sets a critical barrier assumption, and should the project's profitability level fall below this barrier, the project will be abandoned. The barrier may be set after accounting for project stickiness and any other operational issues. The analysis can be solved using the Super Lattice Solver software on the enclosed CD-ROM. In addition, basic barrier options can be solved in a binomial lattice by adding in *IF/AND/OR* statements nested with the regular *MAX* functions in Excel. Chapter 10 has several case examples using barrier-type options. That is, the value of an option at a particular node comes into-the-money or out-of-the-money only *if* the underlying asset value broaches a barrier.

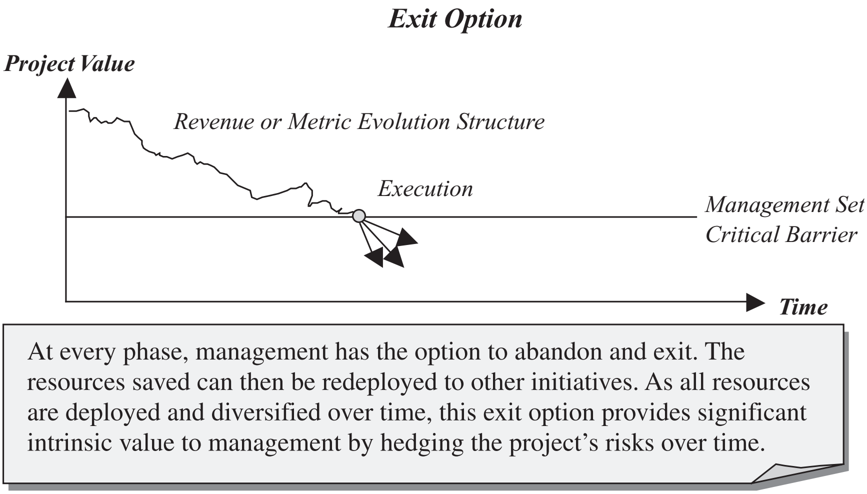


FIGURE 8.3 Exit Option with a Barrier

COMPOUND OPTIONS

In some cases, there exist complex compound options, where the execution of one project provides downstream opportunities. For instance, in Figure 8.4, we see that the infrastructure in place provides a compound option comprising a series of three future phases. Notice that Phase III cannot proceed without the completion and execution of Phase II, which itself cannot proceed without the completion of Phase I. In some cases, these phases in the future can be a combination of different types of options. For example, Phase II options are simply an expansion of Phase I projects, while Phase III projects are only executed if some preset barriers in Phase II are achieved. The Multiple Asset Super Lattice software in the enclosed CD-ROM provides examples of a multiphased sequential compound option as well as a multiple-phase option with different costs, expansion, contraction, and abandonment options at each phase. The cases in Chapters 10 and 11 illustrate how even more complex compound options can be solved easily using the software.

TIMING OPTIONS

Figure 8.5 shows the payoff profile on an option. The static straight line indicates the strategic options value of a project with respect to changes in the underlying variable, the revenues generated by the project assuming no volatility in the cash flows. This is in essence the NPV of the project at termination.

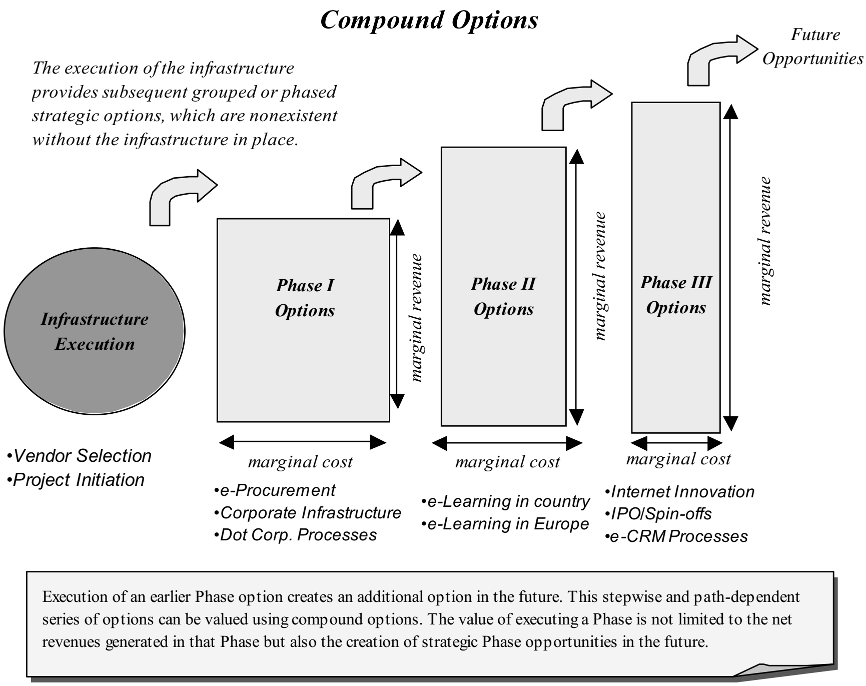


FIGURE 8.4 Multiple-Phased Complex Compound Option

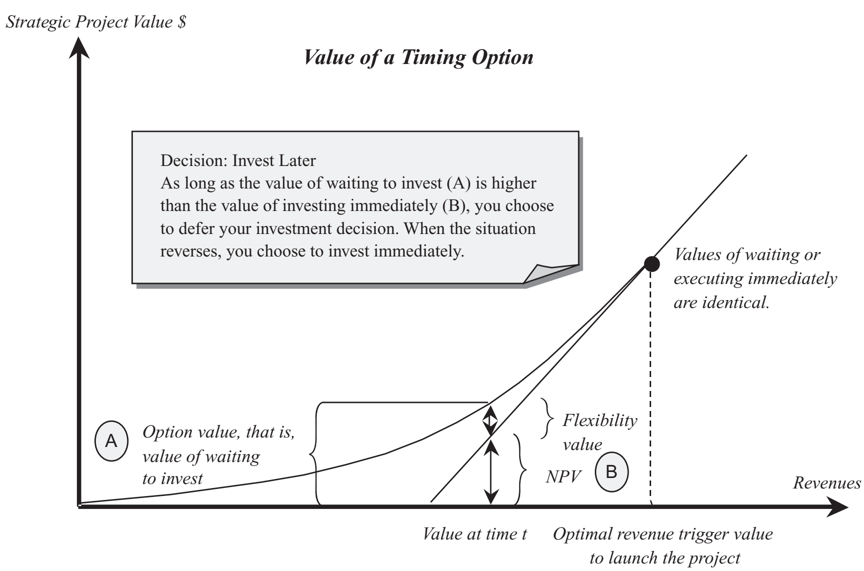


FIGURE 8.5 Timing Option

The curved line above the static payoff line is the strategic options value, assuming there are risks and cash flows may be volatile. Hence, with uncertainty, cash flows can be higher than expected, and with time before expiration, the project is actually worth more than its NPV suggests.

Briefly, a timing option provides the holder the option to defer making an investment decision until a later time without much restriction. That is, competitive or market effects (market share erosion, first to market, strategic positioning, and the like) have negligible effect on the value of the project. Assuming that this holds true, then shifting a project for execution in the future only depends on two factors: the rate of growth of the asset over time and the discount rate or rate of erosion of the time value of money. Of course, this is the simplest case. In reality, optimal timing and trigger values can be obtained by changing the volatility (risk) and dividend rate (cost of waiting and opportunity cost)—these are seen in several cases in Chapter 10.

SOLVING TIMING OPTIONS CALCULATED USING STOCHASTIC OPTIMIZATION

Optimally timing an option execution is a tricky thing because if there are highly risky projects with significant amounts of uncertainty, waiting is sometimes preferred to executing immediately. However, certain projects have an indefinite economic life and during this infinite economic life, certain real options exist. Hence, for an infinite life real option with high volatility, does this mean you wait forever and never do it? In addition, many other factors come into play with analyzing an optimal trigger value and optimal timing on a real option as shown in Figure 8.6. In certain cases, a Game Theory framework incorporating dynamic games competitors may play can be incorporated into the analysis.

To solve the timing option, start by assuming that the value of an underlying asset's process $X = (X_t)$ follows a Geometric Brownian Motion—that is, $dX_t = \alpha X_t dt + \sigma X_t dZ_t$. Then we define the value of a call option to be $\Phi(X) = E\max[(X_T - I)e^{-\rho T}, 0]$, where I is the initial capital investment outlay, X_T is the time value of the underlying asset at the terminal time T, and ρ is the discount rate.

The optimal investment strategy is to maximize the value of the option with respect to time T given the underlying stochastic investment process X—in other words, we want to find $\Phi^*(X) = \max_T E\max[(X_T - I)e^{-\rho T}, 0]$.

First, consider the near-zero volatility case, where there is negligible uncertainty. Next, we require a drift rate, usually measured as the growth rate in the asset value, and defined as α. We will further assume that $\rho > \alpha$, that is, the drift or growth rate of the underlying asset value does not exceed the

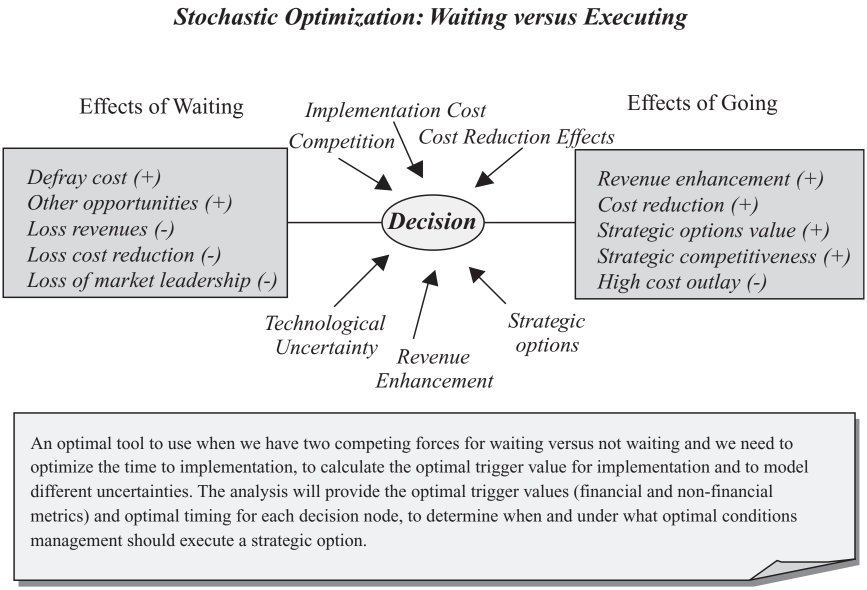


FIGURE 8.6 Stochastic Optimization

discount rate. Otherwise, the process keeps increasing at a much higher rate than can be discounted, the terminal value of the asset becomes infinite, and it is never optimal to exercise the option. Because we have defined α as the growth rate on the underlying investment process, it becomes the growth rate in the deterministic case. Now the foregoing problem simplifies to the condition where $\Phi^*(X) = \max_T \max[(X_0 e^{\alpha T} - I)e^{-\rho T}, 0]$. That is, the underlying asset X_0 at time zero grows at this growth rate α such that at time T, the value of the continuously compounded asset value becomes $X_0 e^{\alpha T}$. In addition, due to the time value of money, the net present value is discounted at a continuous rate of $e^{-\rho T}$. Here we see that delaying the execution of an option creates the marginal benefit of the compounding growth of the asset value over time, while the marginal cost is the time value of money. The optimal timing can then be derived to obtain the equilibrium execution time where the net present value is maximized.

The optimal value of the option can be simply derived through the differential equation of the net present value with respect to time. Starting with $\Phi(X) = \max_T \max[(X_0 e^{\alpha T} - I)e^{-\rho T}, 0]$, we obtain:

$$\frac{d\Phi(X)}{dT} = (\alpha - \rho)X_0 e^{(\alpha - \rho)T} + \rho I e^{-\rho T} = 0$$

for the maximization process, yielding:

$$(\rho - \alpha)X_0 e^{(\alpha-\rho)T} = \rho I e^{-\rho T}$$

$$(\rho - \alpha)X_0 \frac{e^{\alpha T}}{e^{\rho T}} = \frac{\rho I}{e^{\rho T}}$$

$$\alpha T + \ln(\rho - \alpha)X_0 = \ln(\rho I)$$

The *optimal time to execution* is therefore

$$T = \frac{1}{\alpha} \ln\left[\frac{\rho I}{(\rho - \alpha)X_0}\right]$$

Table 8.1 illustrates this example, where if the asset value at time zero is equivalent to the implementation cost \$100, while the discount rate is assumed to be 25 percent and the corresponding risk-free rate is 5.5 percent, the calculated optimal time to execution is 4.52 years, using

$$T = \frac{1}{0.055} \ln\left[\frac{(.25)(\$100)}{(.25 - .055)(\$100)}\right] = 4.52 \text{ years}$$

TABLE 8.1 The Optimal Value of the Option

Assumptions:			
Asset Value at Time 0 (X_0)			\$100
Fixed Implementation Cost I			\$100
Discount Rate			25%
Growth Rate of Underlying Asset			5.5%
Calculated Optimal Time to Execution			4.52
Time	NPV		
1.00	\$4.40		
2.00	7.05		
3.00	8.47		
4.00	9.05		
4.52	**9.12**	This is the maximum NPV value	
5.00	9.07		
6.00	8.72		
7.00	8.16		
8.00	7.48		

Notice that the period 4.52 years provides the maximum NPV. Hence, this maximum NPV of \$9.12 is the option value of waiting, as compared to \$100 − \$100 = \$0 NPV if the project is executed immediately.

Finally, to avoid any negative or undefined values of the optimal timing, we can simply redefine the optimal timing to equal

$$T^* = Max\left[\frac{1}{\alpha}\ln\left(\frac{\rho I}{(\rho-\alpha)X_0}\right);\ 0\right]$$

Using this optimal timing value of

$$T = \frac{1}{\alpha}\ln\left[\frac{\rho I}{(\rho-\alpha)X_0}\right]$$

a very interesting result can be obtained. Specifically, rearranging this equation yields

$$e^{\alpha t} = \frac{\rho I}{(\rho-\alpha)X_0}$$

and we obtain the following:

$$\frac{X_0 e^{\alpha t}}{I} = \frac{\rho}{(\rho-\ \alpha)}$$

which is the *optimal trigger value* of the project. The left-hand-side equation is termed the profitability index, that is, the future value of the underlying asset divided by the implementation cost. If the profitability index exceeds 1.0, this implies that the NPV is positive, because the value of the asset exceeds the implementation cost. An index less than 1.0 implies that the NPV is negative. See Table 8.2. Hence, using this profitability index is akin to making decisions using the NPV analysis.

Table 8.3 shows the optimal timing to execute an option given the respective growth and discount rates. Notice that as discount rates increase, holding the growth rate constant, it is more optimal to execute the option earlier. This is because the time value of money and opportunity cost losses in revenues surpass the growth rate in asset value over longer periods of time. In contrast, holding the discount rate constant and increasing the growth rate, it is clear that waiting is more optimal than immediate execution because the growth rate in asset value appreciation far surpasses the discount rate's opportunity cost of lost revenues. For example, assuming a 10 percent discount rate and a 1 percent growth rate, if a project's asset value exceeds the implementation cost by a ratio of 1.111, or if the net profit exceeds the

TABLE 8.2 Profitability Indexes for Different Growth and Discount Rates

		Growth Rates				
		1.00%	2.00%	3.00%	4.00%	5.00%
Discount Rates	10%	1.111	1.250	1.429	1.667	2.000
	15	1.071	1.154	1.250	1.364	1.500
	20	1.053	1.111	1.176	1.250	1.333
	25	1.042	1.087	1.136	1.190	1.250
	30	1.034	1.071	1.111	1.154	1.200
	35	1.029	1.061	1.094	1.129	1.167
	40	1.026	1.053	1.081	1.111	1.143

TABLE 8.3 Optimal Timing for Different Growth and Discount Rates

		Growth Rates				
		1.00%	2.00%	3.00%	4.00%	5.00%
Discount Rates	10%	10.54	11.16	11.89	12.77	13.86
	15	6.90	7.16	7.44	7.75	8.11
	20	5.13	5.27	5.42	5.58	5.75
	25	4.08	4.17	4.26	4.36	4.46
	30	3.39	3.45	3.51	3.58	3.65
	35	2.90	2.94	2.99	3.03	3.08
	40	2.53	2.56	2.60	2.63	2.67
	Investment Cost		\$100			
	Asset Value		\$100			

implementation cost by 11.1 percent, it is optimal to execute the project immediately; otherwise, it is more optimal to wait.

Tables 8.2 and 8.3 assume negligible uncertainty evolving through time. However, in the uncertain or stochastic case when the growth rate of the underlying asset value is uncertain—that is, α fluctuates at the rate of σ (volatility)—optimal timing can no longer be ascertained. Simulation is preferred in this case. However, the optimal trigger value can still be determined.[1] The optimal trigger value measured in terms of a profitability index value is now as follows:

$$\frac{X_0 e^{\alpha t}}{I} = \frac{\left[\dfrac{2\rho}{\sigma^2} + \left(\dfrac{\alpha}{\sigma^2} - 0.5\right)^2\right]^{0.5} + 0.5 - \dfrac{\alpha}{\sigma^2}}{\left[\dfrac{2\rho}{\sigma^2} + \left(\dfrac{\alpha}{\sigma^2} - 0.5\right)^2\right]^{0.5} - 0.5 - \dfrac{\alpha}{\sigma^2}}$$

TABLE 8.4 Profitability Indexes for Different Growth versus Discount Rates

		Growth Rates				
		1.00%	2.00%	3.00%	4.00%	5.00%
Discount Rates	10%	1.333	1.451	1.616	1.848	2.184
	15	1.250	1.315	1.397	1.500	1.629
	20	1.206	1.250	1.303	1.367	1.442
	25	1.179	1.211	1.250	1.295	1.347
	30	1.160	1.186	1.216	1.250	1.289
	35	1.145	1.167	1.191	1.219	1.250
	40	1.134	1.152	1.173	1.196	1.222
	Volatility		10%			

Table 8.4 illustrates the optimal trigger values with a stochastic 10 percent volatility on growth rates. Notice that the corresponding trigger values measured in terms of profitability indexes are higher for stochastic growth rates than for the deterministic growth rates. This is highly intuitive because the higher the level of uncertainty in the potential future of the underlying asset, the better off it is to wait before executing.

SWITCHING OPTIONS

In the ability to switch from technology 1 to technology 2, the option value is

$$S_2\Phi\left[\frac{\ln\left(\frac{S_2}{(1+X)S_1}\right)+\frac{T\sigma^2}{2}}{\sigma\sqrt{T}}\right]-S_1\Phi\left[\frac{\ln\left(\frac{S_2}{(1+X)S_1}\right)-\frac{T\sigma^2}{2}}{\sigma\sqrt{T}}\right]$$

$$-S_1X\Phi\left[\frac{\ln\left(\frac{S_2}{(1+X)S_1}\right)-\frac{T\sigma^2}{2}}{\sigma\sqrt{T}}\right]$$

where X is the proportional cost with respect to the current technology 1's asset value S_1. Hence, the optimal behavior is such that if the new technology's asset value S_2 exceeds the value of the current technology S_1 plus any associated switching costs S_1X, then it is optimal to switch.

Obviously, if multiple switching options are available, the problem becomes more complicated. Recall from the chooser option example in Chapter 7 that the value of the chooser option is not a simple sum of the individual options to expand, contract, and abandon. This is due to the mutually exclusive and path-dependent nature of these options, where the firm cannot both expand and abandon its business on the same node at the same time, or both expand and contract on the same node at the same time, etc. Valuing these options individually and then adding them together implies that each option is performed independent of one another and that two option executions may occupy the same space. Hence, to obtain the correct results, any crossovers where two options interact in the same space have to be accounted for. The same rule applies here. Thus, when an option exists that allows the switching from technology 1 to technology 2 or 3, the total value of the option is not simply the option to go from 1 to 2 plus the option to go from 1 to 3.

Tables 8.5 through 8.9 illustrate the relationships between the value of a switching option from an old technology to a new technology, and its corresponding input parameters. For example, in Table 8.5, where the present value of both technologies is currently on par with each other and the volatility is very close to 0 percent, with this negligible uncertainty, the value of the option is close to $0, similar to the static net present value of $0 because there is no point in being able to switch technology if the value of both technologies is identical. In contrast, when volatility increases slightly in the second technology, the value of being able to switch to this second technology increases. The rest of the examples are fairly self-explanatory.

TABLE 8.5 The Higher the Volatility of the New Technology, the Greater the Value of the Ability to Switch Technology

PV First Asset	100.00	100.00	100.00	100.00	100.00	100.00
PV Second Asset	100.00	100.00	100.00	100.00	100.00	100.00
First Asset Volatility	0%	1%	1%	1%	1%	1%
Second Asset Volatility	**0%**	**1%**	**2%**	**3%**	**4%**	**5%**
Correlation between Assets	0.00	0.00	0.00	0.00	0.00	0.00
Cost Multiplier	0.00	0.00	0.00	0.00	0.00	0.00
Time to Expiration	1.00	1.00	1.00	1.00	1.00	1.00
Risk-Free Rate	0%	0%	0%	0%	0%	0%
Portfolio Volatility	0.00	0.01	0.02	0.03	0.04	0.05
Switching Option Value	**0.01**	**0.56**	**0.89**	**1.26**	**1.64**	**2.03**
Static NPV	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**

TABLE 8.6 The Higher the Value of the Original Technology, the Lower the Value of the Ability to Switch Technology

PV First Asset	**100.00**	**110.00**	**120.00**	**130.00**	**140.00**	**150.00**
PV Second Asset	100.00	100.00	100.00	100.00	100.00	100.00
First Asset Volatility	10%	10%	10%	10%	10%	10%
Second Asset Volatility	10%	10%	10%	10%	10%	10%
Correlation between Assets	0.00	0.00	0.00	0.00	0.00	0.00
Cost Multiplier	0.00	0.00	0.00	0.00	0.00	0.00
Time to Expiration	1.00	1.00	1.00	1.00	1.00	1.00
Risk-Free Rate	0%	0%	0%	0%	0%	0%
Portfolio Volatility	0.14	0.14	0.14	0.14	0.14	0.14
Switching Option Value	**5.64**	**2.21**	**0.72**	**0.20**	**0.05**	**0.01**
Static NPV	**0.00**	**–10.00**	**–20.00**	**–30.00**	**–40.00**	**–50.00**

TABLE 8.7 The Higher the Value of the New Technology, the Higher the Value of the Ability to Switch Technology

PV First Asset	100.00	100.00	100.00	100.00	100.00	100.00
PV Second Asset	**100.00**	**110.00**	**120.00**	**130.00**	**140.00**	**150.00**
First Asset Volatility	10%	10%	10%	10%	10%	10%
Second Asset Volatility	10%	10%	10%	10%	10%	10%
Correlation between Assets	0.00	0.00	0.00	0.00	0.00	0.00
Cost Multiplier	0.00	0.00	0.00	0.00	0.00	0.00
Time to Expiration	1.00	1.00	1.00	1.00	1.00	1.00
Risk-Free Rate	0%	0%	0%	0%	0%	0%
Portfolio Volatility	0.14	0.14	0.14	0.14	0.14	0.14
Switching Option Value	**5.64**	**12.21**	**20.72**	**30.20**	**40.05**	**50.01**
Static NPV	**0.00**	**10.00**	**20.00**	**30.00**	**40.00**	**50.00**

TABLE 8.8 The Higher the Switching Cost, the Lower the Value of the Ability to Switch Technology

PV First Asset	100.00	100.00	100.00	100.00	100.00	100.00
PV Second Asset	100.00	100.00	100.00	100.00	100.00	100.00
First Asset Volatility	10%	10%	10%	10%	10%	10%
Second Asset Volatility	10%	10%	10%	10%	10%	10%
Correlation between Assets	0.00	0.00	0.00	0.00	0.00	0.00
Cost Multiplier	**0.00**	**0.10**	**0.20**	**0.30**	**0.40**	**0.50**
Time to Expiration	1.00	1.00	1.00	1.00	1.00	1.00
Risk-Free Rate	0%	0%	0%	0%	0%	0%
Portfolio Volatility	0.14	0.14	0.14	0.14	0.14	0.14
Switching Option Value	**5.64**	**2.21**	**0.72**	**0.20**	**0.05**	**0.01**
Static NPV	**0.00**	**–10.00**	**–20.00**	**–30.00**	**–40.00**	**–50.00**

TABLE 8.9 The Longer the Ability to Switch, the Higher the Value of the Ability to Switch Technology

PV First Asset	100.00	100.00	100.00	100.00	100.00	100.00
PV Second Asset	100.00	100.00	100.00	100.00	100.00	100.00
First Asset Volatility	10%	10%	10%	10%	10%	10%
Second Asset Volatility	10%	10%	10%	10%	10%	10%
Correlation between Assets	0.00	0.00	0.00	0.00	0.00	0.00
Cost Multiplier	0.00	0.00	0.00	0.00	0.00	0.00
Time to Expiration	**1.00**	**2.00**	**3.00**	**4.00**	**5.00**	**6.00**
Risk-Free Rate	0%	0%	0%	0%	0%	0%
Portfolio Volatility	0.14	0.14	0.14	0.14	0.14	0.14
Switching Option Value	**5.64**	**7.97**	**9.75**	**11.25**	**12.56**	**13.75**
Static NPV	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**

SUMMARY

Multiple other real options problems requiring more advanced techniques are required in certain circumstances. These models include the applications of stochastic optimization as well as other exotic types of options. In addition, as discussed, decision trees are insufficient when trying to solve real options problems because subjective probabilities are required as well as different discount rates at each node. The difficulties in forecasting the relevant discount rates and probabilities of occurrence are compounded over time, and the resulting values are oftentimes in error. However, decision trees by themselves are great as a depiction of management's strategic initiatives and opportunities over time. Decision trees should be used in conjunction with real options analytics in more complex cases.

CHAPTER 8 QUESTIONS

1. Decision trees are considered inappropriate when used to solve real options problems. Why is this so?
2. What are some of the assumptions required for risk-neutral probabilities to work?
3. What is stochastic optimization?
4. Assuming a 25 percent discount rate, 5.5 percent growth rate, and $100 in both present value of underlying assets and investment cost, change each of these variables at one-unit steps. That is, holding all inputs constant, change discount rate from 25 percent to 26 percent and so forth, and explain what happens to the optimal time to execution. Repeat the steps for growth rate, investment cost, and underlying asset value. Explain your results.

APPENDIX 8A

Stochastic Processes

Throughout the book the author talks about using stochastic processes for establishing simulation structures, risk-neutralizing revenue and cost, and obtaining an evolution of pricing structures. A stochastic process is nothing but a mathematically defined equation that can create a series of outcomes over time, outcomes that are not deterministic in nature. That is, an equation or process that does not follow any simple discernible rule such as price will increase X percent every year or revenues will increase by this factor of X plus Y percent. A stochastic process is by definition nondeterministic, and one can plug numbers into a stochastic process equation and obtain different results every time. For instance, the path of a stock price is stochastic in nature, and one cannot reliably predict the stock price path with any certainty. However, the price evolution over time is enveloped in a process that generates these prices. The process is fixed and predetermined, but the outcomes are not. Hence, by stochastic simulation, we create multiple pathways of prices, obtain a statistical sampling of these simulations, and make inferences on the potential pathways that the actual price may undertake given the nature and parameters of the stochastic process used to generate the time-series.

Four basic stochastic processes are discussed, including the Geometric Brownian Motion, which is the most common and prevalently used process due to its simplicity and wide-ranging applications. The mean-reversion process, barrier long-run process, and jump-diffusion process are also briefly discussed.

SUMMARY MATHEMATICAL CHARACTERISTICS OF GEOMETRIC BROWNIAN MOTIONS

Assume a process X, where $X = [X_t : t \geqslant 0]$ if and only if X_t is continuous, where the starting point is $X_0 = 0$, where X is normally distributed with mean zero and variance one or $X \in N(0, 1)$, and where each increment in time is independent of each other previous increment and is itself normally distributed with mean zero and variance t, such that $X_{t+a} - X_t \in N(0,\ t)$. Then, the process $dX = \alpha X dt + \sigma X dZ$ follows a Geometric Brownian Motion, where α is a drift parameter, σ the volatility measure, $dZ = \varepsilon_t \sqrt{\Delta dt}$ such that

$$\ln\left[\frac{dX}{X}\right] \in N(\mu, \sigma)$$

or X and dX are lognormally distributed. If at time zero, $X(0) = 0$ then the expected value of the process X at any time t is such that $E[X(t)] = X_0 e^{\alpha t}$ and the variance of the process X at time t is $V[X(t)] = X_0^2 e^{2\alpha t}(e^{\sigma^2 t} - 1)$. In the continuous case where there is a drift parameter α, the expected value then becomes

$$E\left[\int_0^\infty X(t)e^{-rt}dt\right] = \int_0^\infty X_0 e^{-(r-\alpha)t}dt = \frac{X_0}{(r - \alpha)}$$

SUMMARY MATHEMATICAL CHARACTERISTICS OF MEAN-REVERSION PROCESSES

If a stochastic process has a long-run attractor such as a long-run production cost or long-run steady state inflationary price level, then a mean-reversion process is more likely. The process reverts to a long-run average such that the expected value is $E[X_t] = \overline{X} + (X_0 - \overline{X})e^{-\eta t}$ and the variance is

$$V[X_t - \overline{X}] = \frac{\sigma^2}{2\eta(1 - e^{-2\eta t})}$$

The special circumstance that becomes useful is that in the limiting case when the time change becomes instantaneous or when $dt \to 0$, we have the condition where $X_t - X_{t-1} = \overline{X}(1 - e^{-\eta}) + X_{t-1}(e^{-\eta} - 1) + \varepsilon_t$, which is the first order autoregressive process, and η can be tested econometrically in a unit root context.

SUMMARY MATHEMATICAL CHARACTERISTICS OF BARRIER LONG-RUN PROCESSES

This process is used when there are natural barriers to prices—for example, floors or caps—or when there are physical constraints like the maximum capacity of a manufacturing plant. If barriers exist in the process, where we define $\overline{X}$ as the upper barrier and $\underline{X}$ as the lower barrier, we have a process where

$$X(t) = \frac{2\alpha}{\sigma^2} \quad \frac{e^{\frac{2\alpha X}{\sigma^2}}}{e^{\frac{2\alpha \bar{X}}{\sigma^2}} - e^{\frac{2\alpha \underline{X}}{\sigma^2}}}$$

SUMMARY MATHEMATICAL CHARACTERISTICS OF JUMP-DIFFUSION PROCESSES

Start-up ventures and research and development initiatives usually follow a jump-diffusion process. Business operations may be status quo for a few months or years, and then a product or initiative becomes highly successful and takes off. An initial public offering of equities, oil price jumps, and price of electricity are textbook examples of this. Assuming that the probability of the jumps follows a Poisson distribution, we have a process $dX = f(X, t)dt + g(X, t)dq$, where the functions f and g are known and where the probability process is

$$dq = \begin{cases} 0 \textit{ with } P(X) = 1 - \lambda dt \\ \mu \textit{ with } P(X) = X dt \end{cases}$$

APPENDIX 8B

Differential Equations for a Deterministic Case

One of the many approaches to solving a real options problem is the use of stochastic *optimization*. This optimization process can be done through a series of simulations or partial-differential equations to obtain a unique closed-form solution, as well as other more advanced optimization algorithms (e.g., simulated annealing, simplex, hill climbing, genetic algorithms, evolutionary solvers, and the like). Following is a very simplistic discussion and example of an optimization problem with constraints. Then a partial-differential equation framework is presented. Chapter 9 briefly illustrates a more complex optimization technique known as stochastic optimization or optimization under uncertainty, used in portfolio optimization and capital resource allocation where the input variables are stochastic and solvable only using Monte Carlo simulation.

A simple optimization process is shown in Figure 8B.1, where we can set up simple optimization problems in an Excel spreadsheet environment. In addition, we can solve optimization problems mathematically, as seen in the simple steps that follow:

- Create an objective function $f(x, y) = 3xy$.
- Set the constraint $c(x, y) = 200 - 5x - 15y$.
- Set the LaGrange Multiplier $\ell(x, y, \lambda) = f(x, y) + \lambda c(x, y) = 3xy + \lambda(200 - 5x - 15y)$.
- Optimize using partial-differentials:
  - $\dfrac{\partial \ell}{\partial \lambda} = 200 - 5x - 15y$
  - $\dfrac{\partial \ell}{\partial x} = 3y - 5\lambda$
  - $\dfrac{\partial \ell}{\partial y} = 3x - 15\lambda$

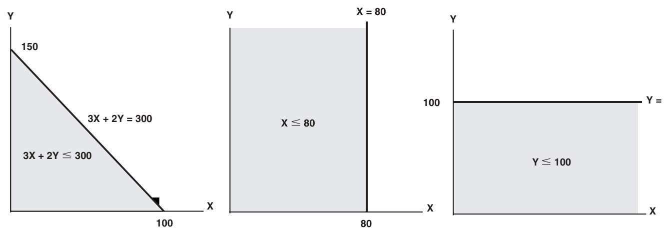
Linear Programming - Graphical Method

Say there are two products X and Y being manufactured. Product X provides a $20 profit and product Y a $15 profit. Product X takes 3 hours to manufacture and product Y takes 2 hours to produce. In any given week, the manufacturing equipment can make both products but has a maximum capacity of 300 hours. In addition, based on market demand, management has determined that they cannot sell more than 80 units of X and 100 units of Y in a given week and prefers not to have any inventory on hand. Therefore, management has set these demand levels as the maximum output for products X and Y, respectively. The issue now becomes what is the optimal production levels of both X and Y such that profits would be maximized in any given week?

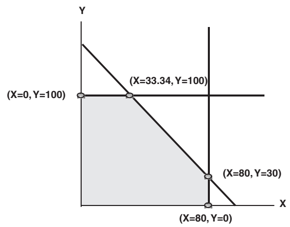
Based on the situation above, we can formulate a linear optimization routine where we have:

The Objective Function:	Max $20X + 15Y$
subject to Constraints:	$3X + 2Y \leq 300$ $X \leq 80$ $Y \leq 100$

We can more easily visualize the constraints by plotting them out one at a time as follows:



The graph below shows the combination of all three constraints. The shaded area shows the feasible area, where all constraints are simultaneously satisfied. Hence, the optimal should fall within this shaded region.



We can easily calculate the intersection points of the constraints. For example, the intersection between Y = 100 and 3X + 2Y = 300 is obtained by solving the equations simultaneously. Substituting, we get 3X + 2(100) = 300. Solving yields X = 33.24 and Y = 100.

Similarly, the intersection between X = 80 and 3X + 2Y = 300 can be obtained by solving the equations simultaneously. Substituting yields 3(80) + 2Y = 300. Solving yields Y = 30 and X = 80.

The other two edges are simply intersections between the axes. Hence, when X = 80, Y = 0 for the X = 80 line and Y = 100 and X = 0 for the Y = 100 line.

From linear programming theory, one of these four intersection edges or extreme values is the optimal solution. One method is simply to substitute each of the end points into the objective function and see which solution set provides the highest profit level.

Using the objective function where Profit = 20X + 15Y and substituting each of the extreme value sets:

When X = 0 and Y = 100:	Profit = \$20 (0) + \$15 (100) = \$1,500
When X = 33.34 and Y = 100:	Profit = \$20 (33.34) + \$15 (100) = \$2,167
When X = 80 and Y = 30:	Profit = \$20 (80) + \$15 (30) = \$2,050
When X = 80 and Y = 0:	Profit = \$20 (80) + \$15 (0) = \$1,600

Here, we see that when X = 33.34 and Y = 100, the profit function is maximized. We can also further verify this by using any combinations of X and Y within the feasible (shaded) area above. For instance, X =10 and Y =10 is a combination that is feasible, but their profit outcome is only \$20 (10) + \$15 (10) = \$350. We can calculate infinite combinations of X and Y sets, but the optimal combination is always going to be at extreme value edges.

We can easily verify which extreme value will be the optimal solution set by drawing the objective function line. If we set the objective function to be:

20X + 15Y = 0	we get X = 20, Y = 15
20X + 15Y = 1000	we get X = 60, Y = 80

If we keep shifting the profit function upward to the right, we will keep intersecting with the extreme value edges. The edge that provides the highest profit function is the optimal solution set.

In our example, point B is the optimal solution, which was verified by our calculations above, where X = 33.34 and Y = 100.

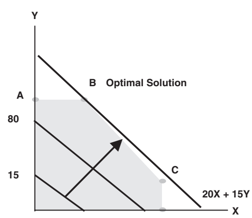


FIGURE 8B.1 Linear Programming

- Solving yields $x = 20$, $y = 6.67$, $\lambda = 4$ and
- Optimal output $f^*(x, y) = 3(20)(6.67) = 400$.
- λ is the constraint relaxation ratio, where an increase of a budget unit increases the optimal output by $\lambda = 4$.
- Using these optimization methods, we can then set up a more complex optimization process.

APPENDIX 8C

Exotic Options Formulae

BLACK AND SCHOLES OPTION MODEL—EUROPEAN VERSION

This is the famous Nobel Prize–winning Black-Scholes model without any dividend payments. It is the European version, where an option can only be executed at expiration and not before. Although it is simple enough to use, care should be taken in estimating its input variable assumptions, especially that of volatility, which is usually difficult to estimate. However, the Black-Scholes model is useful in generating ballpark estimates of the true real options value, especially for more generic-type calls and puts. For more complex real options analysis, different types of exotic options are required.

Definitions of Variables

S	present value of future cash flows (\$)
X	implementation cost (\$)
r	risk-free rate (%)
T	time to expiration (years)
σ	volatility (%)
Φ	cumulative standard-normal distribution

Computation

$$Call = S\Phi\left(\frac{\ln(S/X)+(r+\sigma^2/2)T}{\sigma\sqrt{T}}\right) - Xe^{-rT}\Phi\left(\frac{\ln(S/X)+(r-\sigma^2/2)T}{\sigma\sqrt{T}}\right)$$

$$Put = Xe^{-rT}\Phi\left(-\left[\frac{\ln(S/X)+(r-\sigma^2/2)T}{\sigma\sqrt{T}}\right]\right) - S\Phi\left(-\left[\frac{\ln(S/X)+(r+\sigma^2/2)T}{\sigma\sqrt{T}}\right]\right)$$

BLACK AND SCHOLES WITH DRIFT (DIVIDEND)—EUROPEAN VERSION

This is a modification of the Black-Scholes model and assumes a fixed dividend payment rate of q in percent. This can be construed as the opportunity cost of holding the option rather than holding the underlying asset.

Definitions of Variables

S present value of future cash flows ($)
X implementation cost ($)
r risk-free rate (%)
T time to expiration (years)
σ volatility (%)
Φ cumulative standard-normal distribution
q continuous dividend payout or opportunity cost (%)

Computation

$$Call = Se^{-qT}\Phi\left(\frac{\ln(S/X)+(r-q+\sigma^2/2)T}{\sigma\sqrt{T}}\right) - Xe^{-rT}\Phi\left(\frac{\ln(S/X)+(r-q-\sigma^2/2)T}{\sigma\sqrt{T}}\right)$$

$$Put = Xe^{-rT}\Phi\left(-\left[\frac{\ln(S/X)+(r-q-\sigma^2/2)T}{\sigma\sqrt{T}}\right]\right) - Se^{-qT}\Phi\left(-\left[\frac{\ln(S/X)+(r-q+\sigma^2/2)T}{\sigma\sqrt{T}}\right]\right)$$

BLACK AND SCHOLES WITH FUTURE PAYMENTS—EUROPEAN VERSION

Here, cash flow streams may be uneven over time, and we should allow for different discount rates (risk-free rate should be used) for all future times, perhaps allowing for the flexibility of the forward risk-free yield curve.

Definitions of Variables

S present value of future cash flows ($)
X implementation cost ($)
r risk-free rate (%)

T time to expiration (years)
σ volatility (%)
Φ cumulative standard-normal distribution
q continuous dividend payout or opportunity cost (%)
CF_i cash flow at time i

Computation

$$S^* = S - CF_1 e^{-rt_1} - CF_2 e^{-rt_2} - \ldots - CF_n e^{-rt_n} = S - \sum_{i=1}^{n} CF_i e^{-rt_i}$$

$$Call = S^* e^{-qT}\Phi\left(\frac{\ln(S^*/X) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}}\right) - Xe^{-rT}\Phi\left(\frac{\ln(S^*/X) + (r - q - \sigma^2/2)T}{\sigma\sqrt{T}}\right)$$

$$Put = Xe^{-rT}\Phi\left(-\left[\frac{\ln(S^*/X) + (r - q - \sigma^2/2)T}{\sigma\sqrt{T}}\right]\right) - S^* e^{-qT}\Phi\left(-\left[\frac{\ln(S^*/X) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}}\right]\right)$$

CHOOSER OPTIONS (BASIC CHOOSER)

This is the payoff for a simple chooser option when $t_1 < T_2$, or it doesn't work! In addition, it is assumed that the holder has the right to choose either a call or a put with the same strike price at time t_1 and with the same expiration date T_2. For different values of strike prices at different times, we need a complex variable chooser option.

Definitions of Variables

S present value of future cash flows (\$)
X implementation cost (\$)
r risk-free rate (%)
t_1 time to choose between a call or put (years)
T_2 time to expiration (years)

σ volatility (%)
Φ cumulative standard-normal distribution
q continuous dividend payments (%)

Computation

$$Option\ Value = Se^{-qT_2}\Phi\left[\frac{\ln(S/X) + (r - q + \sigma^2/2)T_2}{\sigma\sqrt{T_2}}\right]$$
$$- Se^{-qT_2}\Phi\left[\frac{-\ln(S/X) + (q - r)T_2 - t_1\sigma^2/2}{\sigma\sqrt{t_1}}\right]$$
$$- Xe^{-rT_2}\Phi\left[\frac{\ln(S/X) + (r - q + \sigma^2/2)T_2}{\sigma\sqrt{T_2}} - \sigma\sqrt{T_2}\right]$$
$$+ Xe^{-rT_2}\Phi\left[\frac{-\ln(S/X) + (q - r)T_2 - t_1\sigma^2/2}{\sigma\sqrt{t_1}} + \sigma\sqrt{t_1}\right]$$

COMPLEX CHOOSER

The holder of the option has the right to choose between a call and a put at different times (T_C and T_P) with different strike levels (X_C and X_P) of calls and puts. Note that some of these equations cannot be readily solved using Excel spreadsheets. Instead, due to the recursive methods used to solve certain bivariate distributions and critical values, the use of programming scripts is required.

Definitions of Variables

S present value of future cash flows ($)
X implementation cost ($)
r risk-free rate (%)
T time to expiration (years) for call (T_C) and put (T_P)
σ volatility (%)
Φ cumulative standard-normal distribution
Ω cumulative bivariate-normal distribution
q continuous dividend payout (%)
I critical value solved recursively
Z intermediate variables (Z_1 and Z_2)

Computation

First, solve recursively for the critical I value as follows:

$$0 = Ie^{-q(T_C-t)}\Phi\left[\frac{\ln(I/X_C)+(r-q+\sigma^2/2)(T_C-t)}{\sigma\sqrt{T_C-t}}\right]$$

$$-X_Ce^{-r(T_C-t)}\Phi\left[\frac{\ln(I/X_C)+(r-q+\sigma^2/2)(T_C-t)}{\sigma\sqrt{T_C-t}}-\sigma\sqrt{T_C-t}\right]$$

$$+Ie^{-q(T_P-t)}\Phi\left[\frac{-\ln(I/X_P)+(q-r-\sigma^2/2)(T_P-t)}{\sigma\sqrt{T_P-t}}\right]$$

$$-X_Pe^{-r(T_P-t)}\Phi\left[\frac{-\ln(I/X_P)+(q-r-\sigma^2/2)(T_P-t)}{\sigma\sqrt{T_P-t}}+\sigma\sqrt{T_P-t}\right]$$

Then using the I value, calculate

$$d_1 = \frac{\ln(S/I)+(r-q+\sigma^2/2)t}{\sigma\sqrt{t}} \quad \text{and } d_2 = d_1-\sigma\sqrt{t}$$

$$y_1 = \frac{\ln(S/X_C)+(r-q+\sigma^2/2)T_C}{\sigma\sqrt{T_C}} \quad \text{and}$$

$$y_2 = \frac{\ln(S/X_P)+(r-q+\sigma^2/2)T_P}{\sigma\sqrt{T_P}}$$

$$\rho_1 = \sqrt{t/T_C} \text{ and } \rho_2 = \sqrt{t/T_P}$$

$$\textit{Option Value} = Se^{-qT_C}\Omega(d_1;\, y_1;\, \rho_1) - X_Ce^{-rT_C}\Omega(d_2;\, y_1-\sigma\sqrt{T_C};\, \rho_1)$$

$$- Se^{-qT_P}\Omega(-d_1;-y_2;\rho_2) + X_Pe^{-rT_P}\Omega(-d_2;-y_2+\sigma\sqrt{T_P};\rho_2)$$

COMPOUND OPTIONS ON OPTIONS

The value of a compound option is based on the value of another option. That is, the underlying variable for the compound option is another option. Again, solving this model requires programming capabilities.

Definitions of Variables

S present value of future cash flows ($)
r risk-free rate (%)

σ volatility (%)
Φ cumulative standard-normal distribution
q continuous dividend payout (%)
I critical value solved recursively
Ω cumulative bivariate-normal distribution
X_1 strike for the underlying ($)
X_2 strike for the option on the option ($)
t_1 expiration date for the option on the option (years)
T_2 expiration for the underlying option (years)

Computation

First, solve for the critical value of I using

$$X_2 = Ie^{-q(T_2-t_1)}\Phi\left(\frac{\ln(I/X_1)+(r-q+\sigma^2/2)(T_2-t_1)}{\sigma\sqrt{(T_2-t_1)}}\right) - X_1e^{-r(T_2-t_1)}\Phi\left(\frac{\ln(I/X_1)+(r-q-\sigma^2/2)(T_2-t_1)}{\sigma\sqrt{(T_2-t_1)}}\right)$$

Solve recursively for the value I above and then input it into

$$\textit{Call on call} = Se^{-qT_2}\Omega\left[\frac{\ln(S/X_1)+(r-q+\sigma^2/2)T_2}{\sigma\sqrt{T_2}};\ \frac{\ln(S/I)+(r-q+\sigma^2/2)t_1}{\sigma\sqrt{t_1}};\sqrt{t_1/T_2}\right]$$

$$-X_1e^{-rT_2}\Omega\left[\frac{\ln(S/X_1)+(r-q+\sigma^2/2)T_2}{\sigma\sqrt{T_2}}-\sigma\sqrt{T_2};\ \frac{\ln(S/I)+(r-q+\sigma^2/2)t_1}{\sigma\sqrt{t_1}}-\sigma\sqrt{t_1};\ \sqrt{t_1/T_2}\right]$$

$$-X_2e^{-rt_1}\Phi\left[\frac{\ln(S/I)+(r-q+\sigma^2/2)t_1}{\sigma\sqrt{t_1}}-\sigma\sqrt{t_1}\right]$$

EXCHANGE ASSET FOR ASSET OPTION

The exchange asset for an asset option is a good application in a mergers and acquisition situation when a firm exchanges one stock for another firm's stock as a means of payment.

Definitions of Variables

- S present value of future cash flows ($) for Asset 1 ($S_1$) and Asset 2 ($S_2$)
- X implementation cost ($)
- Q quantity of Asset 1 to be exchanged for quantity of Asset 2
- r risk-free rate (%)
- T time to expiration (years) for call (T_C) and put (T_P)
- σ volatility (%) of Asset 1 (σ_1) and Asset 2 (σ_2)
- σ^* portfolio volatility after accounting for the assets' correlation ρ
- Φ cumulative standard-normal distribution
- q_1 continuous dividend payout (%) for Asset 1
- q_2 continuous dividend payout (%) for Asset 2

Computation

$$Option =$$

$$Q_1 S_1 e^{-q_1 T}\Phi\left[\frac{\ln(Q_1S_1/Q_2S_2) + (q_2 - q_1 + (\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2)/2)T}{\sqrt{T(\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2)}}\right]$$

$$-Q_2 S_2 e^{-q_2 T}\Phi\left[\frac{\ln(Q_1S_1/Q_2S_2) + (q_2 - q_1 + (\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2)/2)T}{\sqrt{T(\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2)}} - \sqrt{T(\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2)}\right]$$

FIXED STRIKE LOOK-BACK OPTION

The strike price is fixed in advance, and at expiration, the call option pays out the maximum of the difference between the highest observed price in the option's lifetime and the strike X, and 0, that is, $Call = Max[S_{MAX} - X, 0]$. A put at expiration pays out the maximum of the difference between the fixed strike X and the minimum price, and 0, that is, $Put = Max[X - S_{MIN}, 0]$.

Definitions of Variables

- S present value of future cash flows ($)
- X implementation cost ($)
- r risk-free rate (%)
- T time to expiration (years)
- σ volatility (%)
- Φ cumulative standard-normal distribution
- q continuous dividend payout (%)

Computation

Under the fixed strike look-back call option, when we have $X > S_{MAX}$, the call option is

$$Call = Se^{-qT}\Phi\left[\frac{\ln(S/X) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}}\right]$$

$$- Xe^{-rT}\Phi\left[\frac{\ln(S/X) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}} - \sigma\sqrt{T}\right]$$

$$+ Se^{-rT}\frac{\sigma^2}{2(r-q)}\left[\begin{array}{l} -\left(\dfrac{S}{X}\right)^{\frac{-2(r-q)}{\sigma^2}}\Phi\left(\begin{array}{l}\dfrac{\ln(S/X) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}} \\ -\dfrac{2(r-q)}{\sigma}\sqrt{T}\end{array}\right) \\ + e^{(r-q)T}\Phi\left[\dfrac{\ln(S/X) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}}\right]\end{array}\right]$$

However, when $X \leq S_{MAX}$ the call option is

$$Call = e^{-rT}(S_{MAX} - X) + Se^{-qT}\Phi\left[\frac{\ln(S/S_{MAX}) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}}\right]$$

$$- S_{MAX}e^{-rT}\Phi\left[\frac{\ln(S/S_{MAX}) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}} - \sigma\sqrt{T}\right]$$

$$+ Se^{-rT}\frac{\sigma^2}{2(r-q)}\left[\begin{array}{l} -\left(\dfrac{S}{S_{MAX}}\right)^{\frac{-2(r-q)}{\sigma^2}}\Phi\left(\begin{array}{l}\dfrac{\ln(S/S_{MAX}) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}} \\ -\dfrac{2(r-q)}{\sigma}\sqrt{T}\end{array}\right) \\ + e^{(r-q)T}\Phi\left[\dfrac{\ln(S/S_{MAX}) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}}\right]\end{array}\right]$$

FLOATING STRIKE LOOK-BACK OPTIONS

Floating strike look-back options give the call holder the option to buy the underlying security at the lowest observable price and the put holder the option to sell at the highest observable price. That is, we have a $Call = Max(S - S_{MIN}, 0)$ and $Put = Max(S_{MAX} - S, 0)$.

Definitions of Variables

S present value of future cash flows ($)
X implementation cost ($)
r risk-free rate (%)
T time to expiration (years)
σ volatility (%)
Φ cumulative standard-normal distribution
q continuous dividend payout (%)

Computation

$$
\begin{aligned}
Call = {} & Se^{-qT}\Phi\left[\frac{\ln(S/S_{MIN}) + (r-q+\sigma^2/2)T}{\sigma\sqrt{T}}\right] \\
& - S_{MIN}e^{-rT}\Phi\left[\frac{\ln(S/S_{MIN}) + (r-q+\sigma^2/2)T}{\sigma\sqrt{T}} - \sigma\sqrt{T}\right] \\
& + Se^{-rT}\frac{\sigma^2}{2(r-q)}\left[\begin{array}{l}\left(\dfrac{S}{S_{MIN}}\right)^{\frac{-2(r-q)}{\sigma^2}}\Phi\left(\begin{array}{l}\dfrac{-\ln(S/S_{MIN}) - (r-q+\sigma^2/2)T}{\sigma\sqrt{T}} \\ + \dfrac{2(r-q)}{\sigma}\sqrt{T}\end{array}\right) \\ - e^{(r-q)T}\Phi\left[\dfrac{-\ln(S/S_{MIN}) - (r-q+\sigma^2/2)T}{\sigma\sqrt{T}}\right]\end{array}\right]
\end{aligned}
$$

$$
\begin{aligned}
Put = {} & S_{MAX}e^{-rT}\Phi\left[\frac{-\ln(S/S_{MAX}) - (r-q+\sigma^2/2)T}{\sigma\sqrt{T}} + \sigma\sqrt{T}\right] \\
& - Se^{-qT}\Phi\left[\frac{-\ln(S/S_{MAX}) - (r-q+\sigma^2/2)T}{\sigma\sqrt{T}}\right] \\
& + Se^{-rT}\frac{\sigma^2}{2(r-q)}\left[\begin{array}{l}-\left(\dfrac{S}{S_{MAX}}\right)^{\frac{-2(r-q)}{\sigma^2}}\Phi\left(\begin{array}{l}\dfrac{\ln(S/S_{MAX}) + (r-q+\sigma^2/2)T}{\sigma\sqrt{T}} \\ - \dfrac{2(r-q)}{\sigma}\sqrt{T}\end{array}\right) \\ + e^{(r-q)T}\Phi\left[\dfrac{\ln(S/S_{MAX}) + (r-q+\sigma^2/2)T}{\sigma\sqrt{T}}\right]\end{array}\right]
\end{aligned}
$$

FORWARD START OPTIONS

Definitions of Variables

S present value of future cash flows ($)
X implementation cost ($)
r risk-free rate (%)
t_1 time when the forward start option begins (years)
T_2 time to expiration of the forward start option (years)
σ volatility (%)
Φ cumulative standard-normal distribution
q continuous dividend payout (%)

Computation

$$Call = Se^{-qt_1}e^{-q(T_2-t_1)}\Phi\left[\frac{\ln(1/\alpha)+(r-q+\sigma^2/2)(T_2-t_1)}{\sigma\sqrt{T_2-t_1}}\right]$$

$$-Se^{-qt_1}\alpha e^{(-r)(T_2-t_1)}\Phi\left[\frac{\ln(1/\alpha)+(r-q+\sigma^2/2)(T_2-t_1)}{\sigma\sqrt{T_2-t_1}}-\sigma\sqrt{T_2-t_1}\right]$$

$$Put = Se^{-qt_1}\alpha e^{(-r)(T_2-t_1)}\Phi\left[\frac{-\ln(1/\alpha)-(r-q+\sigma^2/2)(T_2-t_1)}{\sigma\sqrt{T_2-t_1}}+\sigma\sqrt{T_2-t_1}\right]$$

$$-Se^{-qt_1}e^{-q(T_2-t_1)}\Phi\left[\frac{-\ln(1/\alpha)-(r-q+\sigma^2/2)(T_2-t_1)}{\sigma\sqrt{T_2-t_1}}\right]$$

where α is the multiplier constant.

Note: If the option starts at X percent out-of-the-money, α will be $(1 + X)$. If it starts at-the-money, α will be 1.0, and $(1 - X)$ if in-the-money.

GENERALIZED BLACK-SCHOLES MODEL

Definitions of Variables

S present value of future cash flows ($)
X implementation cost ($)
r risk-free rate (%)
T time to expiration (years)
σ volatility (%)
Φ cumulative standard-normal distribution
b carrying cost (%)
q continuous dividend payout (%)

Computation

$$Call = Se^{(b-r)T}\Phi\left(\frac{\ln(S/X) + (b + \sigma^2/2)T}{\sigma\sqrt{T}}\right)$$

$$-Xe^{-rT}\Phi\left(\frac{\ln(S/X) + (b - \sigma^2/2)T}{\sigma\sqrt{T}}\right)$$

$$Put = Xe^{-rT}\Phi\left(-\left[\frac{\ln(S/X) + (b - \sigma^2/2)T}{\sigma\sqrt{T}}\right]\right)$$

$$- Se^{(b-r)T}\Phi\left(-\left[\frac{\ln(S/X) + (b + \sigma^2/2)T}{\sigma\sqrt{T}}\right]\right)$$

Notes:

$b = 0$	Futures options model
$b = r - q$	Black-Scholes with dividend payment
$b = r$	Simple Black-Scholes formula
$b = r - r^*$	Foreign currency options model

OPTIONS ON FUTURES

The underlying security is a forward or futures contract with initial price F. Here, the value of F is the forward or futures contract's initial price, replacing S with F as well as calculating its present value.

Definitions of Variables

X	implementation cost ($)
F	futures single-point cash flows ($)
r	risk-free rate (%)
T	time to expiration (years)
σ	volatility (%)
Φ	cumulative standard-normal distribution
q	continuous dividend payout (%)

Computation

$$Call = Fe^{-rT}\Phi\left(\frac{\ln(F/X) + (\sigma^2/2)T}{\sigma\sqrt{T}}\right) - Xe^{-rT}\Phi\left(\frac{\ln(F/X) - (\sigma^2/2)T}{\sigma\sqrt{T}}\right)$$

$$Put = Xe^{-rT}\Phi\left(-\left[\frac{\ln(F/X) - (\sigma^2/2)T}{\sigma\sqrt{T}}\right]\right) - Fe^{-rT}\Phi\left(-\left[\frac{\ln(F/X) + (\sigma^2/2)T}{\sigma\sqrt{T}}\right]\right)$$

SPREAD OPTION

The payoff on a spread option depends on the spread between the two futures contracts less the implementation cost.

Definitions of Variables

X implementation cost ($)
r risk-free rate (%)
T time to expiration (years)
σ volatility (%)
Φ cumulative standard-normal distribution
F_1 price for futures contract 1
F_2 price for futures contract 2
ρ correlation between the two futures contracts

Computation

First, calculate the portfolio volatility:

$$\sigma = \sqrt{\sigma_1^2 + \left[\sigma_2 \frac{F_2}{F_2 + X}\right]^2 - 2\rho\sigma_1\sigma_2 \frac{F_2}{F_2 + X}}$$

Then, obtain the call and put option values:

$$Call = (F_2 + X)\left[e^{-rT}\left\{\begin{array}{l}\dfrac{F_1}{F_2 + X}\Phi\left[\dfrac{\ln\left[\dfrac{F_1}{F_2 + X}\right] + (\sigma^2/2)T}{\sigma\sqrt{T}}\right] \\ -\Phi\left[\dfrac{\ln\left[\dfrac{F_1}{F_2 + X}\right] + (\sigma^2/2)T}{\sigma\sqrt{T}} - \sigma\sqrt{T}\right]\end{array}\right\}\right]$$

$$Put = (F_2 + X)\left[e^{-rT}\left\{\begin{array}{l}\Phi\left[\dfrac{-\ln\left[\dfrac{F_1}{F_2 + X}\right] - (\sigma^2/2)T}{\sigma\sqrt{T}} + \sigma\sqrt{T}\right] \\ -\dfrac{F_1}{F_2 + X}\Phi\left[\dfrac{-\ln\left[\dfrac{F_1}{F_2 + X}\right] - (\sigma^2/2)T}{\sigma\sqrt{T}}\right]\end{array}\right\}\right]$$

DISCRETE TIME SWITCH OPTIONS

The discrete time switch option holder will receive an amount equivalent to $A\Delta t$ at maturity T for each time interval of Δt where the corresponding asset price $S_{i\Delta t}$ has exceeded strike price X. The put option provides a similar payoff every time $S_{i\Delta t}$ is below the strike price.

Definitions of Variables

- S present value of future cash flows ($)
- X implementation cost ($)
- r risk-free rate (%)
- T time to expiration (years)
- σ volatility (%)
- Φ cumulative standard-normal distribution
- b carrying cost (%), usually the risk-free rate less any continuous dividend payout rate

Computation

$$Call = Ae^{-rT}\sum_{i=1}^{n}\Phi\left(\frac{\ln(S/X)+(b-\sigma^2/2)i\Delta t}{\sigma\sqrt{i\Delta t}}\right)\Delta t$$

$$Put = Ae^{-rT}\sum_{i=1}^{n}\Phi\left(\frac{-\ln(S/X)-(b-\sigma^2/2)i\Delta t}{\sigma\sqrt{i\Delta t}}\right)\Delta t$$

TWO-CORRELATED-ASSETS OPTION

The payoff on an option depends on whether the other correlated option is in-the-money. This is the continuous counterpart to a correlated quadranomial model.

Definitions of Variables

- S present value of future cash flows ($)
- X implementation cost ($)
- r risk-free rate (%)
- T time to expiration (years)
- σ volatility (%)
- Ω cumulative bivariate-normal distribution function
- ρ correlation (%) between the two assets
- q_1 continuous dividend payout for the first asset (%)
- q_2 continuous dividend payout for the second asset (%)

Computation

$$Call = S_2 e^{-q_2 T}\Omega\left[\begin{array}{l}\dfrac{\ln(S_2/X_2)+(r-q_2-\sigma_2^2/2)T}{\sigma_2\sqrt{T}}+\sigma_2\sqrt{T};\\[2ex] \dfrac{\ln(S_1/X_1)+(r-q_1-\sigma_1^2/2)T}{\sigma_1\sqrt{T}}+\rho\sigma_2\sqrt{T};\ \rho\end{array}\right]$$

$$-X_2 e^{-rT}\Omega\left[\begin{array}{l}\dfrac{\ln(S_2/X_2)+(r-q_2-\sigma_2^2/2)T}{\sigma_2\sqrt{T}};\\[2ex] \dfrac{\ln(S_1/X_1)+(r-q_1-\sigma_1^2/2)T}{\sigma_1\sqrt{T}};\ \rho\end{array}\right]$$

$$Put = X_2 e^{-rT}\Omega\left[\begin{array}{l}\dfrac{-\ln(S_2/X_2)-(r-q_2-\sigma_2^2/2)T}{\sigma_2\sqrt{T}};\\[2ex] \dfrac{-\ln(S_1/X_1)-(r-q_1-\sigma_1^2/2)T}{\sigma_1\sqrt{T}};\ \rho\end{array}\right]$$

$$-S_2 e^{-q_2 T}\Omega\left[\begin{array}{l}\dfrac{-\ln(S_2/X_2)-(r-q_2-\sigma_2^2/2)T}{\sigma_2\sqrt{T}}-\sigma_2\sqrt{T};\\[2ex] \dfrac{-\ln(S_1/X_1)-(r-q_1-\sigma_1^2/2)T}{\sigma_1\sqrt{T}}-\rho\sigma_2\sqrt{T};\ \rho\end{array}\right]$$

PART Three

Software Applications

CHAPTER 9

Introduction to the Real Options Valuation's Super Lattice Software and Risk Simulator Software

Now that you are confident with the applicability of real options and its intricate mathematical constructs, it is time to move on and use the Real Options Valuation's Super Lattice Solver (SLS) and Risk Simulator software in the enclosed CD-ROM. As shown in Chapters 7 and 8, applying real options is not an easy task. The use of software-based models allows the analyst to apply a consistent, well-tested, and replicable set of models. It reduces computational errors and allows the user to focus more on the process and problem at hand rather than on building potentially complex and mathematically intractable models. This chapter starts with an introduction to the Super Lattice Solver software and continues with the Risk Simulator software.

The enclosed CD-ROM has a 30-day trial version of the Super Lattice Solver and Risk Simulator software. For professors, contact the author for complimentary year-long licenses for you and your students for installation in computer labs. The remainder of this book and relevant examples require the use of these software applications. To install the Super Lattice Solver software, insert the CD and wait for the setup program to start. If it does not start automatically, browse the content of the CD and double-click on the *CDAutorun.exe* file and follow the simple on-screen instructions. You must be first connected to the Internet before you can download and install the latest version of the software. Install the Super Lattice Solver software and then the Risk Simulator software. When prompted, enter the following user name and license key for a 30-day trial of the SLS software:

Name: **30 Day License** License Key: **513C-27D2-DC6B-9666**

Another license key is required to permanently unlock and use the software, and the license can be purchased by contacting the author.

After successfully installing the software, verify that the installation was successful by clicking on and making sure that the following folder exists: *Start* | *Programs* | *Real Options Valuation* | *Real Options Super Lattice Solver*. Note that the SLS software will work on most international Windows operating systems but requires a quick change in settings by clicking on *Start* | *Control Panel* | *Regional and Language Options*. Select *English (United States)*. This is required because the numbering convention is different in foreign countries (e.g., one thousand dollars and fifty cents is written as 1,000.50 in the United States versus 1.000,50 in certain European countries).

INTRODUCTION TO THE SUPER LATTICE SOLVER SOFTWARE

The Real Options Super Lattice Software (SLS) comprises several modules, including the Single Super Lattice Solver (SSLS), Multiple Super Lattice Solver (MSLS), Multinomial Lattice Solver (MNLS), SLS Excel Solution, and SLS Functions. These modules are highly powerful and customizable binomial and multinomial lattice solvers and can be used to solve many types of options (including the three main families of options: *real options*, which deals with physical and intangible assets; *financial options*, which deals with financial assets and the investments of such assets; and *employee stock options*, which deals with financial assets provided to employees within a corporation). This text illustrates some sample real options, financial options, and employee stock options applications that users will most frequently encounter.

- The SSLS is used primarily for solving options with a *single underlying asset* using binomial lattices. Even highly complex options with a single underlying asset can be solved using the SSLS.
- The MSLS is used for solving options with *multiple underlying assets* and sequential compound options with *multiple phases* using binomial lattices. Highly complex options with multiple underlying assets and phases can be solved using the MSLS.
- The MNLS uses *multinomial lattices* (trinomial, quadranomial, pentanomial) to solve specific options that cannot be solved using binomial lattices.
- The SLS Excel Solution implements the SSLS and MSLS computations within the Excel environment, allowing users to access the SSLS and MSLS functions directly in Excel. This feature facilitates model building,

formula and value linking and embedding, as well as running simulations, and provides the user sample templates to create such models.

- The SLS Functions are additional real options and financial options models accessible directly through Excel. This facilitates model building, linking and embedding, and running simulations.

The SLS software is created by the author and accompanies the materials presented at different training courses on real options, simulation, and employee stock options valuation taught by Dr. Mun. While the software and its models are based on his books, the training courses cover the real options subject matter in more depth, including the solution of sample business cases and the framing of real options of actual cases. It is highly suggested that the reader familiarizes him- or herself with the fundamental concepts of real options in Chapters 6 and 7 prior to attempting an in-depth real options analysis using this software.

Note: The first edition of *Real Options Analysis: Tools and Techniques* published in 2002 shows the Real Options Analysis Toolkit software, an older precursor to the Super Lattice Solver, also created by Dr. Johnathan Mun. The Super Lattice Solver version 1.1 supersedes the Real Options Analysis Toolkit by providing the following enhancements, and is introduced in this second edition:

- All inconsistencies, computation errors, and bugs fixed and verified.
- Allowance of changing input parameters over time (customized options).
- Allowance of changing volatilities over time.
- Incorporation of Bermudan (vesting and blackout periods) and Customized Options.
- Flexible modeling capabilities in creating or engineering your own customized options.
- General enhancements to accuracy, precision, and analytical prowess.

As the creator of both the Super Lattice Solver and Real Options Analysis Toolkit software, the author suggests that the reader focuses on using the Super Lattice Solver as it provides many powerful enhancements and analytical flexibility over its predecessor, the older, less powerful, and less flexible Real Options Analysis Toolkit software.

SINGLE SUPER LATTICE SOLVER

Figure 9.1 illustrates the SSLS module. After installing the software, the user can access the SSLS by clicking on *Start* | *Programs* | *Real Options Valuation* | *Real Options Super Lattice Solver* | *Single Super Lattice Solver*. The SSLS

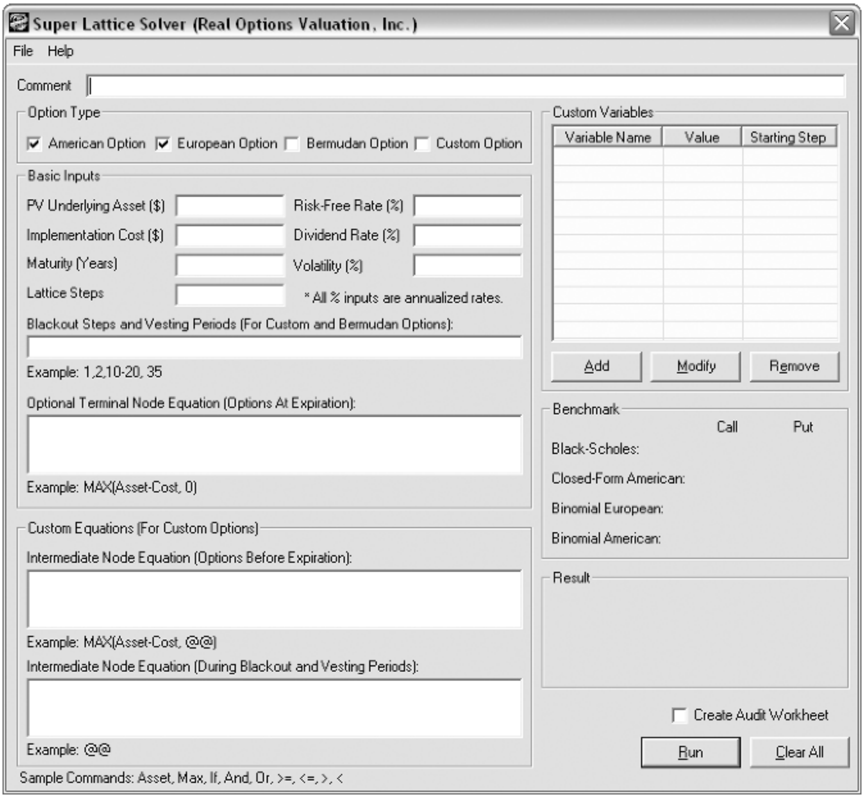


FIGURE 9.1 Single Super Lattice Solver (SSLS)

has several sections: Option Type, Basic Inputs, Custom Equations, Custom Variables, Benchmark, Result, and Create Audit Worksheet.

SSLS Examples

To help you get started, several simple examples are in order. A simple European call option is computed in this example using SSLS. To follow along, start this example file by selecting *Start* | *Programs* | *Real Options Valuation* | *Real Options Super Lattice Solver* | *Sample Files* | *Plain Vanilla Call Option I*. This example file will be loaded into the SSLS software as seen in Figure 9.2. The starting PV Underlying Asset or starting stock price is $100, and the Implementation Cost or strike price is $100 with a five-year maturity. The annualized risk-free rate of return is 5 percent, and the historical, comparable, or future expected annualized volatility is 10 percent. Click on *RUN* (or Alt-R) and a 100-step binomial lattice is computed and the results indicate a value of $23.3975 for both the European and American call options. Benchmark

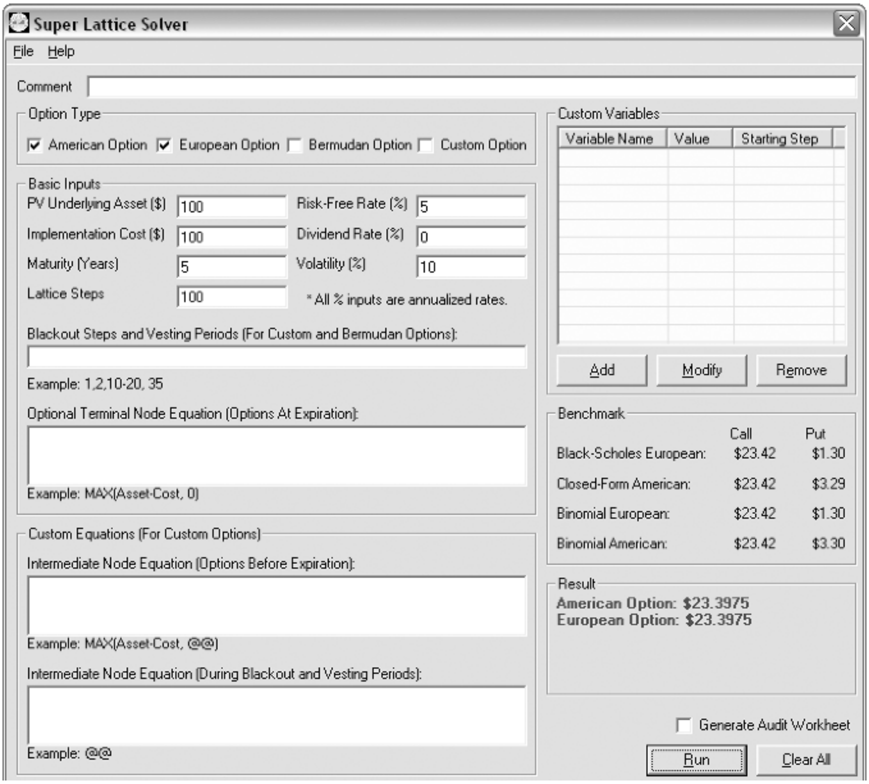


FIGURE 9.2 SSLS Results of a Simple European and American Call Option

values using Black-Scholes and Closed-Form American approximation models as well as standard plain-vanilla Binomial American and Binomial European Call and Put Options with 1,000-step binomial lattices are also computed. Notice that only the American and European Options are selected and the computed results are for these simple plain-vanilla American and European call options.

The benchmark results use both closed-form models (Black-Scholes and Closed-Form Approximation models) and 1,000-step binomial lattices on plain-vanilla options. You can change the steps to *1000* in the basic inputs section to verify that the answers computed are equivalent to the benchmarks as seen in Figure 9.3. Notice that, of course, the values computed for the American and European options are identical to each other and identical to the benchmark values of $23.4187, as it is never optimal to exercise a standard plain-vanilla call option early if there are no dividends. Be aware that the higher the lattice step, the longer it takes to compute the results. It is advisable

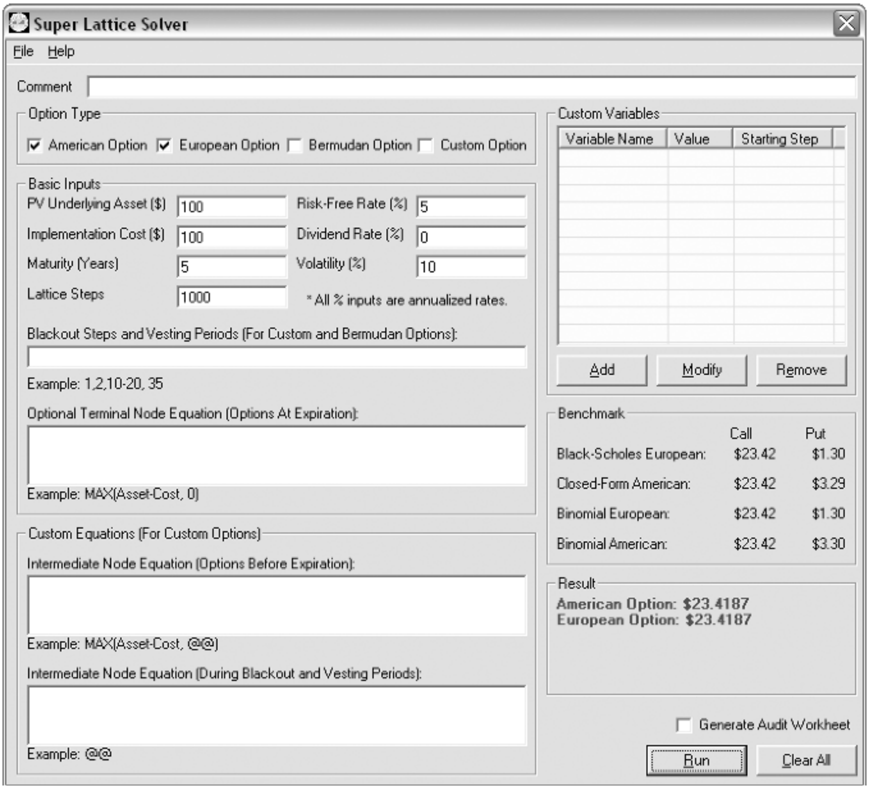


FIGURE 9.3 SSLS Comparing Results with Benchmarks

to start with lower lattice steps to make sure the analysis is robust and then progressively increase lattice steps to check for results convergence. See Chapter 6 on convergence criteria on lattices for more details about binomial lattice convergence as to how many lattice steps are required for a robust option valuation.

Alternatively, you can enter Terminal and Intermediate Equations for a call option to obtain the same results. Notice that using 100 steps and creating your own Terminal Equation of *Max(Asset-Cost,0)* and Intermediate Equation of *Max(Asset-Cost,@@)* will yield the same answer. When entering your own equations, make sure that Custom Option is first checked.

When entering your own equations, make sure that Custom Option is first checked.

Figure 9.4 illustrates how the analysis is done. The example file used in this example is: *Plain Vanilla Call Option III*. Notice that the value $23.3975 in Figure 9.4 agrees with the value in Figure 9.2. The Terminal Node Equation is the computation that occurs at maturity, while the Intermediate Node Equation is the computation that occurs at all periods prior to maturity, and is computed using backward induction. The symbol "@@" represents "keeping the option open," and is often used in the Intermediate Node Equation when analytically representing the fact that the option is not executed but kept open for possible future execution. Therefore, in Figure 9.4, the Intermediate Node Equation *Max(Asset-Cost,@@)* represents the profit maximization decision of either executing the option or leaving it open for possible future execution. In contrast, the Terminal Node Equation of *Max(Asset-Cost,0)* represents the profit maximization decision at maturity of either executing the option if it is in-the-money, or allowing it to expire worthless if it is at-the-money or out-of-the-money.

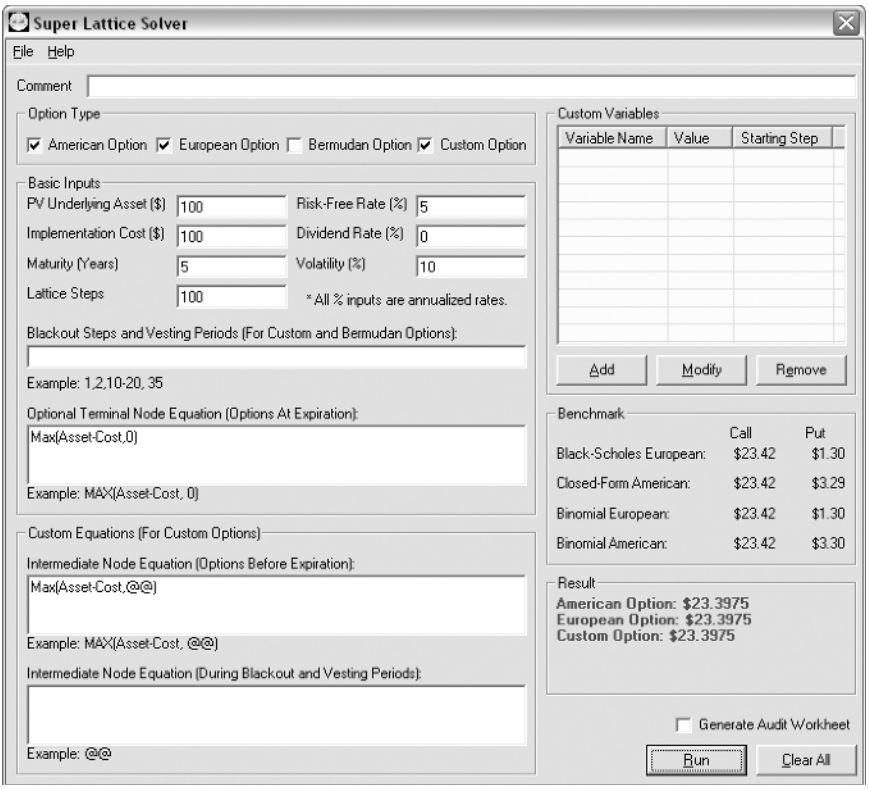


FIGURE 9.4 Custom Equation Inputs

In addition, you can create an Audit Worksheet in Excel to view a sample 10-step binomial lattice by checking the box *Generate Audit Worksheet*. For instance, loading the example file *Plain Vanilla Call Option I* and selecting the box creates a worksheet as seen in Figure 9.5. Several items on this audit worksheet are noteworthy:

- The audit worksheet generated will show the first 10 steps of the lattice, regardless of how many you enter. That is, if you enter 1,000 steps, the first 10 steps will be generated. If a complete lattice is required, simply enter 10 steps in the SSLS and the full 10-step lattice will be generated instead. The Intermediate Computations and Results are for the Super Lattice, based on the number of lattice steps entered, and not based on the 10-step lattice generated. To obtain the Intermediate Computations for 10-step lattices, simply rerun the analysis inputting 10 as the lattice steps. This way, the audit worksheet generated will be for a 10-step lattice, and the results from SSLS will now be comparable (Figure 9.6).
- The worksheet only provides values as it is assumed that the user was the one who entered in the terminal and intermediate equations, hence there is really no need to recreate these equations in Excel again. The user can always reload the SSLS file and view the equations or print out the form if required (by clicking on *File* | *Print*).

The software also allows you to save or open analysis files. That is, all the inputs in the software will be saved and can be retrieved for future use. The results will not be saved because you may accidentally delete or change an input and the results will no longer be valid. In addition, rerunning the super lattice computations will only take a few seconds, and it is always advisable for you to always rerun the model when opening an old analysis file.

You may also enter in Blackout Steps. These are the steps on the super lattice that will have different behaviors than the terminal or intermediate steps. For instance, you can enter *1000* as the lattice steps, and enter *0-400* as the blackout steps, and some Blackout Equation (e.g., @@). This means that for the first 400 steps, the option holder can only keep the option open. Other examples include entering: 1, 3, 5, 10 if these are the lattice steps where blackout periods occur. You will have to calculate the relevant steps within the lattice where the blackout exists. For instance, if the blackout exists in years 1 and 3 on a 10-year, 10-step lattice, then steps 1, 3 will be the blackout dates. This blackout step feature comes in handy when analyzing options with holding periods, vesting periods, or periods where the option cannot be executed. Employee stock options have blackout and vesting periods, and certain contractual real options have periods during which the option cannot be executed (e.g., cooling-off periods, or proof of concept periods).

Option Valuation Audit Sheet

Assumptions		***Intermediate Computations***	
PV Asset Value ($)	$100.00	*Stepping Time (dt)*	0.0500
Implementation Cost ($)	$100.00	*Up Step Size (up)*	1.0226
Maturity (Years)	5.00	*Down Step Size (down)*	0.9779
Risk-free Rate (%)	5.00%	*Risk-neutral Probability*	0.5504
Dividends (%)	0.00%		
Volatility (%)	10.00%	***Results***	
Lattice Steps	100	*Lattice Result*	23.40
Option Type	European		

Terminal Equation	MAX(Asset-Cost, 0)
Intermediate Equation	@@
Intermediate Equation (Blackouts)	@@

Underlying Asset Lattice

										125.06
									122.29	
								119.59		119.59
							116.94		116.94	
						114.36		114.36		114.36
					111.83		111.83		111.83	
				109.36		109.36		109.36		109.36
			106.94		106.94		106.94		106.94	
		104.57		104.57		104.57		104.57		104.57
	102.26		102.26		102.26		102.26		102.26	
100.00		100.00		100.00		100.00		100.00		100.00
	97.79		97.79		97.79		97.79		97.79	
		95.63		95.63		95.63		95.63		95.63
			93.51		93.51		93.51		93.51	
				91.44		91.44		91.44		91.44
					89.42		89.42		89.42	
						87.44		87.44		87.44
							85.51		85.51	
								83.62		83.62
									81.77	
										79.96

Option Valuation Lattice

										45.33
									42.81	
								40.35		39.96
							37.97		37.58	
						35.66		35.27		34.87
					33.43		33.04		32.64	
				31.27		30.88		30.49		30.09
			29.18		28.80		28.41		28.02	
		27.18		26.79		26.41		26.02		25.64
	25.25		24.87		24.49		24.11		23.73	
23.40		23.03		22.65		22.28		21.90		21.52
	21.26		20.90		20.53		20.16		19.79	
		19.22		18.86		18.50		18.14		17.77
			17.28		16.93		16.58		16.22	
				15.45		15.10		14.76		14.41
					13.71		13.38		13.05	
						12.09		11.77		11.45
							10.58		10.27	
								9.19		8.89
									7.91	
										6.74

FIGURE 9.5 SSLS-Generated Audit Worksheet

If equations are entered into the Terminal Equation box and American, European, or Bermudan Options are chosen, the terminal equation you entered will be the one used in the super lattice for the terminal nodes. However, for the intermediate nodes, the American option assumes the same terminal equation plus the ability to keep the option open; the European option assumes that the option can only be kept open and not executed; while

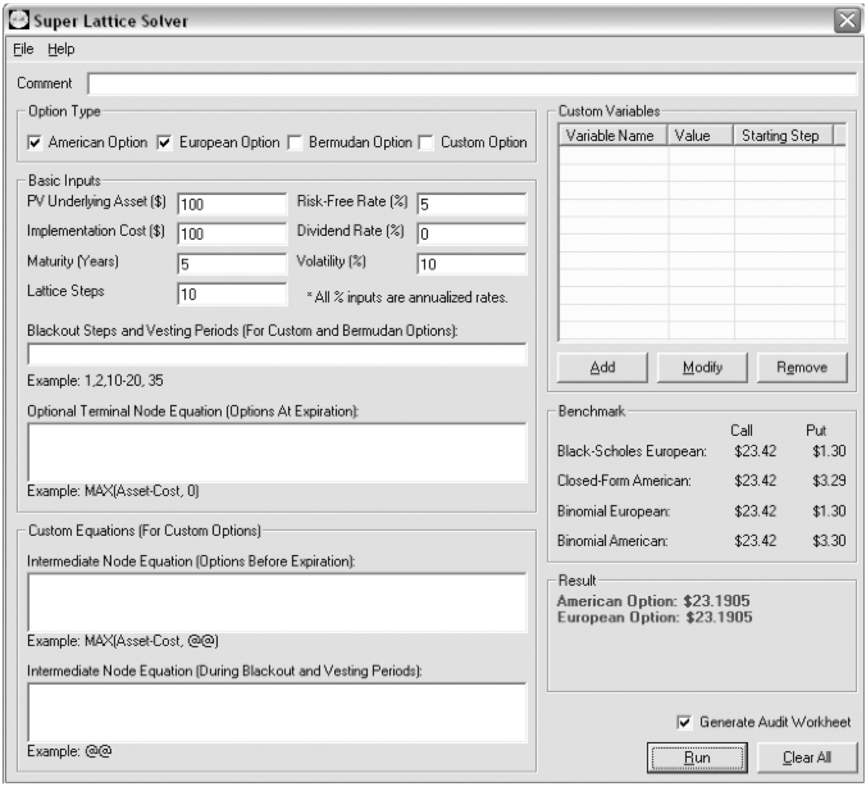


FIGURE 9.6 SSLS Results with a 10-Step Lattice

the Bermudan option assumes that during the blackout lattice steps, the option will be kept open and cannot be executed. If you also enter the Intermediate Equation, the Custom Option should be first chosen (otherwise you cannot use the Intermediate Equation box). The Custom Option result uses all the equations you have entered in Terminal, Intermediate, and Intermediate with Blackout sections.

The Custom Variables list is where you can add, modify, or delete custom variables, the variables that are required beyond the basic inputs. For instance, when running an abandonment option, you need the salvage value. You can add this in the Custom Variables list, provide it a name (a variable name must be a single word), the appropriate value, and the starting step when this value becomes effective. That is, if you have multiple salvage values (i.e., if salvage values change over time), you can enter the same variable name (e.g., *salvage*) several times, but each time, its value changes and you can specify when the appropriate salvage value becomes effective. For instance, in a 10-year, 100-step super lattice problem where there are two salvage values—$100 occurring within the first 5 years and increases to $150 at the beginning of Year 6—you can enter two salvage variables with the same name, $100 with a starting step of 0, and $150 with a starting step of 51. Be careful here as Year 6 starts at step 51 and not 61. That is, for a 10-year option with a 100-step lattice, we have: Steps 1–10 = Year 1; Steps 11–20 = Year 2; Steps 21–30 = Year 3; Steps 31–40 = Year 4; Steps 41–50 = Year 5; Steps 51–60 = Year 6; Steps 61–70 = Year 7; Steps 71–80 = Year 8; Steps 81–90 = Year 9; and Steps 91–100 = Year 10. Finally, incorporating 0 as a blackout step indicates that the option cannot be executed immediately. See Chapter 10 for more details on using Custom Variables.

MULTIPLE SUPER LATTICE SOLVER

The MSLS is an extension of the SSLS in that the MSLS can be used to solve options with multiple underlying assets and multiple phases. The MSLS allows the user to enter multiple underlying assets as well as multiple valuation lattices (Figure 9.7). These valuation lattices can call to user-defined custom

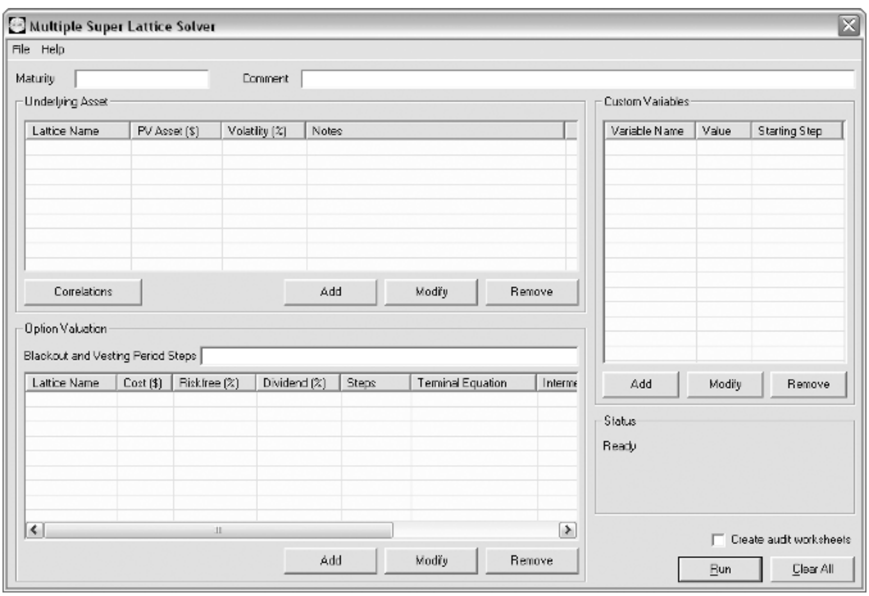


FIGURE 9.7 Multiple Super Lattice Solver

variables. Some examples of the types of options that the MSLS can be used to solve include:

- Sequential Compound Options (two-, three-, and multiple-phased sequential options).
- Simultaneous Compound Options (multiple assets with multiple simultaneous options).
- Chooser and Switching Options (choosing among several options and underlying assets).
- Floating Options (choosing between calls and puts).
- Multiple Asset Options (3D binomial option models).

The MSLS software has several areas including a *Maturity* and *Comment* area. The Maturity value is a global value for the entire option, regardless of how many underlying or valuation lattices exist. The Comment field is for your personal notes describing the model you are building. There is also a *Blackout and Vesting Period Steps* section and a *Custom Variables* list similar to the SSLS. The MSLS also allows you to create Audit Worksheets.

To illustrate the power of the MSLS, a simple illustration is in order. Click on *Start* | *Programs* | *Real Options Valuation* | *Real Options Super Lattice Solver* | *Sample Files* | *MSLS — Two-Phased Sequential Compound Option*. Figure 9.8 shows the MSLS example loaded. In this simple example, a single underlying asset is created with two valuation phases.

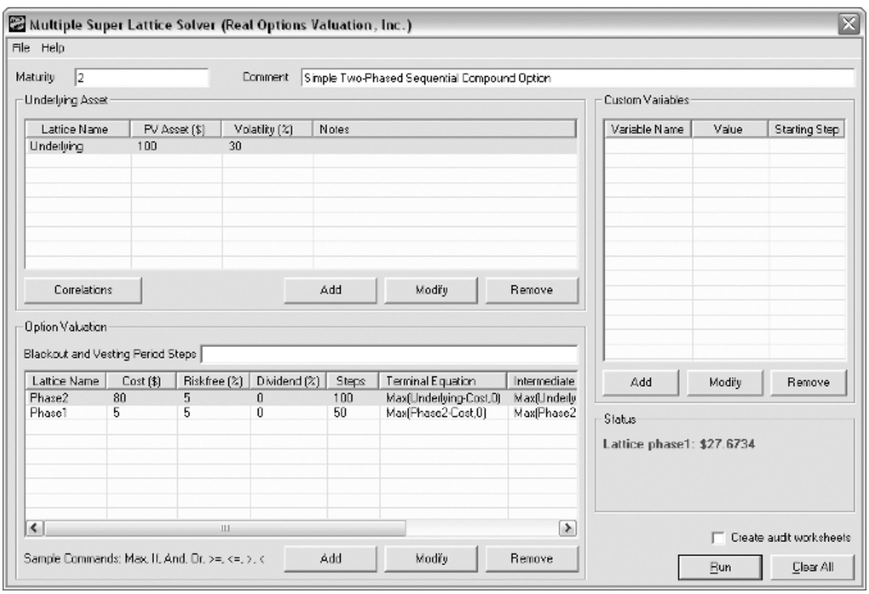


FIGURE 9.8 MSLS Solution to a Simple Two-Phased Sequential Compound Option

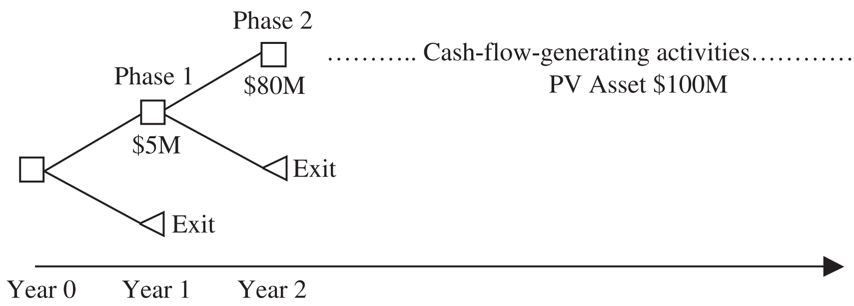


FIGURE 9.9 Strategy Tree for Two-Phased Sequential Compound Option

The strategy tree for this option is seen in Figure 9.9. The project is executed in two phases—the first phase within the first year costs \$5 million, while the second phase occurs within two years but only after the first phase is executed, and costs \$80 million, both in present value dollars. The PV Asset of the project is \$100 million (NPV is therefore \$15 million), and faces 30 percent volatility in its cash flows (see Appendix 7A on Volatility for the relevant volatility computations). The computed strategic value using the MSLS is \$27.67 million, indicating that there is a \$12.67 million in option value. That is, spreading out and staging the investment into two phases has significant value (an expected value of \$12.67 million to be exact). See the sections on compound options in Chapter 10 for more examples and results interpretation.

MULTINOMIAL LATTICE SOLVER

The MNLS is another module of the Real Options Valuation's Super Lattice Solver software. It applies multinomial lattices—where multiple branches stem from each node—such as trinomials (three branches), quadranomials (four branches), and pentanomials (five branches). Figure 9.10 illustrates the MNLS module. The module has a Basic Inputs section, where all of the common inputs for the multinomials are listed. Then, there are four sections with four different multinomial applications complete with the additional required inputs and results for both American and European call and put options.

Figure 9.11 shows an example call and put option computation using trinomial lattices. To follow along, open the example file *MNLS—Simple Calls and Puts using Trinomial Lattices*. Note that the results shown in Figure 9.11 using a 50-step lattice is equivalent to the results shown in Figure 9.2 using a 100-step binomial lattice. In fact, a trinomial lattice or any other multinomial lattice provides identical answers to the binomial lattice at the limit, but convergence is achieved faster at lower steps. To illustrate, Table 9.1 shows

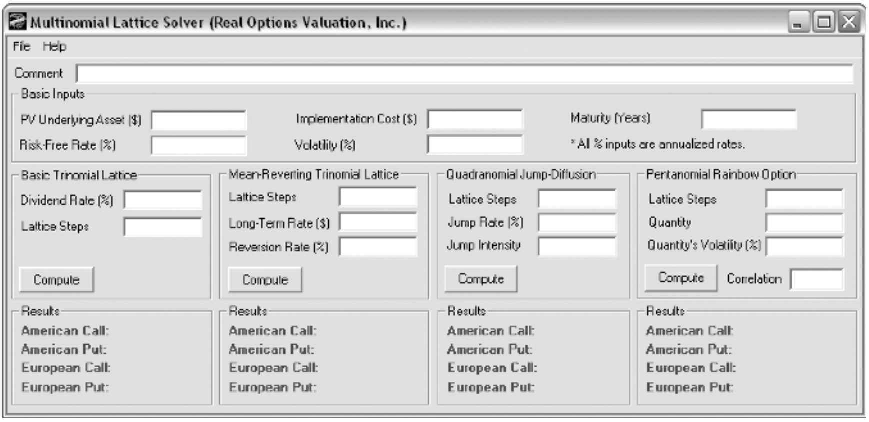


FIGURE 9.10 Multinomial Lattice Solver

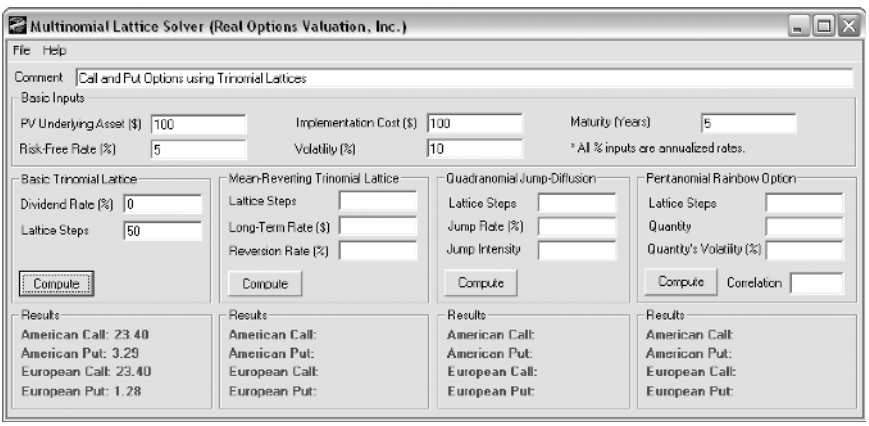


FIGURE 9.11 A Simple Call and Put Using Trinomial Lattices

TABLE 9.1 Binomial versus Trinomial Lattices

Steps	5	10	100	1,000	5,000
Binomial Lattice	$30.73	$29.22	$29.72	$29.77	$29.78
Trinomial Lattice	29.22	29.50	29.75	29.78	29.78

how the trinomial lattice of a certain set of input assumptions yields the correct option value with fewer steps than it takes for a binomial lattice. Because both yield identical results at the limit but trinomials are much more difficult to calculate and take a longer computation time, the binomial lattice is usually used instead. However, a trinomial is required only under one special circumstance: when the underlying asset follows a mean-reverting process.

With the same logic, quadranomials and pentanomials yield identical results as the binomial lattice with the exception that these multinomial lattices can be used to solve the following different special limiting conditions:

- Trinomials: Results are identical to binomials and are most appropriate when used to solve mean-reverting underlying assets.
- Quadranomials: Results are identical to binomials and are most appropriate when used to solve options whose underlying assets follow jump-diffusion processes.
- Pentanomials: Results are identical to binomials and are most appropriate when used to solve two underlying assets that are combined, called rainbow options (e.g., price and quantity are multiplied to obtain total revenues, but price and quantity each follows a different underlying lattice with its own volatility but both underlying parameters could be correlated to one another).

See the sections on Mean-Reverting, Jump-Diffusion, and Rainbow Options in Chapter 10 for more details, examples, and results interpretation.

SLS EXCEL SOLUTION (SSLS, MSLS, AND CHANGING VOLATILITY MODELS IN EXCEL)

The SLS software also allows you to create your own models in Excel using customized functions. This is an important functionality because certain models may require linking from other spreadsheets or databases, run certain Excel macros and functions, or certain inputs need to be simulated, or inputs may change over the course of modeling your options. This Excel compatibility allows you the flexibility to innovate within the Excel spreadsheet environment. Specifically, the sample worksheet included in the software solves the SSLS, MSLS, and Changing Volatility model.

To illustrate, Figure 9.12 shows a Customized Abandonment Option solved using SSLS. The same problem can be solved using the *SLS Excel Solution* by clicking on *Start* | *Programs* | *Real Options Valuation* | *Real Options Super Lattice Solver* | *SLS Excel Solution*. The sample solution is seen in Figure 9.13. Notice the same results using the SSLS versus the SLS Excel Solution

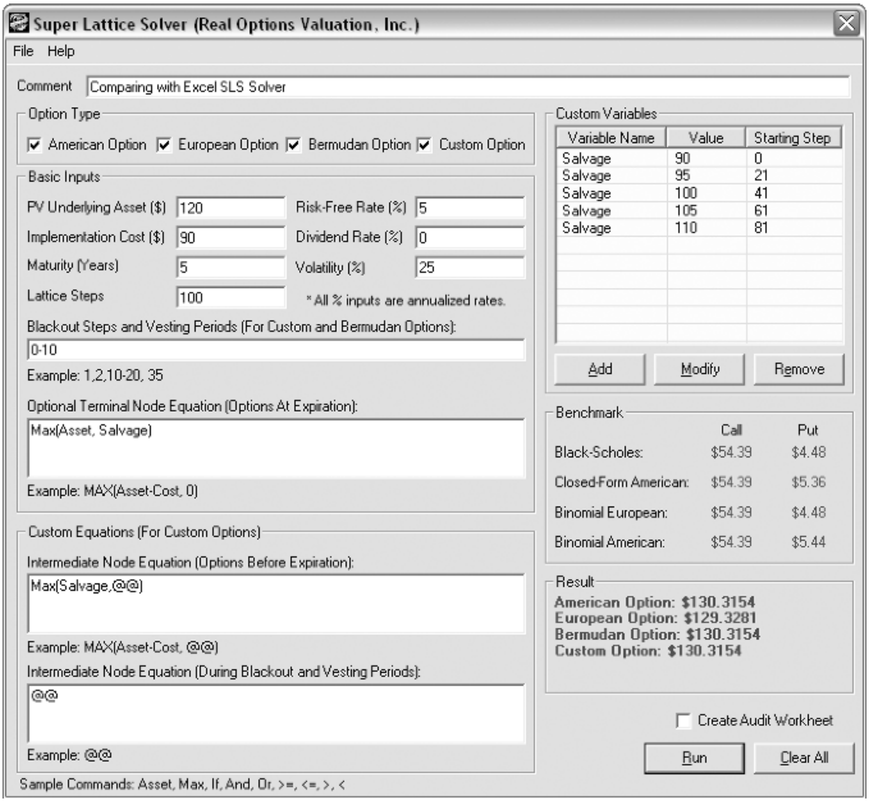


FIGURE 9.12 Customized Abandonment Option Using SSLS

SUPER LATTICE SOLVER (SINGLE ASSET)

Option Type	0
PV Underlying Asset	$120.00
Annualized Volatility	25.00%
Maturity (Years)	5.00
Implementation Cost	$0.00
Risk-Free Rate	5.00%
Dividend Yield	0.00%
Lattice Steps	100
Terminal Equation	MAX(Asset, Salvage)
Intermediate Equation	MAX(Salvage, @@)
Intermediate Equation During Blackout	@@
Blackout Steps	0-10
Super Lattice Solver Result	***$130.3154***

Custom Variables List

Variable Name	*Value*	*Starting Steps*
Salvage	*90.00*	*0*
Salvage	*95.00*	*21*
Salvage	*100.00*	*41*
Salvage	*105.00*	*61*
Salvage	*110.00*	*81*

Note: This is the Excel version of the Super Lattice Solver, useful when running simulations or when linking to and from other spreadsheets. Use this sample spreadsheet for your models. You can simply click on File, Save As to save as a different file and start using the model. For the option type, set 0 = American, 1 = European, 2 = Bermudan, 3 = Custom

FIGURE 9.13 Customized Abandonment Option Using SLS Excel Solution file. You can use the template provided by simply clicking on *File | Save As* in Excel and use the new file for your own modeling needs.

Similarly, the MSLS can also be solved using the SLS Excel Solver. Figure 9.14 shows a complex multiple-phased sequential compound option solved using the SLS Excel Solver. The results shown here are identical to the results generated from the MSLS module (example file: *MSLS — Multiple Phased Complex Sequential Compound Option*). One small note of caution here is that if you add or reduce the number of option valuation lattices, make sure you change the function's link for the *MSLS Result* to incorporate the right number of rows otherwise the analysis will not compute properly. For example, the default shows three option valuation lattices and by selecting the *MSLS Results* cell in the spreadsheet and clicking on *Insert | Function*, you will see that the function links to cells A24:H26 for these three rows for the *OVLattices* input in the function. If you add another option valuation lattice, change the link to A24:H27, and so forth. You can also leave the list of custom variables as is. The results will not be affected if these variables are not used in the custom equations.

Finally, Figure 9.15 shows a Changing Volatility and Changing Risk-free Rate Option. In this model, the volatility and risk-free yields are allowed to change over time and a nonrecombining lattice is required to solve the option. In most cases, it is recommended that you create option models without the changing volatility term structure because getting a single volatility is difficult enough let alone a series of changing volatilities over time. If different volatilities that are uncertain need to be modeled, run a Monte Carlo simulation using the Risk Simulator software on volatilities instead. This model should only be used when the volatilities are modeled robustly and the volatilities are rather certain and changes over time. The same advice applies to a changing risk-free rate term structure.

SLS FUNCTIONS

The software also provides a series of SLS functions that are directly accessible in Excel. To illustrate its use, start the SLS Functions by clicking on *Start | Programs | Real Options Valuation | Real Options Super Lattice Solver | SLS Functions*, and Excel will start. When in Excel, you can click on the function wizard icon or simply select an empty cell and click on *Insert | Function*. While in Excel's equation wizard, either select the *All* category or *Real Options Valuation*, the name of the company that developed the software. Here you will see a list of SLS functions (with SLS prefixes) that are ready for use in Excel. Figure 9.16 shows the Excel equation wizard.

Suppose you select the first function, *SLSBinomialAmericanCall* and hit OK. Figure 9.17 shows how the function can be linked to an existing Excel

MULTIPLE SUPER LATTICE SOLVER (MULTIPLE ASSET & MULTIPLE PHASES)

Maturity (Years)	5.00
Blackout Steps	0-20
Correlation*	

MSLS Result $134.0802

Underlying Asset Lattices

Lattice Name	PV Asset	Volatility
Underlying	100.00	25.00

Custom Variables

Name	Value	Starting Steps
Salvage	100.00	31
Salvage	90.00	11
Salvage	80.00	0
Contract	0.90	0
Expansion	1.50	0
Savings	20.00	0

Option Valuation Lattices

Lattice Name	Cost	Riskfree	Dividend	Steps	Terminal Equation	Intermediate Equation	Intermediate Equation for Blackout
Phase3	50.00	5.00	0.00	50	Max(Underlying*Expansion-Cost,Underlying,Salvage)	Max(Underlying*Expansion-Cost,Salvage,@@)	@@
Phase2	0.00	5.00	0.00	30	Max(Phase3,Phase3*Contract+Savings,Salvage,0)	Max(Phase3*Contract+Savings,Salvage,@@)	@@
Phase1	0.00	5.00	0.00	10	Max(Phase2,Salvage,0)	Max(Salvage,@@)	@@

Note: This is the Excel version of the Multiple Super Lattice Solver, useful when running simulations or when linking to and from other spreadsheets.
Use this sample spreadsheet for your models. You can simply click on File, Save As to save as a different file and start using the model.
*Because this is an Excel solution, the correlation function is not supported and is linked to an empty cell.

FIGURE 9.14 Complex Sequential Compound Option Using SLS Excel Solver

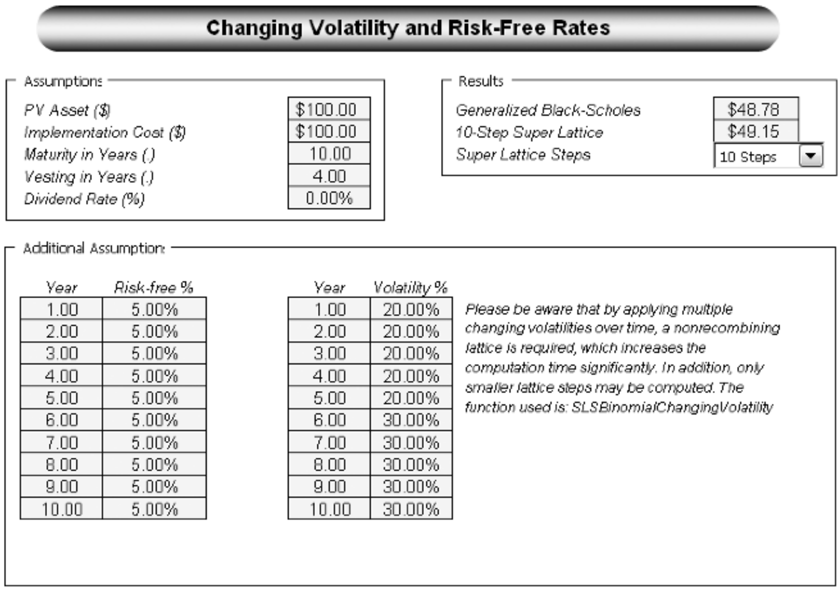


FIGURE 9.15 Changing Volatility and Risk-Free Rate Option

model. The values in cells B1 to B7 can be linked from other models or spreadsheets, or can be created using Excel's Visual Basic for Applications (VBA) macros, or can be dynamic and changing as in when running a simulation. Another quick note of caution here is that certain SLS functions require many input variables, and Excel's equation wizard can only show five variables at

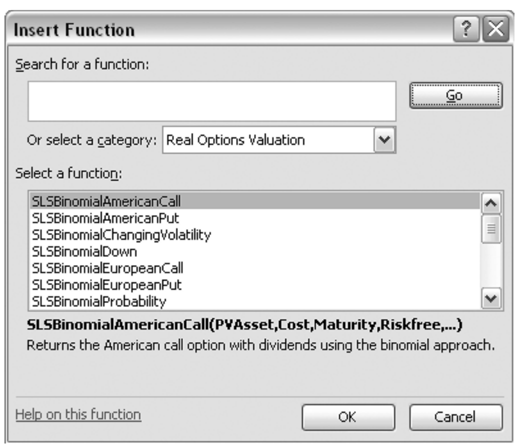


FIGURE 9.16 Excel's Equation Wizard

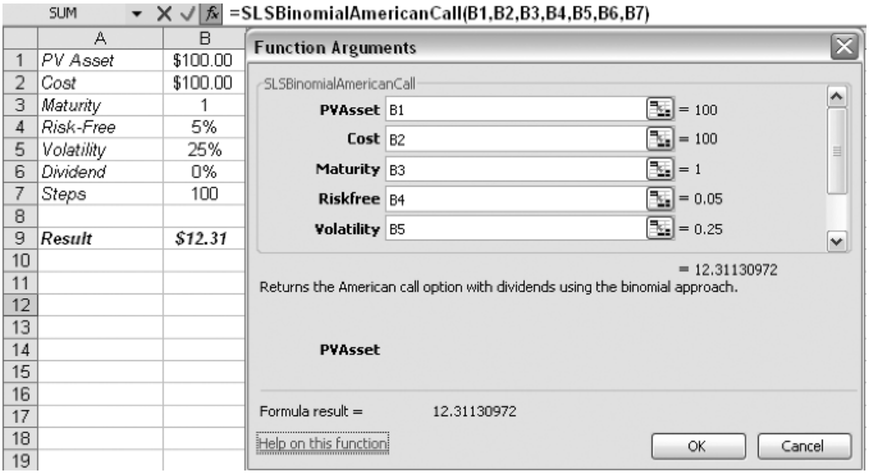


FIGURE 9.17 Using SLS Functions in Excel

a time. Therefore, remember to scroll down the list of variables by clicking on the vertical scroll bar to access the rest of the variables.

You are now equipped to start using the SLS software in building and solving real options, financial options, and employee stock options problems. These applications are introduced starting in Chapter 10.

LATTICE MAKER

Finally, the full version of the software comes with an advanced binomial *Lattice Maker* module. This Lattice Maker is capable of generating binomial lattices and decision lattices with visible formulas in an Excel spreadsheet. Figure 9.18 illustrates an example option generated using this module. The illustration shows the module inputs (you can obtain this module by clicking on *Start* | *Programs* | *Real Options Valuation* | *Real Options Super Lattice Solver* | *Lattice Maker*) and the resulting output lattice. Notice that the visible equations are linked to the existing spreadsheet, which means this module will come in handy when running Monte Carlo simulations or when used to link to and from other spreadsheet models. The results can also be used as a presentation and learning tool to peep inside the analytical black box of binomial lattices. Last but not least, a decision lattice with specific decision nodes indicating expected optimal times of execution of certain options is also available in this module. The results generated from this module are identical to those generated using the SLS and Excel functions, but has the added

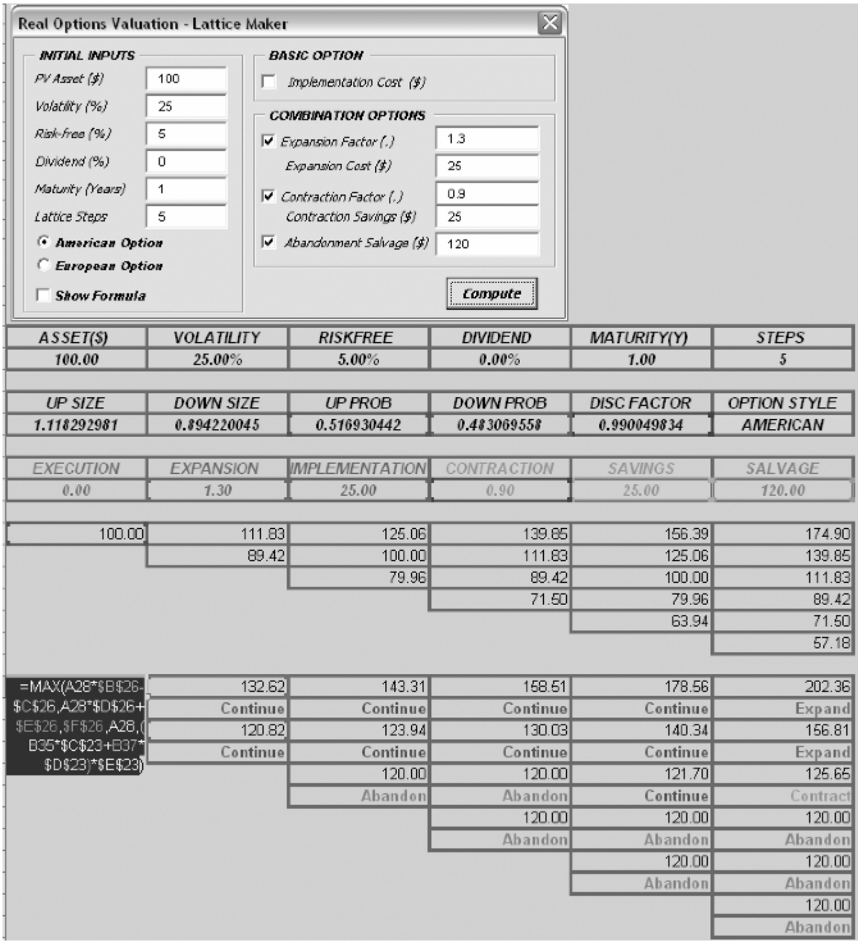


FIGURE 9.18 Lattice Maker

advantage of a visible lattice (lattices of up to 200 steps can be generated using this module).

INTRODUCTION TO THE RISK SIMULATOR SOFTWARE

Risk Simulator is a simulation, forecasting, and optimization program written in Microsoft .NET C# and functions together with Excel. To install the software, insert the CD and click on *Install Risk Simulator*. You need to be connected to the Internet to download the latest version for installation. If the installer does not appear after you insert the CD, browse the contents of the CD and double-click on *CDAutorun.exe*. The minimum requirements for this software are:

- Pentium III processor or later.
- Windows 2000, XP, or later.
- Microsoft Excel XP, 2003, or later.
- Microsoft .NET Framework.
- 30MB free space.
- 256MB RAM recommended.

Most new computers come with Microsoft .NET Framework already preinstalled. However, if an error message pertaining to requiring .NET Framework occurs during the installation of Risk Simulator, exit the installation. Then, install the relevant .NET Framework software found on the CD (*DOT NET Framework* folder) or on the Microsoft website (go to www.microsoft.com and enter in the following search term: "*get .net framework*"). Complete the .NET installation, restart the computer and then reinstall the Risk Simulator software.

Once installation is complete, start Microsoft Excel and if the installation was successful, you should see an additional *Simulation* item on the menu bar in Excel. You are now ready to start using the software. The following sections provide step-by-step instructions for using the software. Be aware that this installation process will always download the latest version of the software—it automatically installs the full version but is accessible for 30 days only and you can purchase the license key to permanently unlock the software by visiting www.realoptionsvaluation.com—and therefore some of the examples in this book might look slightly different than what you see on your screen as the software is continually updated and enhanced.

MONTE CARLO SIMULATION

Monte Carlo simulation, named for the famous gambling capital of Monaco, is a very potent methodology. Statisticians and mathematicians sometimes dislike it because it solves difficult and often intractable problems with too much simplicity and ease. Instead, mathematical purists would prefer the more elegant approach: the old-fashioned way. Solving a fancy stochastic mathematical model provides a sense of accomplishment and completion as opposed to the brute force method used in simulation. However, for the practitioner, simulation opens the door for solving difficult and complex but practical problems with great ease. Monte Carlo creates artificial futures by generating thousands and even millions of sample paths of outcomes and looks at their prevalent characteristics. For analysts in a company, taking graduate level advanced math courses is just not logical or practical. A brilliant analyst would use all available tools at his or her disposal to obtain the same answer the easiest and most practical way possible. And in all cases, when modeled correctly, Monte Carlo simulation provides similar answers to the more mathematically elegant methods. So, what is Monte Carlo simulation and how does it work?

What Is Monte Carlo Simulation?

Monte Carlo simulation in its simplest form is a random number generator that is useful for forecasting, estimation, and risk analysis. A simulation calculates numerous scenarios of a model by repeatedly picking values from a user-predefined *probability distribution* for the uncertain variables and using those values for the model. As all those scenarios produce associated results in a model, where each scenario can have a *forecast*. Forecasts are events (usually with formulas or functions) that you define as important outputs of the model. These usually are events such as totals, net profit, or gross expenses.

Simplistically, think of the Monte Carlo simulation approach as picking golf balls out of a large basket repeatedly with replacement. The size and shape of the basket depend on the distributional *input assumption* (e.g., a normal distribution with a mean of 100 and a standard deviation of 10 versus a uniform distribution or a triangular distribution) where some baskets are deeper or more symmetrical than others, allowing certain balls to be pulled out more frequently than others. The number of balls pulled repeatedly depends on the number of *trials* simulated. For a large model with multiple related assumptions, imagine the large model as a very large basket, where many baby baskets reside. Each baby basket has its own set of golf balls that are bouncing around. Sometimes these baby baskets are holding hands with each other (if there is a *correlation* between the variables) and the golf balls are bouncing in tandem while others are bouncing independently of one another. The balls that are picked each time from these interactions within the model (the large central basket) are tabulated and recorded, providing a *forecast output* result of the simulation.

Getting Started with Risk Simulator

As software updates occur more frequently than book editions, it is always advisable to check www.realoptionsvaluation.com for the latest version of the Super Lattice Solver software and Risk Simulator software than the one included with this book. E-mail the author at JohnathanMun@cs.com or visit the Real Options Valuation, Inc. web site (www.realoptionsvaluation.com) to obtain full commercial and academic versions of the software. The following sections briefly illustrate how the software can be used in a very fundamental sense. See the software's user manual for more details about using the software as the following pages are not meant to replace the software's user manual.

A High-Level Overview of the Software

The Risk Simulator software has several different applications including Monte Carlo simulation, forecasting, and optimization.

- The simulation application allows users to run simulations in their existing Excel-based models, generate simulation forecasts (distributions of results), perform distributional fitting (automatically finding the best-fitting statistical distribution), compute correlations (maintain relationships among variables), identify sensitivities (creating tornado and sensitivity charts), as well as run custom and nonparametric simulations (simulations using historical data without specifying any distributions or their parameters).
- The forecasting application can be used to generate automatic time-series forecasts (with seasonality and trend), multivariate regressions (linear and nonlinear regressions), ARIMA forecasts (an advanced econometric forecasting model called the Autoregressive Integrated Moving Average), nonlinear extrapolations (curve fitting), and stochastic processes (random walks, mean-reversions, and jump-diffusion processes).
- The optimization application is used for optimizing multiple decision variables subject to constraints to maximize or minimize an objective, and can be run either as a static optimization or as a dynamic optimization under uncertainty together with Monte Carlo simulation.

Running a Monte Carlo Simulation

Typically, to run a simulation in your existing Excel model, the following steps have to be performed:

- Start a new simulation profile.
- Define input assumptions in the relevant cells.
- Define output forecasts in the relevant cells.
- Check or change any simulation preferences.
- Run simulation.
- Interpret the results.

1. Starting a New Simulation Profile To start a new simulation, you first need to create a simulation profile:

- Start Excel and create a new or open an existing model.
- Click on *Simulation* and select *New Simulation Profile*.

- Specify a title for your simulation as well as all other pertinent information (Figure 9.19).
  - ***Title:*** Specifying a simulation title allows you to create multiple simulation profiles in a single Excel model. This means that you can now save different simulation scenario profiles within the same model without having to delete existing assumptions and changing them each time a new simulation scenario is required.
  - ***Number of trials:*** This is where the number of simulation trials required is entered. That is, running 1,000 trials means that 1,000 different iterations of outcomes based on the input assumptions will be generated.
  - ***Pause on simulation error:*** If checked, the simulation stops every time an error is encountered in the Excel model. That is, if your model encounters a computation error (e.g., some input values generated in a simulation trial may yield a divide by zero error in one of your spreadsheet cells), the simulation stops. This is important to help audit your model to make sure there are no computational errors in your Excel model. However, if you are sure the model works, then there is no need for this preference to be checked.
  - ***Turn on correlations:*** If checked, correlations between paired input assumptions will be computed, yielding slightly slower simulations. Otherwise, correlations will all be set to zero and a simulation is run assuming no cross-correlations between input assumptions. Applying correlations will yield more accurate results if indeed correlations exist, and will tend to yield a lower forecast confidence if negative correlations exist.

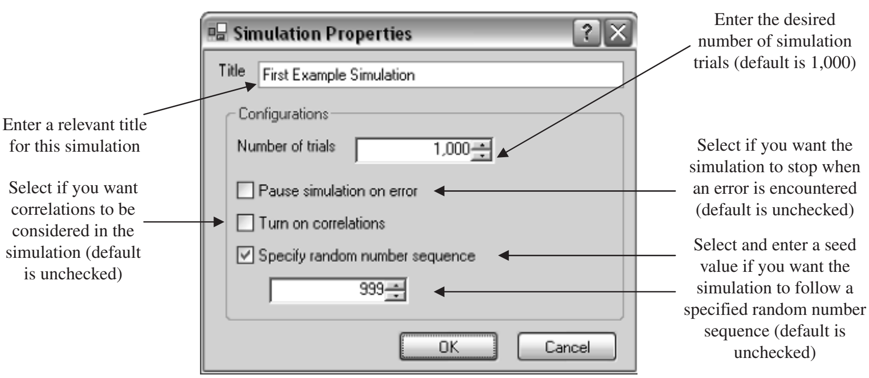


FIGURE 9.19 New Simulation Profile

- *Specify a random number sequence:* Simulation by definition will yield slightly different results every time a simulation is run. This is by virtue of the random number generation routine in Monte Carlo simulation. However, when making presentations, sometimes you may require the same results (especially when the report you are presenting shows one set of results and during a live presentation you would like to show the same results being generated, or when you are sharing models with others and would like the same results to be obtained every time), then check this preference and enter in an initial seed number. The seed number can be any positive integer. Using the same initial seed value, the same number of trials, and the same input assumptions will always yield the same sequence of random numbers, guaranteeing the same final set of results.

2. Defining Input Assumptions The next step is to set input assumptions in your model. Note that assumptions can only be assigned to cells without any equations or functions, that is, inputs in a model, whereas output forecasts can only be assigned to cells with equations and functions, that is, outputs of a model. Do the following to set new input assumptions in your model:

- Click on *Simulation* and select *Set Input Assumption*.
- Select the relevant distribution you want and enter the relevant distribution parameters and hit *OK* to insert the input assumption into your model (Figure 9.20).

Notice that in the Assumption Properties, there are several key areas worthy of mention. Figure 9.21 shows the different areas:

- ***Enter assumption name:*** This optional area allows you to enter in unique names for the assumptions to help track what each of the assumptions represents.
- ***Distribution Gallery:*** This area to the left shows all of the different distributions available in the software. To change the views, right click anywhere in the gallery and select large icons, small icons, or list. There are about two dozen distributions available for use.
- ***Input parameters:*** Depending on the distribution selected, the required relevant parameters are shown. You may either enter the parameters directly or link them to specific cells in your worksheet. Hard coding or typing the parameters is useful when the assumption parameters are assumed not to change. Linking to worksheet cells is useful when the input parameters need to be visible or are allowed to be changed (click on the link icon to link input parameter to a worksheet cell).
- ***Distributional boundaries:*** These are typically not used by the average analyst but exist for truncating the distributional assumptions. For

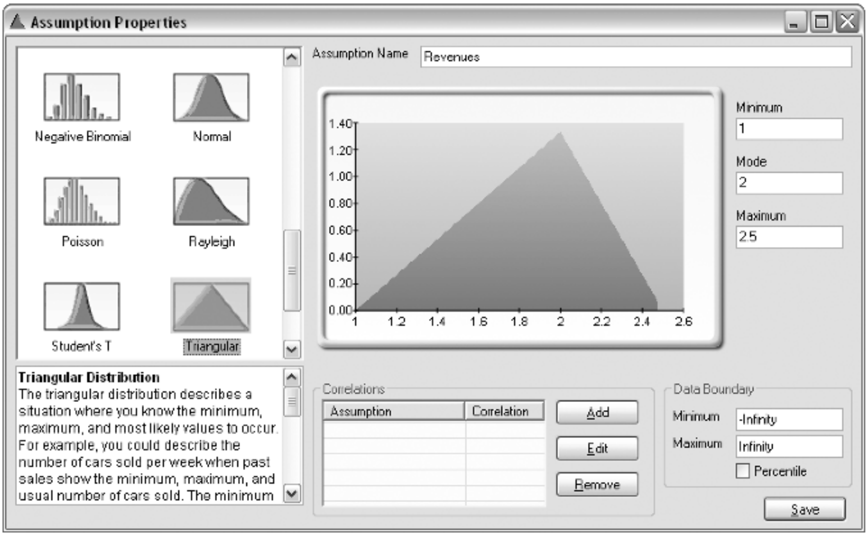


FIGURE 9.20 Setting an Input Assumption

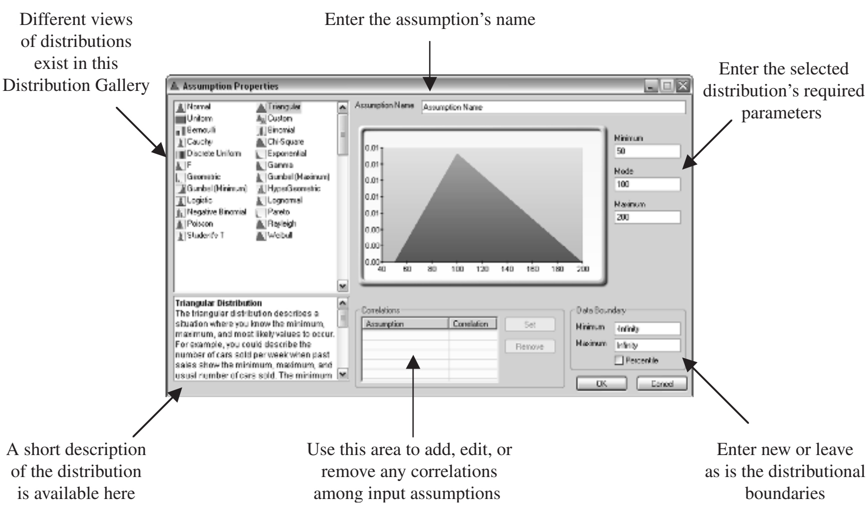


FIGURE 9.21 Assumption Properties

instance, if a normal distribution is selected, the theoretical boundaries are between negative infinity and positive infinity. However, in practice, the simulated variable exists only within some smaller range and this range can then be entered to truncate the distribution appropriately.

- ***Correlations:*** Pairwise correlations can be assigned to input assumptions here. If assumptions are required, remember to check the *turn on correlations* preference by clicking on *Simulation | Edit Simulation Profile*.
- ***Short description:*** These exist for each of the distributions in the gallery. The short descriptions explain when a certain distribution is used as well as the input parameter requirements. See Appendix 9B for details on each distribution type available in the software.

3. Defining Output Forecasts The next step is to define output forecasts in the model. Forecasts can only be defined on output cells with equations or functions. The following describes the set forecast process:

- Click on *Simulation* and select *Set Output Forecast*
- Enter the relevant information and click *OK*

Figure 9.22 illustrates the set forecast properties.

- ***Forecast name:*** Specify the name of the forecast cell. This is important because when you have a large model with multiple forecast cells, naming the forecast cells individually allows you to access the right results.
- ***Forecast precision:*** Instead of relying on a guesstimate of how many trials to run in your simulation, you can set up precision and error controls.

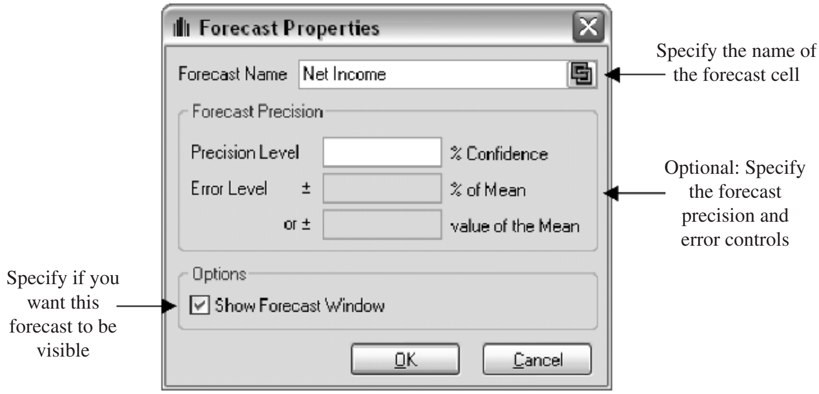


FIGURE 9.22 Set Output Forecast

When an error-precision combination has been achieved in the simulation, the simulation pauses and informs you of the precision achieved, making the number of simulation trials an automated process and does not require user guesses on the required number of trials to simulate.

- ***Show forecast window:*** Allows the user to show or not show a particular forecast window.

4. Check Simulation Preferences Before running a simulation, the run preferences should always be verified:

- Click on *Simulation* and select *Simulation* | *Edit Simulation Profile*.
- Verify the settings and hit *OK*.

Figure 9.23 shows the various combinations of preferences available. In addition, Figure 9.24 shows the Risk Simulator icon toolbar. Most of the features available under the *Simulation* menu item in Figure 9.25 are also available in the icon toolbar.

5. Run Simulation If everything looks right, simply click on the *Run* icon or *Simulation* | *Run Simulation* and the simulation will proceed. You may also *Reset* a simulation after it has run to rerun it, or to *Pause* it during a run. Also, the *Step* function allows you to simulate a single trial, one at a time, useful for educating others on simulation (i.e., you can show that at each trial, all the values in the assumption cells are being replaced and the entire model is recalculated each time).

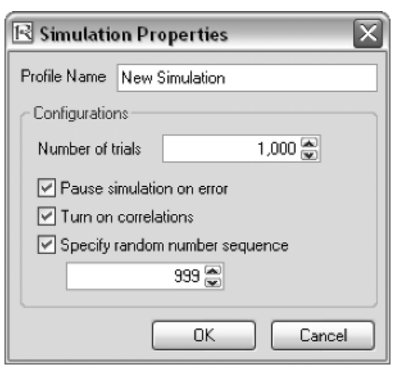


FIGURE 9.23 Run Preferences

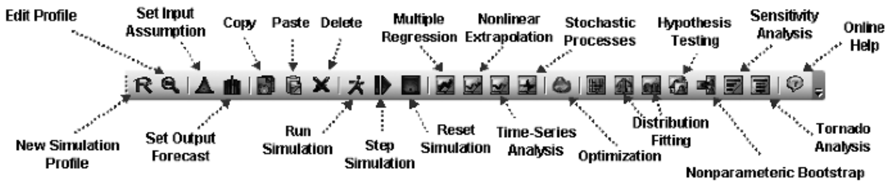


FIGURE 9.24 Risk Simulator Icon Toolbar

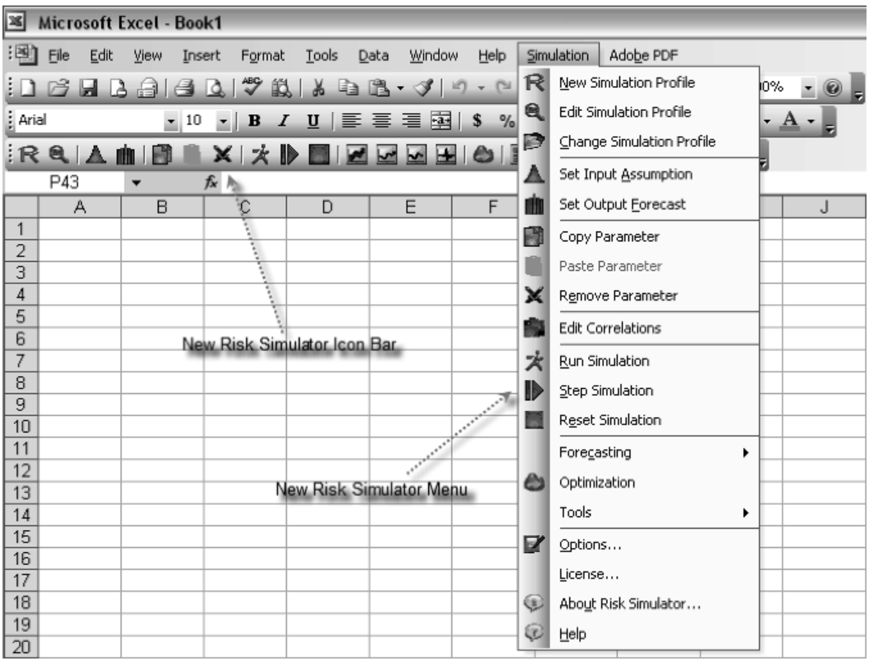


FIGURE 9.25 Risk Simulator Menu

6. Interpreting the Forecast Results The final step in Monte Carlo simulation is to interpret the resulting forecast charts. Figures 9.26 to 9.28 show a sample forecast chart and the corresponding statistics. Typically, the following information is important in interpreting the results of a simulation:

- *Forecast histogram:* The histogram shows the frequency counts of values occurring in the total number of trials simulated. The vertical bars show the frequency of a particular x value occurring out of the total number of

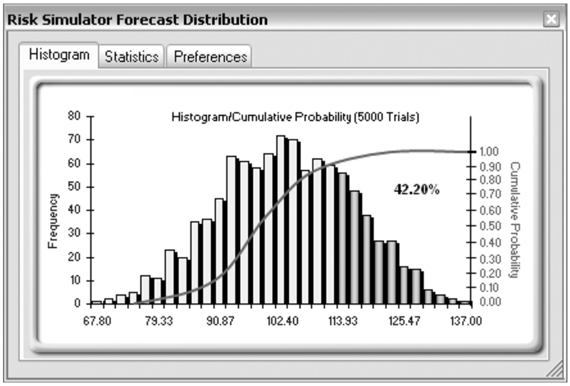


FIGURE 9.26 Forecast Chart

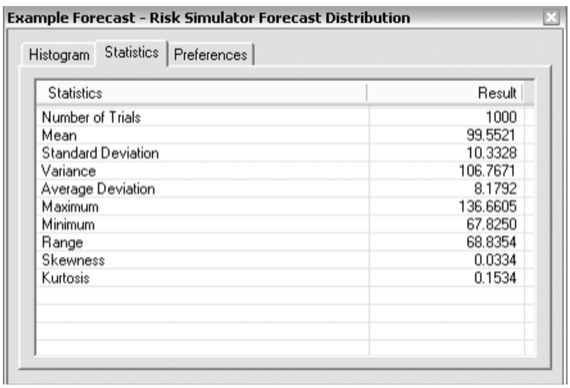


FIGURE 9.27 Forecast Statistics

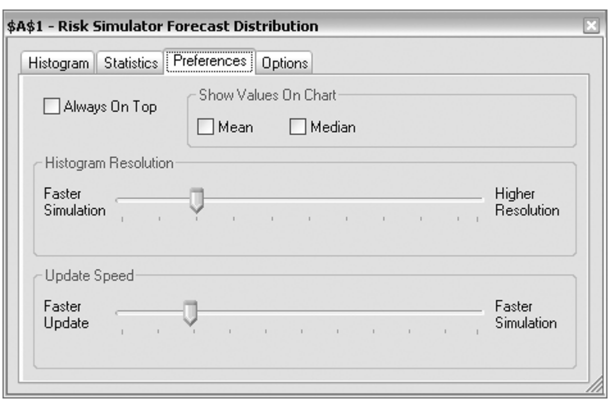


FIGURE 9.28 Forecast Chart Preferences

trials, while the cumulative frequency (smooth line) shows the total probabilities of all values at and below *x* occurring in the forecast.

- ***Forecast statistics:*** The forecast statistics summarizes the distribution of the forecast values in terms of the four moments of a distribution. See the section on *Understanding the Forecast Statistics* for more details on what some of these statistics mean.

The *Preferences* tab in the forecast chart allows you to change the look and feel of the charts. For instance, if *Always On Top* is selected, the forecast charts will always be visible regardless of what other software is running on your computer. *Histogram Resolution* allows you to change the number of bins of the histogram, anywhere from 5 bins to 100 bins. Also, the *Update Speed* section allows you to control how fast the simulation runs versus how often the forecast chart is updated. That is, if you wish to see the forecast chart updated at almost every trial, this updating will slow down the simulation as more memory is being allocated to updating the chart versus running the simulation. This is merely a user preference and in no way changes the results of the simulation, just the speed of completing the simulation. To further increase the speed of the simulation, you can minimize Excel while the simulation is running, thereby reducing the memory required to visibly update the Excel spreadsheet and freeing up the memory to run the simulation. Finally, the *Options* tab allows you to filter in/out values that fall within a particular range.

Finally, Figure 9.29 shows the view of the probability that the results exceed 90. That is, the probability $P(X \geq 90) = 96.50$ percent. This means that there is a 96.50 percent probability that the outcome will exceed 90 and a 3.50 percent probability it will fall below 90. In contrast, Figure 9.30 shows the 90 percent confidence interval of the results fall between 91.20 and

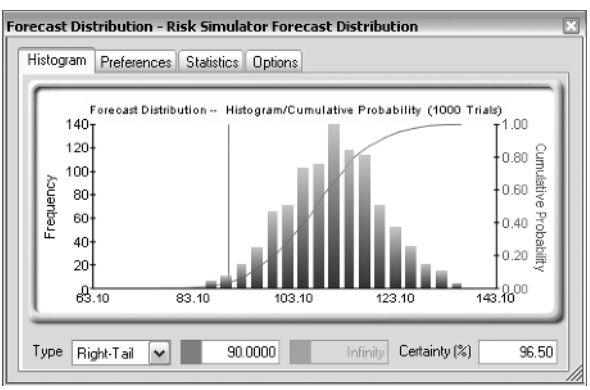


FIGURE 9.29 Forecast Chart

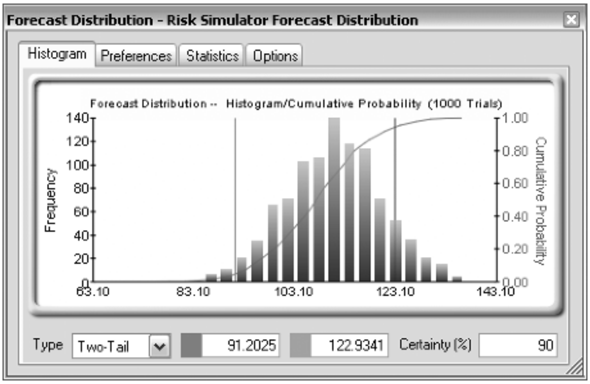


FIGURE 9.30 Forecast Chart

122.93, which means there is a 5 percent chance the result will be below 91.20 and a 5 percent chance it will exceed 122.93. This is done by first choosing the confidence type, enter the relevant inputs, and hitting *Tab*.

Understanding the Forecast Statistics

Most distributions can be defined up to four moments. The first moment describes its location or central tendency (expected returns), the second moment describes its width or spread (risks), the third moment its directional skew (most probable events), and the fourth moment its peakedness or thickness in the tails (catastrophic losses or gains). All four moments should be calculated in practice and interpreted to provide a more comprehensive view of the project under analysis. Risk Simulator provides the results of all four moments in its *Statistics* view in the forecast charts.

Measuring the Center of the Distribution—the First Moment The first moment of a distribution measures the expected rate of return on a particular project. It measures the location of the project's scenarios and possible outcomes on average. The common statistics for the first moment include the mean (average), median (center of a distribution), and mode (most commonly occurring value). Figure 9.31 illustrates the first moment—where, in this case, the first moment of this distribution is measured by the mean (μ) or average value.

Measuring the Spread of the Distribution—the Second Moment The second moment measures the spread of a distribution, which is a measure of risk. The spread or width of a distribution measures the variability of a variable, that is, the potential that the variable can fall into different regions of the distribution—in other words, the potential scenarios of outcomes. Figure 9.32 illustrates two distributions with identical first moments (identical

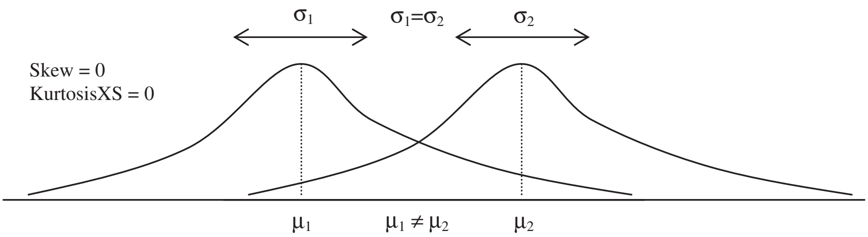


FIGURE 9.31 First Moment

means) but very different second moments or risks. The visualization becomes clearer in Figure 9.33. As an example, suppose there are two stocks and the first stock's movements (illustrated by the darker line) with the smaller fluctuation is compared against the second stock's movements (illustrated by the dotted line) with a much higher price fluctuation. Clearly an investor would view the stock with the wilder fluctuation as riskier because the outcomes of the more risky stock are relatively more unknown than the less risky stock. The vertical axis in Figure 9.33 measures the stock prices, thus, the more risky stock has a wider range of potential outcomes. This range is translated into a distribution's width (the horizontal axis) in Figure 9.32, where the wider distribution represents the riskier asset. Hence, width or spread of a distribution measures a variable's risks.

Notice that in Figure 9.32, both distributions have identical first moments or central tendencies but clearly the distributions are very different. This dif-

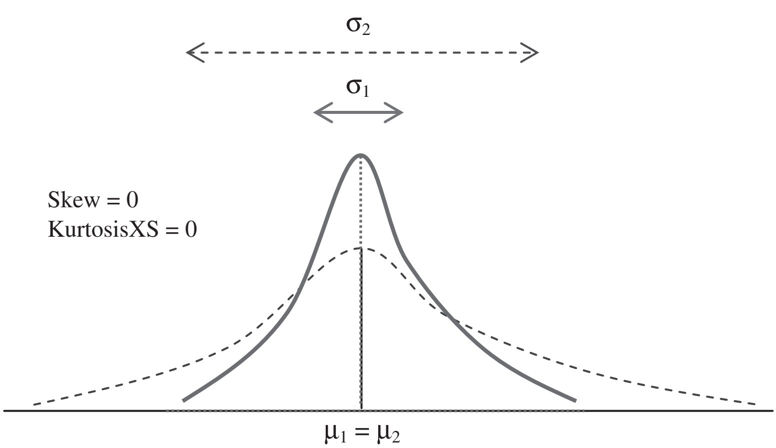


FIGURE 9.32 Second Moment

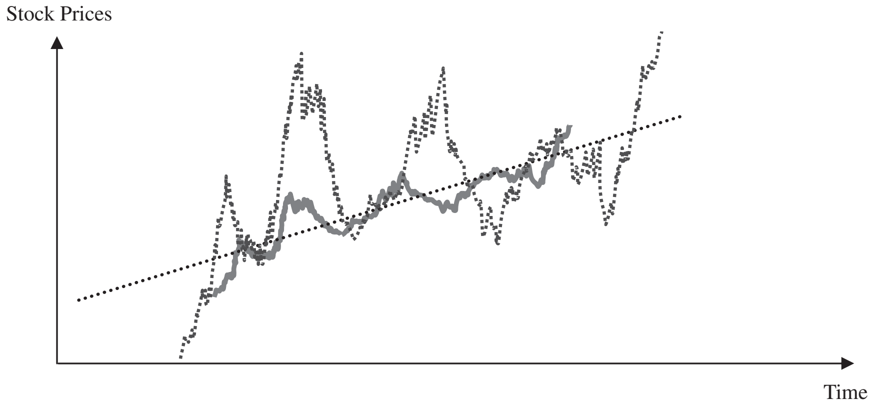


FIGURE 9.33 Stock Price Fluctuations

ference in the distributional width is measurable. Mathematically and statistically, the width or risk of a variable can be measured through several different statistics, including the range, standard deviation (σ), variance, coefficient of variation, and percentiles.

Measuring the Skew of the Distribution—the Third Moment The third moment measures a distribution's skewness, that is, how the distribution is pulled to one side or the other. Figure 9.34 illustrates a negative or left skew (the tail of the distribution points to the left) and Figure 9.35 illustrates a positive or right skew (the tail of the distribution points to the right). The mean is always skewed toward the tail of the distribution while the median remains constant. Another way of seeing this is that the mean moves but the standard deviation, variance, or width may still remain constant. If the third moment is not considered, then looking only at the expected returns (e.g., median or mean) and risk (standard deviation), a positively skewed project might be incorrectly chosen! For example, if the horizontal axis represents the net revenues of a project, then clearly a left or negatively skewed distribution might be preferred as there is a higher probability of greater returns (Figure 9.34) as compared to a higher probability for lower level returns (Figure 9.35). Thus, in a skewed distribution, the median is a better measure of returns, as the medians for both Figures 9.34 and 9.35 are identical, risks are identical, and hence, a project with a negatively skewed distribution of net profits is a better choice. Failure to account for a project's distributional skewness may mean that the incorrect project may be chosen (e.g., two projects may have identical first and second moments, that is, they both have identical returns and risk profiles, but their distributional skews may be very different).

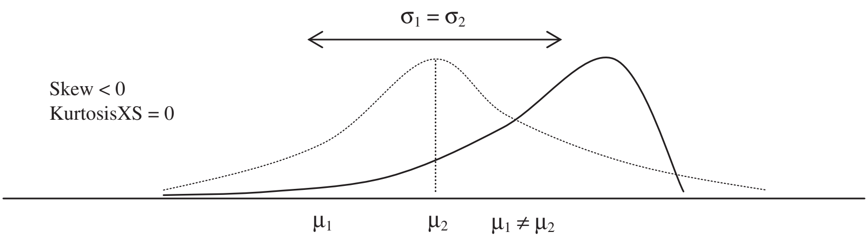


FIGURE 9.34 Third Moment (Left Skew)

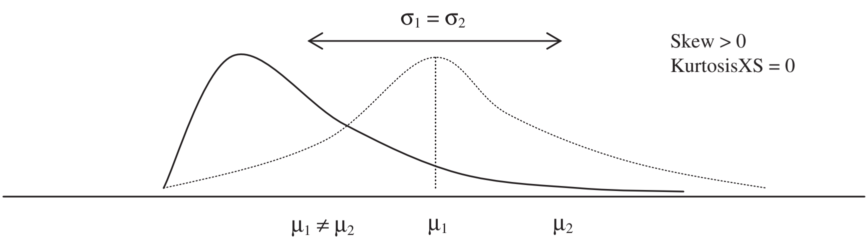


FIGURE 9.35 Third Moment (Right Skew)

Measuring the Catastrophic Tail Events in a Distribution—the Fourth Moment The fourth moment or kurtosis, measures the peakedness of a distribution. Figure 9.36 illustrates this effect. The background (denoted by the dotted line) is a normal distribution with a kurtosis of 3.0, or an excess kurtosis (Kurtosis XS) of 0.0. Risk Simulator's results show the KurtosisXS (simply listed as Kurtosis) value, using 0 as the normal level of kurtosis, which means that a negative KurtosisXS indicates flatter tails (platykurtic distributions like the Uniform distribution), while positive values indicate fatter tails (leptokurtic distributions like the Student's T or Lognormal distributions). The distribution depicted by the bold line has a higher excess kurtosis, thus the area under the curve is thicker at the tails with less area in the central body. This condition has major impacts on risk analysis as for the two distributions in Figure 9.36, the first three moments (mean, standard deviation, and skewness) can be identical but the fourth moment (kurtosis) is different. This condition means that, although the returns and risks are identical, the probabilities of extreme and catastrophic events (potential large losses or large gains) occurring are higher for a high kurtosis distribution (e.g., stock market returns are leptokurtic or have high kurtosis). Ignoring a project's kurtosis may be detrimental. Typically, a higher excess kurtosis value indicates that the downside risks are higher (e.g., the Value at Risk or VaR of a project might be significant).

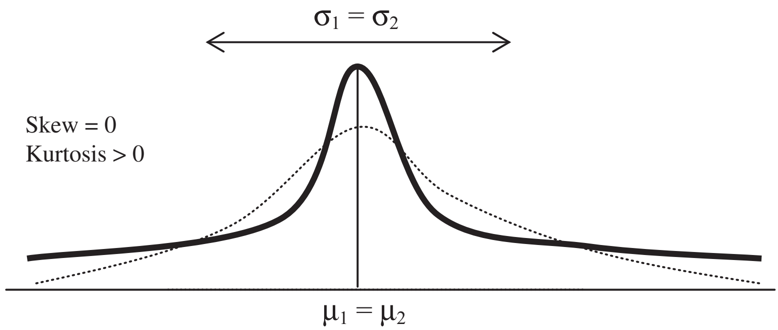


FIGURE 9.36 Fourth Moment

FORECASTING

Forecasting is the act of predicting the future, whether it is based on historical data or speculation about the future when no history exists. When historical data exist, a quantitative or statistical approach is best, but if no historical data exist, then potentially a qualitative or judgmental approach is usually the only recourse. Figure 9.37 lists the most common methodologies for forecasting.

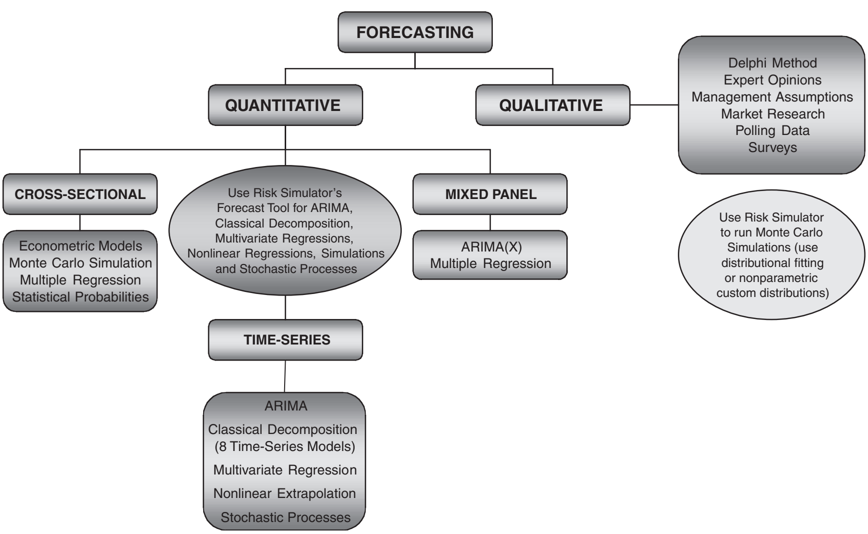


FIGURE 9.37 Forecasting Methods

Forecasting Techniques

Generally, forecasting can be divided into quantitative and qualitative. Qualitative forecasting is used when little to no reliable historical, contemporaneous, or comparable data exists. Several qualitative methods exist such as the Delphi or expert opinion approach (a consensus-building forecast by field experts, marketing experts, or internal staff members), management assumptions (target growth rates set by senior management), as well as market research or external data or polling and surveys (data obtained through third-party sources, industry and sector indexes, or from active market research). These estimates can be either single-point estimates (an average consensus) or a set of forecast values (a distribution of forecasts). The latter can be entered into Risk Simulator as a custom distribution and the resulting forecasts can be simulated, that is, a nonparametric simulation using the estimated data points themselves as the distribution.

On the quantitative side of forecasting, the available data or data that needs to be forecasted can be divided into time-series (values that have a time element to them, such as revenues at different years, inflation rates, interest rates, market share, failure rates), cross-sectional (values that are time-independent, such as the grade point average of sophomore students across the nation in a particular year, given each student's levels of SAT scores, IQ, and number of alcoholic beverages consumed per week), or mixed panel (mixture between time-series and panel data, e.g., predicting sales over the next 10 years given budgeted marketing expenses and market share projections. This means that the sales data is time-series but exogenous variables such as marketing expenses and market share exist to help to model the forecast predictions).

The Risk Simulator software provides the user several forecasting methodologies:

- Time-Series Analysis
- ARIMA Advanced Time-Series Modeling
- Multivariate Regression
- Stochastic Forecasting
- Nonlinear Extrapolation

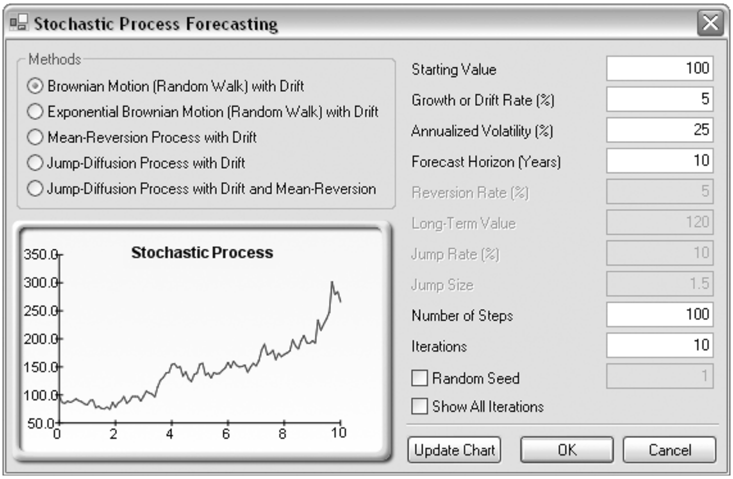
Running the Forecasting Tool in Risk Simulator

In order to create forecasts, several quick steps are required:

- Start Excel and enter in or open your existing historical data.
- Select the data and click on *Simulation* and select *Forecasting*.
- Select the relevant sections (Time-Series Analysis, ARIMA, Multivariate Regression, Stochastic Forecasting, or Nonlinear Extrapolation) and enter the relevant inputs.

Figure 9.38 illustrates the *Forecasting* tool and the various methodologies: Stochastic Processes, Time-Series Analysis, Nonlinear Extrapolation, and Cross-Sectional and Panel Data Multivariate Regression.

The following provides a quick review of each methodology and several quick getting-started examples in using the software. The example data file used to create these examples is included in the Risk Simulator software and can be accessed through: *Start* | *Programs* | *Real Options Valuation* | *Risk Simulator* | *Examples*.



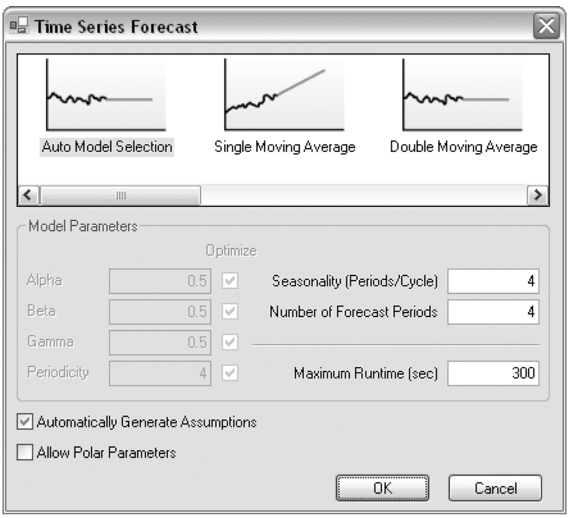


FIGURE 9.38 The Forecasting Methods in Risk Simulator

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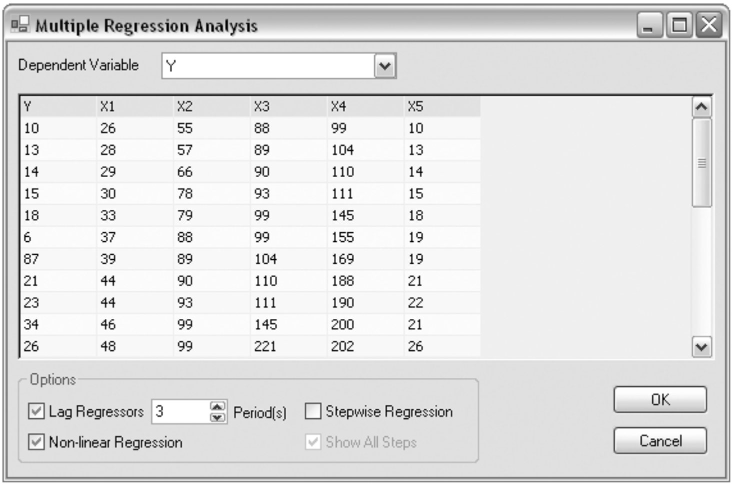


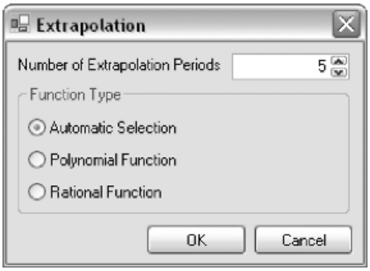


FIGURE 9.38 (*Continued*)

Time-Series Analysis Figure 9.39 lists the eight most common time-series models, segregated by seasonality and trend. For instance, if the data variable has no trend or seasonality, then a single moving-average model or a single exponential-smoothing model would suffice. However, if seasonality exists but no discernable trend is present, either a seasonal additive or seasonal multiplicative model would be better, and so forth. See *Modeling Risk: Applying Monte Carlo Simulation, Real Options Analysis, Forecasting, and Optimization Techniques, Second Edition* (Johnathan Mun, Wiley, Hoboken, NJ, 2006), for more detailed analysis and discussion of these eight time-series analyses and time-series forecasting.

Figure 9.40 illustrates the sample results generated by using the *Forecasting* tool. The model used was a Holt-Winters' Multiplicative model. Notice that in Figure 9.40, the model-fitting and forecast chart indicates that the trend and seasonality are picked up nicely by the Holt-Winters' Multiplicative model.

	No Seasonality	With Seasonality
No Trend	*Single Moving Average* *Single Exponential Smoothing*	*Seasonal Additive* *Seasonal Multiplicative*
With Trend	*Double Moving Average* *Double Exponential Smoothing*	*Holt-Winter's Additive* *Holt-Winter's Multiplicative*

FIGURE 9.39 The Eight Most Common Time-Series Methods

ARIMA Advanced Time-Series Modeling One very powerful advanced times-series forecasting tool is the Box-Jenkins ARIMA or Auto Regressive Integrated Moving Average approach. ARIMA forecasting assembles three separate tools into a comprehensive model. The first tool segment is the autoregressive or AR term, which corresponds to the number of the lagged value of the residual in the unconditional forecast model. In essence, the model captures the historical variation of actual data to a forecasting model and uses this variation or residual to create a better predicting model. The second tool segment is the integration order or the I term. This integration term corresponds to the number of differencing the time-series to be forecasted goes through. This element accounts for any nonlinear growth rates existing in the data and makes the time-series data stationary. The third tool segment is the moving average or MA term, which is essentially the moving average of lagged forecast errors. By incorporating these lagged forecast errors, the model in essence learns from its forecast errors or mistakes and corrects for them through a moving average calculation. In addition, ARIMA models can be mixed with exogenous variables, but make sure that the exogenous variables have enough data points to cover the additional number of periods to forecast. Finally, be aware that due to the complexity of the models, this module may take several minutes to run.

ARIMA(p,d,q) models are the extension of the AR model that uses three components for modeling the serial correlation in the time series data. The first component is the autoregressive (AR) term. The AR(p) model uses the p lags

Time-Series Analysis (Holt-Winters Seasonal Multiplicative)

Summary Statistics

Alpha, Beta, Gamma	RMSE	Alpha, Beta, Gamma	RMSE
0.00, 0.00, 0.00	914.824	0.60, 0.60, 0.60	113.974
0.10, 0.10, 0.10	415.322	0.70, 0.70, 0.70	138.884
0.20, 0.20, 0.20	187.202	0.80, 0.80, 0.80	171.881
0.30, 0.30, 0.30	118.795	0.90, 0.90, 0.90	202.578
0.40, 0.40, 0.40	101.794	1.00, 1.00, 1.00	319.759
0.50, 0.50, 0.50	102.143		

Best Fit (Alpha, Beta, Gamma) = 0.2431, 0.9990, 0.7798

Time-Series Analysis Summary

When both seasonality and trend exist, more advanced models are required to decompose the data into their base elements: a base-case level (L) weighted by the alpha parameter; a trend component (b) weighted by the beta parameter; and a seasonality component (S) weighted by the gamma parameter. Several methods exist but the two most common are the Holt-Winters' additive seasonality and Holt-Winters' multiplicative seasonality methods. In the Holt-Winter's multiplicative model, the base case level and trend are added together and multiplied by the seasonality factor to obtain the forecast fit.

The best-fitting test for the moving average forecast uses the root mean squared errors (RMSE). The RMSE calculates the square root of the average squared deviations of the fitted values versus the actual data points.

Mean Squared Error (MSE) is an absolute error measure that squares the errors (the difference between the actual historical data and the forecast-fitted data predicted by the model) to keep the positive and negative errors from canceling each other out. This measure also tends to exaggerate large errors by weighting the large errors more heavily than smaller errors by squaring them, which can help when comparing different time-series models. Root Mean Square Error (RMSE) is the square root of MSE and is the most popular error measure, also known as the quadratic loss function. RMSE can be defined as the average of the absolute values of the forecast errors and is highly appropriate when the cost of the forecast errors is proportional to the absolute size of the forecast error. The RMSE is used as the selection criteria for the best-fitting time-series model.

Mean Absolute Percentage Error (MAPE) is a relative error statistic measured as an average percent error of the historical data points and is most appropriate when the cost of the forecast error is more closely related to the percentage error than the numerical size of the error. Finally, an associated measure is the Theil's U statistic, which measures the naivety of the model's forecast. That is, if the Theil's U statistic is less than 1.0, then the forecast method used provides an estimate that is statistically better than guessing.

Period	Actual	Forecast Fit
1	684.20	
2	584.10	
3	765.40	
4	892.30	
5	885.40	684.20
6	677.00	667.57
7	1006.60	935.44
8	1122.10	1198.07
9	1163.40	1112.43
10	993.20	887.91
11	1312.50	1348.38
12	1545.30	1546.54
13	1596.20	1572.45
14	1260.40	1299.19
15	1735.20	1704.74
16	2029.70	1976.22
17	2107.80	2026.03
18	1650.30	1637.29
19	2304.40	2245.93
20	2639.40	2643.07
Forecast 21		2713.66
Forecast 22		2114.75
Forecast 23		2900.37
Forecast 24		3293.75
Forecast 25		3346.46
Forecast 26		2580.73
Forecast 27		3506.09
Forecast 28		3947.49
Forecast 29		3979.26
Forecast 30		3046.71
Forecast 31		4111.81
Forecast 32		4601.23

Error Measurements	
RMSE	71.8164
MSE	5157.5910
MAD	53.4084
MAPE	4.50%
Theil's U	0.3055

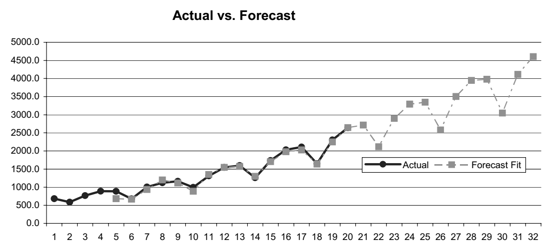


FIGURE 9.40 Example Holt-Winters' Forecast Report

of the time series in the equation. An AR(p) model has the form: $y_t = a_1 y_{t-1} + \ldots + a_p y_{t-p} + e_t$. The second component is the integration (d) order term. Each integration order corresponds to differencing the time series. I(*1*) means differencing the data once. I (d) means differencing the data d times. The third component is the moving average (MA) term. The MA(q) model uses the q lags of the forecast errors to improve the forecast. An MA(q) model has the form: $y_t = e_t + b_1 e_{t-1} + \ldots + b_q e_{t-q}$. Finally, an ARMA($p$,$q$) model has the combined form: $y_t = a_1 y_{t-1} + \ldots + a_p y_{t-p} + e_t + b_1 e_{t-1} + \ldots + b_q e_{t-q}$. Figure 9.41 illustrates the results of an ARIMA (2,1,0).

Multivariate Regression It is assumed that the user is sufficiently knowledgeable about the fundamentals of regression analysis. The general bivariate linear regression equation takes the form of $Y = \beta_0 + \beta_1 X + \varepsilon$ where β_0 is the intercept, β_1 is the slope, and ε is the error term. It is bivariate as there are only two variables, a Y or dependent variable, and an X or independent

ARIMA (Autoregressive Integrated Moving Average)

Regression Statistics

R-Squared (Coefficient of Determination)	*0.8921*	*Akaike Infomation Criterion (AIC)*	*6.0714*
Adjusted R-Squared	*0.8886*	*Schwarz Criterion (SC)*	*6.1775*
Multiple R (Multiple Correlation Coefficient)	*0.9445*	*Log Likelihood*	*-290.4610*
Standard Error of the Estimates (SEy)	*14.7890*	*Durbin-Watson statistic*	*2.0361*
Observations	*97*	*Number of Iterations*	*0*

*Autoregressive Integrated Moving Average(ARIMA(p,d,q)) models are the extension of the AR model that use three components for modeling the serial correlation in the time series data. The first component is the autoregressive(AR) term. The AR(p) model uses the p lags of the time series in the equation. An AR(p) model has the form: y(t)=a(1)*y(t-1)+...+a(p)*y(t-p)+e(t).The second component is the integration(d) order term. Each integration order corresponds to differencing the time series. I(1) means differencing the data once. I(d) means differencing the data d times.The third component is the moving average(MA) term. The MA(q) model uses the q lags of the forecast errors to improve the forecast. An MA(q) model has the form: y(t)=e(t)+b(1)*e(t-1)+...+b(q)*e(t-q).Finally, an ARMA(p,q) model has the combined form: y(t)=a(1)*y(t-1)+...+a(p)*y(t-p)+e(t)+b(1)*e(t-1)+...+b(q)*e(t-q).*

The R-Squared or Coefficient of Determination indicates that of the variation in the dependent variable can be explained and accounted for by the independent variables in this regression analysis. However, in a multiple regression, the Adjusted R-Squared takes into account the existence of additional independent variables or regressors and adjusts this R-Squared value to a more accurate view the regression's explanatory power. Hence, only 88.86% of the variation in the dependent variable can be explained by the regressors. However, under some circumstances, it tends to be unreliable.

The Multiple Correlation Coefficient (Multiple R) measures the correlation between the actual dependent variable (Y) and the estimated or fitted (Y) based on the regression equation. This is also the square root of the Coefficient of Determination (R-Squared).

The Standard Error of the Estimates (SEy) describes the dispersion of data points above the below the regression line or plane. This value is used as part of the calculation to obtain the confidence interval of the estimates later.

The AIC and SC are often used in model selection. SC imposes a greater penalty for additional coefficients. Generally, the user should select a model with the lowest value of the AIC and SC.

The Durbin-Watson statistic measures the serial correlation in the residuals. Generally, DW less than 2 implies positive serial correlation.

Regression Results

	Intercept	*Y(-1)*	*Y(-2)*	*EX(1)*
Coefficients	*76.3147*	*0.9852*	*-0.2470*	*-0.3309*
Standard Error	*12.9248*	*0.0958*	*0.0673*	*0.4610*
t-Statistic	*5.9045*	*10.2884*	*-3.6726*	*-0.7179*
p-Value	***0.0000***	***0.0000***	***0.0004***	*0.4746*
Lower 5%	*50.6487*	*0.7951*	*-0.3806*	*-1.2463*
Upper 95%	*101.9808*	*1.1754*	*-0.1135*	*0.5845*

Degrees of Freedom		***Hypothesis Test***	
Degrees of Freedom for Regression	3	*Critical t-Statistic (99% confidence with df of 93)*	*5.8409*
Degrees of Freedom for Residual	93	*Critical t-Statistic (95% confidence with df of 93)*	*1.9858*
Total Degrees of Freedom	96	*Critical t-Statistic (90% confidence with df of 93)*	*1.6609*

The Coefficients provide the estimated regression intercept and slopes. For instance, the coefficients are the b values in the following regression equation: Y = b(0) + b(1)X(1) + b(2)X(2) + ... + b(n)X(n). The Standard Errors measure how accurate the predicted Coefficients are, and the t-Statistics are the ratios of each predicted Coefficient to its Standard Error.

The t-Statistic is used in hypothesis testing, where we set the null hypothesis (Ho) such that the real mean of the Coefficient = 0, and the alternate hypothesis (Ha) such that the real mean of the Coefficient is not equal to 0. A t-test is is performed and the calculated t-Statistic is compared to the critical values at the relevant Degrees of Freedom for Residual. The t-test is very important as it calculates if each of the coeffients is statistically significant in the presence of the other regressors. This means that the t-test statistically verifies whether a regressor or independent variable should remain in the regression or it should be dropped.

The Coefficient is statistically significant if its calculated t-Statistic exceeds the Critical t-Statistic at the relevant degrees of freedom (df). The three main confidence levels used to test for significance are 90%, 95% and 99%. If a Coefficient's t-Statistic exceeds the Critical level, it is considered statistically significant. Alternatively, the p-Value calculates each t-Statistic's probability of occurrence, which means that the smaller the p-Value, the more significant the Coefficient. The usual critical levels for the p-Value are 0.01, 0.05, and 0.10, corresponding to the 99%, 95%, and 99% confidence levels.

The Coefficients with their p-Values highlighted in blue indicate that they are statistically significant at the 95% confidence or 0.05 alpha level, while those highlighted in red indicate that they are not statistically significant at any of the alpha levels.

Analysis of Variance

	Sums of Squares	Mean of Squares	F-Statistic	P-Value	***Hypothesis Test***	
Regression	*18730.7410*	*6243.5803*	*256.2591*	***0.0000***	*Critical F-statistic (99% confidence with df of 3 and 93)*	*3.9994*
Residual	*2265.8827*	*24.3643*			*Critical F-statistic (95% confidence with df of 3 and 93)*	*2.7025*
Total	*20996.6237*	*6267.9447*			*Critical F-statistic (90% confidence with df of 3 and 93)*	*2.1436*

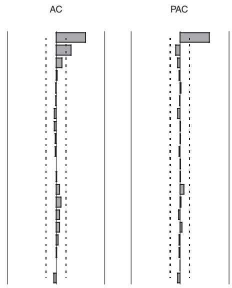
The Analysis of Variance (ANOVA) table provides an F-test of the regression model's overall statistical significance. Instead of looking at individual regressors as in the t-test, the F-test looks at all the estimated Coefficient's statistical properties. The F-statistic is calculated as the ratio of the Regression's Mean of Squares to the Residual's Mean of Squares. The numerator measures how much of the regression is explained, while the denominator measures how much is unexplained. Hence, the larger the F-statistic, the more significant the model. The corresponding P-Value is calculated to test the null hypothesis (Ho) where all the Coefficients are simultaneously equal to zero, versus the alternate hypothesis (Ha) that they are all simultaneously different from zero, indicating a significant overall regression model. If the P-Value is smaller than the 0.01, 0.05, or 0.10 alpha significance, then the regression is significant. The same approach can be applied to the F-statistic.

FIGURE 9.41 ARIMA Report

(*continues*)

Autocorrelation

Time Lag	AC	PAC	LBound	UBound	Q-Stat	Prob
1	0.6042	0.6042	(0.2010)	0.2010	36.5167	0.0000
2	0.3109	(0.0852)	(0.2010)	0.2010	46.2900	0.0000
3	0.1215	(0.0490)	(0.2010)	0.2010	47.7980	0.0000
4	0.0208	(0.0194)	(0.2010)	0.2010	47.8428	0.0000
5	(0.0009)	0.0241	(0.2010)	0.2010	47.8429	0.0000
6	(0.0036)	(0.0020)	(0.2010)	0.2010	47.8442	0.0000
7	(0.0322)	(0.0502)	(0.2010)	0.2010	47.9546	0.0000
8	(0.0303)	0.0147	(0.2010)	0.2010	48.0539	0.0000
9	(0.0145)	0.0141	(0.2010)	0.2010	48.0769	0.0000
10	(0.0118)	(0.0143)	(0.2010)	0.2010	48.0922	0.0000
11	0.0005	0.0115	(0.2010)	0.2010	48.0923	0.0000
12	0.0147	0.0156	(0.2010)	0.2010	48.1166	0.0000
13	0.0718	0.0879	(0.2010)	0.2010	48.7060	0.0000
14	0.1018	0.0232	(0.2010)	0.2010	49.9040	0.0000
15	0.0789	(0.0251)	(0.2010)	0.2010	50.6335	0.0000
16	0.0770	0.0477	(0.2010)	0.2010	51.3374	0.0000
17	0.0502	(0.0126)	(0.2010)	0.2010	51.6403	0.0000
18	0.0156	(0.0197)	(0.2010)	0.2010	51.6699	0.0000
19	0.0021	0.0031	(0.2010)	0.2010	51.6705	0.0001
20	(0.0386)	(0.0498)	(0.2010)	0.2010	51.8561	0.0001



If autocorrelation AC(1) is nonzero, it means that the series is first order serially correlated. If AC(k) dies off more or less geometrically with increasing lag , it implies that the series follows a low-order autoregressive process. If AC(k) drops to zero after a small number of lags, it implies that the series follows a low-order moving-average process. Partial correlation PAC(k) measures the correlation of values that are k periods apart after removing the correlation from the intervening lags. If the pattern of autocorrelation can be captured by an autoregression of order less than k, then the partial autocorrelation at lag k will be close to zero. Ljung-Box Q-statistics and their p-values at lag k has the null hypothesis that there is no autocorrelation up to order k. The dotted lines in the plots of the autocorrelations are the approximate two standard error bounds. If the autocorrelation is within these bounds, it is not significantly different from zero at (approximately) the 5% significance level.

Forecasting

Period	Actual (Y)	Forecast (F)	Error (E)
1	160.8074	166.2524	(5.4449)
2	203.5786	200.9466	2.6320
3	233.0943	230.4672	2.6271
4	253.7002	249.3845	4.3157
5	265.0096	262.6057	2.4039
6	265.5111	268.2399	(2.7289)
7	259.7946	266.1787	(6.3841)
8	261.9179	260.4460	1.4719
9	258.9343	263.8008	(4.8665)
10	258.6440	260.5955	(1.9516)
11	261.9522	260.5557	1.3965
12	265.2055	263.7053	1.5002
13	268.8561	265.9864	2.8697
14	264.8347	269.1998	(4.3651)
15	259.3739	264.3989	(5.0250)
16	262.8198	259.6577	3.1621
17	261.3541	264.8365	(3.4824)
18	259.3486	262.3699	(3.0213)
19	261.1928	260.5876	0.6052
20	257.6445	263.2992	(5.6547)
21	262.4407	259.2615	3.1792
22	270.5320	264.2012	6.3307
23	274.1573	270.9806	3.1767
24	268.5451	272.7102	(4.1651)
25	265.4515	265.8555	(0.4040)
26	270.3259	264.5467	5.7792
27	267.2530	270.5950	(3.3419)
28	267.0962	266.1008	0.9955
29	269.9099	267.1213	2.7886
30	266.5013	270.3758	(3.8744)
31	264.5603	266.0822	(1.5219)
32	273.1362	264.6128	8.5234
33	278.5752	273.1320	5.4432
34	278.4910	276.4994	1.9916
35	282.5558	275.3362	7.2196
36	280.8297	279.7848	1.0450
37	275.2929	276.5562	(1.2632)
38	276.1229	271.6341	4.4888
39	269.2611	273.6364	(4.3753)
40	258.8014	266.7868	(7.9854)
41	254.8353	258.4373	(3.6020)
42	258.3237	256.6011	1.7226
43	266.8104	261.7407	5.0697

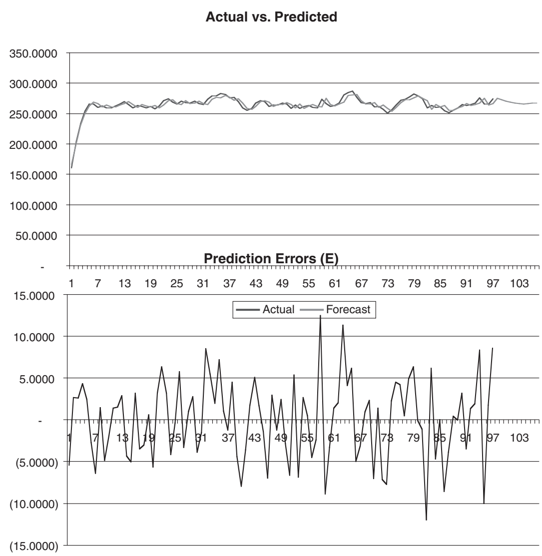


FIGURE 9.41 (*Continued*)

variable, where X is also known as the regressor (sometimes a bivariate regression is also known as a univariate regression as there is only a single independent variable X). The dependent variable is named as such as it *depends* on the independent variable, for example, sales revenue depends on the amount of marketing costs expended on a product's advertising and promotion, making the dependent variable sales and the independent variable mar-

44	270.5966	268.9070	1.6896
45	269.1559	270.5482	(1.3923)
46	261.4754	268.4569	(6.9815)
47	264.1149	261.1396	2.9753
48	263.3904	264.6314	(1.2409)
49	266.8035	264.3419	2.4616
50	264.9749	267.8389	(2.8640)
51	258.2496	264.9203	(6.6706)
52	264.0457	258.7109	5.3348
53	258.4755	265.3473	(6.8718)
54	261.1958	258.5245	2.6713
55	263.0192	262.5353	0.4839
56	259.9390	264.4747	(4.5357)
57	258.9255	261.3517	(2.4261)
58	273.2430	260.7856	12.4575
59	265.9005	274.8010	(8.9005)
60	261.4165	264.7250	(3.3085)
61	263.2812	261.8876	1.3936
62	267.1203	265.0858	2.0345
63	279.9807	268.6572	11.3235
64	283.9324	279.8530	4.0794
65	286.3158	280.1589	6.1569
66	276.3647	281.3454	(4.9807)
67	267.9716	271.1187	(3.1470)
68	266.0794	265.1361	0.9433
69	267.4657	265.1509	2.3148
70	260.2724	267.3225	(7.0501)
71	261.5311	260.1404	1.3907
72	256.7619	263.9492	(7.1873)
73	250.2610	257.9907	(7.7296)
74	255.4071	253.1641	2.2429
75	264.1499	259.6649	4.4850
76	271.8074	267.6056	4.2018
77	273.3329	272.8663	0.4666
78	277.0364	272.1534	4.8831
79	281.6071	275.2806	6.3265
80	278.5538	278.6524	(0.0986)
81	273.8826	275.0039	(1.1213)
82	258.8083	270.8064	(11.9981)
83	263.0429	256.8896	6.1532
84	259.6296	264.3199	(4.6903)
85	260.8564	260.8400	0.0164
86	254.1241	262.7052	(8.5811)
87	251.4270	255.2938	(3.8668)
88	255.3470	254.9317	0.4153
89	258.9861	259.0495	(0.0634)
90	264.6729	261.4809	3.1919
91	262.8745	266.3508	(3.4763)
92	264.3228	263.0024	1.3204
93	266.5885	264.6794	1.9091
94	275.2350	266.8922	8.3427
95	265.0402	275.0985	(10.0583)
96	265.4645	263.7103	1.7542
97	274.2390	265.6979	8.5412
98		274.6053	
99		272.3880	
100		269.9274	
101		268.2171	
102		266.9680	
103		265.9658	
104		265.6253	
105		265.7847	
106		266.8176	
107		266.8470	

FIGURE 9.41 (*Continued*)

keting costs. An example of a bivariate regression is seen as simply inserting the best-fitting line through a set of data points in a two-dimensional plane as seen on the left panel in Figure 9.42. In other cases, a multivariate regression can be performed, where there are multiple or n number of independent X variables, where the general regression equation will now take the form of $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 \ldots \beta_n X_n + \varepsilon$. In this case, the best-fitting line will be within an $n + 1$ dimensional plane.

However, fitting a line through a set of data points in a scatter plot as in Figure 9.42 may result in numerous possible lines. The best-fitting line is defined as the single unique line that minimizes the total vertical errors, that is, the sum of the absolute distances between the actual data points (Y_i) and the estimated line ($\hat{Y}$) as shown on the right panel of Figure 9.42. In order to find the best-fitting line that minimizes the errors, a more sophisticated

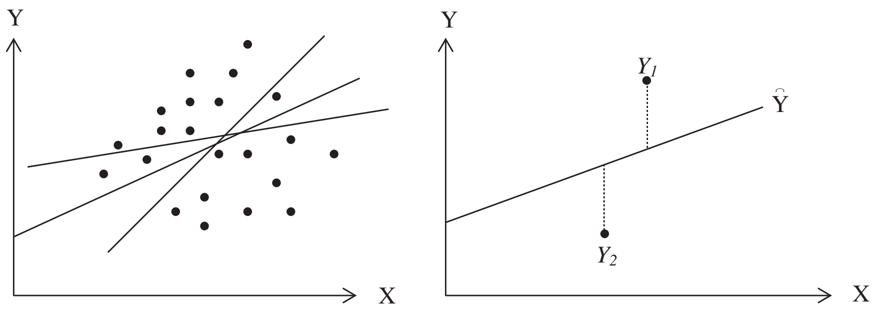


FIGURE 9.42 Bivariate Regression

approach is required, that is, regression analysis. Regression analysis, therefore, finds the unique best-fitting line by requiring that the total errors be minimized, or by calculating

$$Min \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2$$

where only one unique line minimizes this sum of squared errors. The errors (vertical distance between the actual data and the predicted line) are squared to avoid the negative errors from canceling out the positive errors. Solving this minimization problem with respect to the slope and intercept requires calculating a first derivative and setting them equal to zero:

$$\frac{d}{d\beta_0} \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2 = 0 \text{ and } \frac{d}{d\beta_1} \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2 = 0$$

which yields the bivariate regression's least squares equations:

$$\beta_1 = \frac{\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y})}{\sum_{i=1}^{n} (X_i - \overline{X})^2} = \frac{\sum_{i=1}^{n} X_i Y_i - \dfrac{\sum_{i=1}^{n} X_i \sum_{i=1}^{n} Y_i}{n}}{\sum_{i=1}^{n} X_i^2 - \dfrac{\left[\sum_{i=1}^{n} X_i\right]^2}{n}}$$

$$\beta_0 = \overline{Y} - \beta_1 \overline{X}$$

For multivariate regression, the analogy is expanded to account for multiple independent variables, where $Y_i = \beta_2 + \beta_2 X_{2,i} + \beta_3 X_{3,i} + \varepsilon_i$ and the estimated slopes can be calculated by:

$$\hat{\beta}_2 = \frac{\sum Y_i X_{2,i} \sum X_{3,i}^2 - \sum Y_i X_{3,i} \sum X_{2,i} X_{3,i}}{\sum X_{2,i}^2 \sum X_{3,i}^2 - \left(\sum X_{2,i} X_{3,i}\right)^2}$$

$$\hat{\beta}_3 = \frac{\sum Y_i X_{3,i} \sum X_{2,i}^2 - \sum Y_i X_{2,i} \sum X_{2,i} X_{3,i}}{\sum X_{2,i}^2 \sum X_{3,i}^2 - \left(\sum X_{2,i} X_{3,i}\right)^2}$$

In running multivariate regressions, great care must be taken to set up and interpret the results. For instance, a good understanding of econometric modeling is required (e.g., identifying regression pitfalls such as structural breaks, multicollinearity, heteroskedasticity, autocorrelation, specification tests, nonlinearities, and so forth) before a proper model can be constructed. See *Modeling Risk: Applying Monte Carlo Simulation, Real Options Analysis, Forecasting, and Optimization Techniques, Second Edition* (Johnathan Mun, Wiley, Hoboken, NJ, 2006) for more detailed analysis and discussion of multivariate regression as well as how to identify these regression pitfalls. Figure 9.43 illustrates a sample multivariate regression result report.

Stochastic Forecasting A stochastic process is nothing but a mathematically defined equation that can create a series of outcomes over time, outcomes that are not deterministic in nature. That is, an equation or process that does not follow any simple discernible rule such as price will increase X percent every year or revenues will increase by this factor of X plus Y percent. A stochastic process is by definition nondeterministic, and one can plug numbers into a stochastic process equation and obtain different results every time. For instance, the path of a stock price is stochastic in nature, and one cannot reliably predict the stock price path with any certainty. However, the price evolution over time is enveloped in a process that generates these prices. The process is fixed and predetermined, but the outcomes are not. Hence, by stochastic simulation, we create multiple pathways of prices, obtain a statistical sampling of these simulations, and make inferences on the potential pathways that the actual price may undertake given the nature and parameters of the stochastic process used to generate the time series. Three basic stochastic processes are included in Risk Simulator's Forecasting tool, including Geometric Brownian motion or random walk, which is the most common and prevalently used process due to its simplicity and wide-ranging applications. The other two stochastic processes are the mean-reversion process and jump-diffusion process.

The interesting thing about stochastic process simulation is that historical data are not necessarily required. That is, the model does not have to fit any sets of historical data. Simply compute the expected returns and the volatility of the historical data (see Appendix 7A for details on returns and volatility estimates) or estimate them using comparable external data or make assumptions about these values. Figure 9.44 shows the results using Risk Simulator's Forecasting tool to generate 10 iterations of a 100-step stochastic process (assuming a starting value of 100, expected annualized returns of 5 percent, volatility of 25 percent, and a ten-year forecast horizon). Other sets of stochastic processes can be constructed using the *Forecasting* tool's *Stochastic Processes* module.

Please note that there are many other powerful functionalities that are not listed in this chapter but could be found in the Risk Simulator User Manual (e.g., tornado analysis, sensitivity charts, distributional fitting, hypothesis testing, nonlinear extrapolation, and many more).

Regression Analysis Report

Regression Statistics	
R-Squared (Coefficient of Determination)	*0.5541*
Adjusted R-Squared	*0.4527*
Multiple R (Multiple Correlation Coefficient)	*0.7444*
Standard Error of the Estimates (SEy)	*14.7150*
nObservations	*28*

The R-Squared or Coefficient of Determination indicates that 55.41% of the variation in the dependent variable can be explained and accounted for by the independent variables in this regression analysis. However, in a multiple regression, the Adjusted R-Squared takes into account the existence of additional independent variables or regressors and adjusts this R-Squared value to a more accurate view of the regression's explanatory power. Hence, only 45.27% of the variation in the dependent variable can be explained by the regressors.

The Multiple Correlation Coefficient (Multiple R) measures the correlation between the actual dependent variable (Y) and the estimated or fitted (Y) based on the regression equation. This is also the square root of the Coefficient of Determination (R-Squared).

The Standard Error of the Estimates (SE_y) describes the dispersion of data points above and below the regression line or plane. This value is used as part of the calculation to obtain the confidence interval of the estimates later.

Regression Results

	Intercept	X1	X2	X3	X4	X5
Coefficients	8.0795	-0.2446	0.1659	-0.0095	0.0178	0.4952
Standard Error	8.1593	0.4730	0.3061	0.1122	0.0418	1.6267
t-Statistic	0.9902	-0.5171	0.5418	-0.0848	0.4259	0.3044
p-Value	0.3328	0.6103	0.5934	0.9332	0.6743	0.7637
Lower 5%	-8.8419	-1.2256	-0.4690	-0.2422	-0.0688	-2.8783
Upper 95%	25.0009	0.7365	0.8007	0.2232	0.1044	3.8687

Degrees of Freedom		Hypothesis Test	
Degrees of Freedom for Regression	5	Critical t-Statistic (99% confidence with df of 22)	2.8188
Degrees of Freedom for Residual	22	Critical t-Statistic (95% confidence with df of 22)	2.0739
Total Degrees of Freedom	27	Critical t-Statistic (90% confidence with df of 22)	1.7171

The Coefficients provide the estimated regression intercept and slopes. For instance, the coefficients are estimates of the true population β values in the following regression equation: $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + ... + \beta_n X_n$. The Standard Error measures how accurate the predicted Coefficients are, and the t-Statistics are the ratios of each predicted Coefficient to its Standard Error.

The t-Statistic is used in hypothesis testing, where we set the null hypothesis (Ho) such that the real mean of the Coefficient = 0, and the alternate hypothesis (Ha) such that the real mean of the Coefficient is not equal to 0. A t-test is performed and the calculated t-Statistic is compared to the critical values at the relevant Degrees of Freedom for Residual. The t-test is very important as it calculates if each of the coeffients is statistically significant in the presence of the other regressors. This means that the t-test statistically verifies whether a regressor or independent variable should remain in the regression or it should be dropped.

The Coefficient is statistically significant if its calculated t-Statistic exceeds the Critical t-Statistic at the relevant degrees of freedom (df). The three main confidence levels used to test for significance are 90%, 95% and 99%. If a Coefficient's t-Statistic exceeds the Critical level, it is considered statistically significant. Alternatively, the p-Value calculates each t-Statistic's probability of occurrence, which means that the smaller the p-Value, the more significant the Coefficient. The usual significant levels for the p-Value are 0.01, 0.05, and 0.10, corresponding to the 99%, 95%, and 99% confidence levels.

The Coefficients with their p-Values highlighted in blue indicate that they are statistically significant at the 95% confidence or 0.05 alpha level, while those highlighted in red indicate that they are not statistically significant at any of the alpha levels.

FIGURE 9.43 Multivariate Regression Results

Analysis of Variance

	Sums of Squares	Mean of Squares	F-Statistic	P-Value	Hypothesis Test	
Regression	*5919.2453*	*1183.8491*	*5.4673*	*0.0020*	*Critical F-statistic (99% confidence with df of 4 and 3)*	*3.9880*
Residual	*4763.7189*	*216.5327*			*Critical F-statistic (95% confidence with df of 4 and 3)*	*2.6613*
Total	*10682.9643*				*Critical F-statistic (90% confidence with df of 4 and 3)*	*2.1279*

The Analysis of Variance (ANOVA) table provides an F-test of the regression model's overall statistical significance. Instead of looking at individual regressors as in the t-test, the F-test looks at all the estimated Coefficients' statistical properties. The F-statistic is calculated as the ratio of the Regression's Mean of Squares to the Residual's Mean of Squares. The numerator measures how much of the regression is explained, while the denominator measures how much is unexplained. Hence, the larger the F-statistic, the more significant the model. The corresponding p-Value is calculated to test the null hypothesis (Ho) where all the Coefficients are simultaneously equal to zero, versus the alternate hypothesis (Ha) that they are all simultaneously different from zero, indicating a significant overall regression model. If the p-Value is smaller than the 0.01, 0.05, or 0.10 alpha significance, then the regression is significant. The same approach can be applied to the F-statistic by comparing the calculated F-statistic with the critical F values at various significance levels.

Forecasting

Period	Actual (Y)	Forecast (F)	Error (E)
1	10	16.7176	(6.7176)
2	13	18.1252	(5.1252)
3	14	19.9657	(5.9657)
4	15	22.1958	(7.1958)
5	18	23.6613	(5.6613)
6	6	24.8487	(18.8487)
7	87	24.7268	62.2732
8	21	24.9410	(3.9410)
9	23	25.9599	(2.9599)
10	34	25.8248	8.1752
11	26	27.1239	(1.1239)
12	28	27.9043	0.0957
13	29	31.0906	(2.0906)
14	30	34.3457	(4.3457)
15	33	28.9797	4.0203
16	23	36.2009	(13.2009)
17	39	37.2167	1.7833
18	44	46.1075	(2.1075)
19	44	43.8360	0.1640
20	46	48.3004	(2.3004)
21	48	48.3328	(0.3328)
22	55	53.6713	1.3287
23	57	54.3234	2.6766
24	66	67.1361	(1.1361)
25	48	48.3328	(0.3328)
26	55	53.6713	1.3287
27	57	54.3234	2.6766
28	66	67.1361	(1.1361)

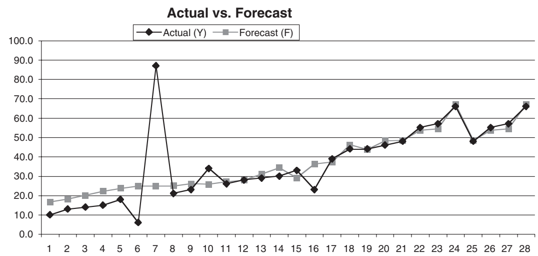


FIGURE 9.43 (*Continued*)

OPTIMIZATION

Before embarking on solving an optimization problem, it is vital to understand the terminology of optimization—the terms used to describe certain attributes of the optimization process. These terms include: decision variables, constraints, and objectives.

Decision variables are quantities over which you have control; for example, the amount of a product to make, the number of dollars to allocate among different investments, or which projects to select from among a limited set. As an example, portfolio optimization analysis includes a go or no-go decision on particular projects. In addition, the dollar or percentage budget allocation across multiple projects also can be structured as decision variables.

Constraints describe relationships among decision variables that restrict the values of the decision variables. For example, a constraint might ensure that the total amount of money allocated among various investments cannot exceed a specified amount or at most one project from a certain group can be selected. Budgets or constraints may take the form of timing restrictions, minimum returns, or risk tolerance levels.

Objectives give a mathematical representation of the model's desired outcome, such as maximizing profit or minimizing cost, in terms of the decision variables. In financial analysis, for example, the objective may be to maximize returns while minimizing risks (maximizing the Sharpe's ratio or returns-to-risk ratio).

The solution to an optimization model provides a set of values for the decision variables that optimizes (maximizes or minimizes) the associated objective. If the real business conditions were simple and the future were

Stochastic Process Forecasting

Statistical Summary

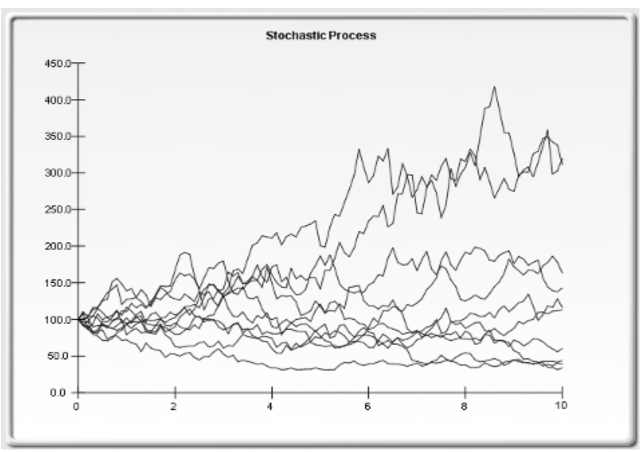
A stochastic process is a sequence of events or paths generated by probabilistic laws. That is, random events can occur over time but are governed by specific statistical and probabilistic rules. The main stochastic processes include Random Walk or Brownian Motion, Mean-Reversion, and Jump-Diffusion. These processes can be used to forecast a multitude of variables that seemingly follow random trends but yet are restricted by probabilistic laws.

The Random Walk Brownian Motion process can be used to forecast stock prices, prices of commodities, and other stochastic time-series data given a drift or growth rate and a volatility around the drift path. The Mean-Reversion process can be used to reduce the fluctuations of the Random Walk process by allowing the path to target a long-term value, making it useful for forecasting time-series variables that have a long-term rate such as interest rates and inflation rates (these are long-term target rates by regulatory authorities or the market). The Jump-Diffusion process is useful for forecasting time-series data when the variable can occasionally exhibit random jumps, such as oil prices or price of electricity (discrete exogenous event shocks can make prices jump up or down). Finally, these three stochastic processes can be mixed and matched as required.

The results on the right indicate the mean and standard deviation of all the iterations generated at each time step. If the Show All Iterations option is selected, each iteration pathway will be shown in a separate worksheet. The graph generated below shows a sample set of the iteration pathways.

Stochastic Process: Brownian Motion (Random Walk) with Drift

Start Value	*100*	*Steps*	*100.00*	*Jump Rate*	*N/A*
Drift Rate	*5.00%*	*Iterations*	*10.00*	*Jump Size*	*N/A*
Volatility	*25.00%*	*Reversion Rate*	*N/A*	*Random Seed*	*1431155157*
Horizon	*10*	*Long-Term Value*	*N/A*		



Time	*Mean*	*Stdev*
0.0000	100.00	0.00
0.1000	99.10	7.47
0.2000	96.03	7.22
0.3000	94.97	13.59
0.4000	97.39	15.57
0.5000	99.50	17.01
0.6000	97.79	20.92
0.7000	102.23	25.54
0.8000	106.54	26.54
0.9000	102.34	21.16
1.0000	102.77	20.86
1.1000	103.30	22.41
1.2000	103.27	19.23
1.3000	103.02	23.61
1.4000	97.78	19.65
1.5000	96.84	20.53
1.6000	100.92	25.22
1.7000	105.18	26.90
1.8000	100.75	30.33
1.9000	101.20	29.71
2.0000	103.67	36.95
2.1000	108.09	42.76
2.2000	111.58	42.61
2.3000	111.25	41.54
2.4000	108.47	35.22
2.5000	107.13	32.56
2.6000	108.95	32.95
2.7000	114.64	38.78
2.8000	114.13	36.61
2.9000	114.97	35.91
3.0000	114.33	39.90
3.1000	112.69	39.94
3.2000	115.11	39.89
3.3000	117.64	42.82
3.4000	114.70	39.91
3.5000	115.52	43.45
3.6000	117.60	49.89
3.7000	120.21	51.94
3.8000	116.64	53.52
3.9000	118.70	56.12
4.0000	113.19	56.71
4.1000	109.09	58.33
4.2000	103.70	52.23
4.3000	108.41	53.12
4.4000	108.67	56.30
4.5000	105.96	52.42
4.6000	106.12	55.80
4.7000	107.70	55.11
4.8000	109.43	58.43
4.9000	114.50	59.64
5.0000	110.44	53.91
5.1000	109.68	53.96

FIGURE 9.44 Stochastic Process Forecast Results

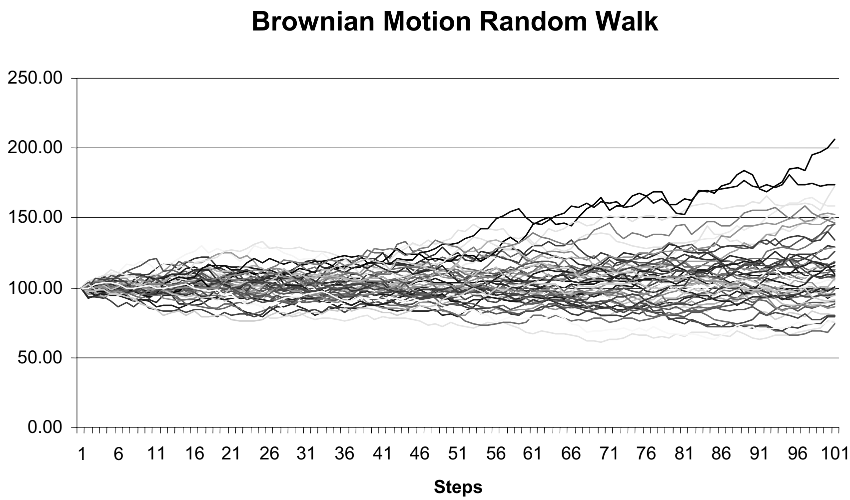


FIGURE 9.44 (*Continued*)

predictable, all data in an optimization model would be constant, making the model deterministic. In many cases, however, a deterministic optimization model cannot capture all the relevant intricacies of a practical decision-making environment. When a model's data are uncertain and can only be described probabilistically, the objective will have some probability distribution for any chosen set of decision variables. You can find this probability distribution by simulating the model using Risk Simulator. An optimization model under uncertainty has several additional elements, including assumptions, forecasts, forecast statistics, and requirements.

Assumptions capture the uncertainty of model data using probability distributions, whereas forecasts are the frequency distributions of possible results for the model. Forecast statistics are summary values of a forecast distribution, such as the mean, standard deviation, and variance. The optimization process controls the optimization by maximizing, minimizing, or restricting forecast statistics. Finally, requirements are additional restrictions on forecast statistics. Upper and lower limits can be set for any statistic of a forecast distribution and a range of requirement values can be obtained by defining a variable requirement.

Each optimization model has one objective, a forecast variable that mathematically represents the model's objective in terms of the assumption and decision variables. Optimization's job is to find the optimal value of the objective by selecting and improving different values for the decision variables. When model data are uncertain and can only be described using probability distributions, the objective itself will have some probability distribution for any set of decision variables.

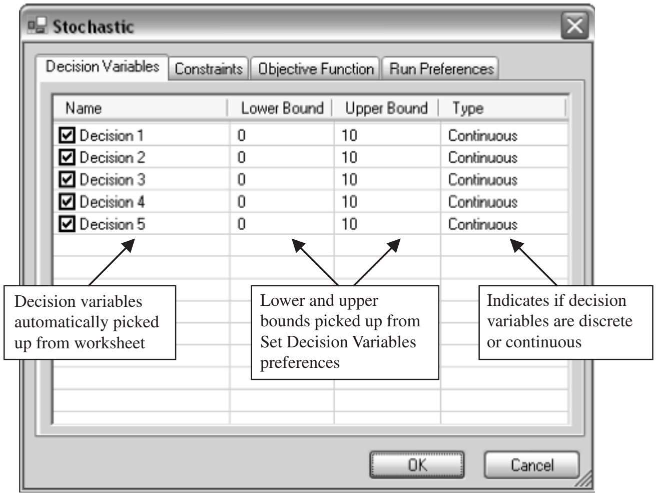


FIGURE 9.45 Optimization Tool in Risk Simulator (Decision Variables)

Figures 9.45 to 9.48 show the *Optimization* tool in Risk Simulator that can be obtained by starting Excel and clicking on *Simulation* and selecting *Optimization*.

For additional hands-on examples of optimization in action, see the case study in Chapter 11 on Integrated Risk Analysis. This case study illustrates

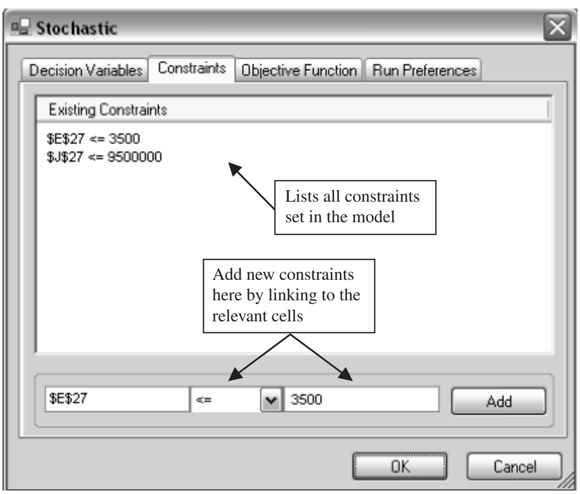


FIGURE 9.46 Optimization Tool in Risk Simulator (Constraints)

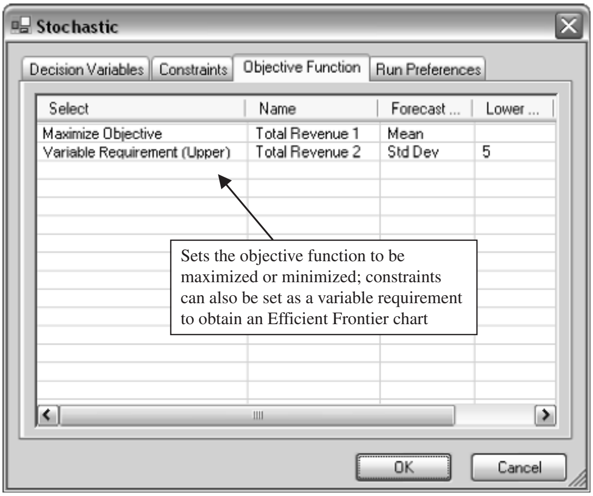


FIGURE 9.47 Optimization Tool in Risk Simulator (Objective Function)

how an efficient frontier can be generated and how forecasting, simulation, optimization, and real options can be combined into a seamless analytical process. Appendix 9D provides more technical discussions of optimization while the Risk Simulator's User Manual provides more hands-on step-by-step procedures for running optimization.

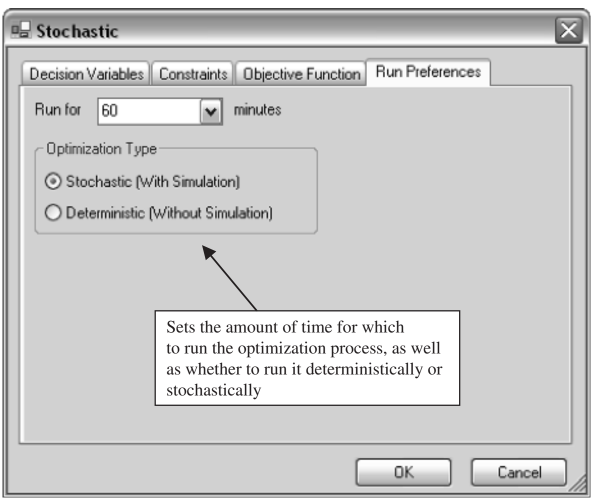


FIGURE 9.48 Optimization Tool in Risk Simulator (Run Preferences)

APPENDIX **9A**

Financial Options

Below are several key points to note in financial options, which are fairly similar to real options, because the underlying theoretical justifications are identical in nature.

DEFINITIONS

- An *option* is a contract that gives the owner/holder the right but not the legal obligation to conduct a transaction involving an underlying asset—for example, the purchase or sale of the asset—at a predetermined future date or within a specified period of time and at a predetermined price (the exercise or strike price); but the option only provides the long position the right to decide whether to trade and the seller the obligation to perform.
- A *call option* is an option to buy a specified number of shares of a security within some future period at prespecified prices.
- A *put option* is an option to sell a specified number of shares of a security within some future period at prespecified prices.
- The *exercise price* is the strike price or the price stated in the option contract at which the security can be bought or sold.
- The *option price* is the market price of the option contract.
- The *expiration date* is the date the option expires or matures.
- The *formula value* is the extrinsic value of an option or value of a call option if it were exercised today, which is equal to the current stock price minus the strike price.
- A *naked option* is an option sold without the stock to back it up.
- A *derivative* is a security whose value is derived from the values of other assets.

BLACK-SCHOLES MODEL

Basic options pricing models like the Black-Scholes make the following assumptions:

- That the stocks underlying the call options provide no dividends during the life of the options.
- That there are no transaction costs involved with the sale or purchase of either the stock or the option.
- That the short-term risk-free interest rate is known and is constant during the life of the option.
- That the security buyers may borrow any fraction of the purchase price at the short-term risk-free rate.
- That short-term selling is permitted without penalty and sellers receive immediately the full cash proceeds at today's price for securities sold short.
- That a call or put option can be exercised only on its expiration date.
- That security trading takes place in continuous time and stock prices move in continuous time.

Using the Black-Scholes paradigm:

- The value of a call option increases (decreases) as the current stock price increases (decreases).
- As the call option's exercise price increases (decreases), the option's value decreases (increases).
- As the term to maturity lengthens, the option's value increases.
- As the risk-free rate increases, the option's value tends to increase.
- The greater the volatility of the underlying stock price, the greater the possibility the stock's price will exceed the call option's exercise price—thus, the more valuable the option will be.

OTHER KEY POINTS

- The owner of a *call option* has the right to purchase the underlying good at a specific price for a specific time period, while the owner of a *put option* has the right to sell the underlying stock within a specified time period. To acquire these rights, owners of options must buy them by paying a price called the *premium* to the seller of the option.
- For every owner of an option, there must be a seller, called the *option writer*.
- Notice there are four possible positions: buyer of the call option, the seller or writer of the call option, the buyer of a put option, and the seller or writer of a put option.

- At termination, if the stock price S is above the strike price X, a call option has value and is said to be in-the-money—that is, when $S - X > 0$, a call option is in-the-money.
- At termination, if the stock price S is at the strike price X, a call option has no value and is said to be at-the-money—that is, when $S - X = 0$, a call option is at-the-money.
- At termination, if the stock price S is below the strike price X, a call option has no value and is said to be out-of-the-money—that is, when $S - X < 0$, a call option is out-of-the-money.
- At termination, if the stock price S is above the strike price X, a put option has no value and is said to be out-of-the-money—that is, when $S - X > 0$, a put option is out-of-the-money.
- At termination, if the stock price S is at the strike price X, a put option has no value and is said to be at-the-money—that is, when $S - X = 0$, a put option is at-the-money.
- At termination, if the stock price S is below the strike price X, a put option has value and is said to be in-the-money—that is, when $S - X < 0$, a put option is in-the-money.
- American options allow the owner to exercise the option at any time before or at expiration.
- European options can only be exercised at expiration.
- Bermudan options can be exercised at any time before or at expiration except during vesting or blackout periods.
- If both options have the same characteristics, an American call option is worth ≥ Bermudan call option ≥ European call option if there are dividend outflows. Otherwise, it is never optimal to exercise an American call option early and therefore the American call option and the Bermudan call option value reverts to the European call option value when dividend outflows are negligible. These apply only to plain-vanilla call options.

APPENDIX 9B

Probability Distributions for Monte Carlo Simulation

UNDERSTANDING PROBABILITY DISTRIBUTIONS

This chapter demonstrates the power of Monte Carlo simulation but in order to get started with simulation, one first needs to understand the concept of probability distributions. This appendix continues with the use of the author's Risk Simulator software and shows how simulation can be very easily and effortlessly implemented in an existing Excel model. A limited trial version of the Risk Simulator software is available in the enclosed CD-ROM. A limited version is included because software versions change rapidly over time, and to obtain the latest version of this software, please visit the author's website at www.RealOptionsValuation.com to download the latest version. Professors can obtain free licenses for their students and themselves if this book and the simulation/options valuation software are used in class.

To begin to understand probability, consider this example: You want to look at the distribution of nonexempt wages within one department of a large company. First, you gather raw data—in this case, the wages of each nonexempt employee in the department. Second, you organize the data into a meaningful format and plot the data as a frequency distribution on a chart. To create a frequency distribution, you divide the wages into group intervals and list these intervals on the chart's horizontal axis. Then you list the number or frequency of employees in each interval on the chart's vertical axis. Now you can easily see the distribution of nonexempt wages within the department.

A glance at the chart illustrated in Figure 9B.1 reveals that most of the employees (approximately 60 out of a total of 180) earn from \$7.00 to \$9.00 per hour.

You can chart this data as a probability distribution. A probability distribution shows the number of employees in each interval as a fraction of the total number of employees. To create a probability distribution, you divide the

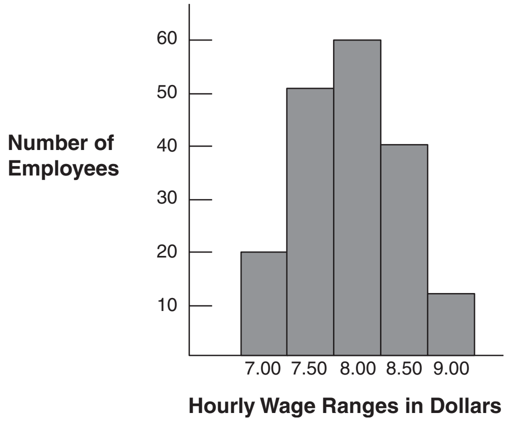


FIGURE 9B.1 Frequency Histogram I

number of employees in each interval by the total number of employees and list the results on the chart's vertical axis.

The chart in Figure 9B.2 shows the number of employees in each wage group as a fraction of all employees; you can estimate the likelihood or probability that an employee drawn at random from the whole group earns a wage within a given interval. For example, assuming the same conditions exist at the time the sample was taken, the probability is 0.33 (a one in three chance) that an employee drawn at random from the whole group earns between \$8.00 and \$8.50 an hour.

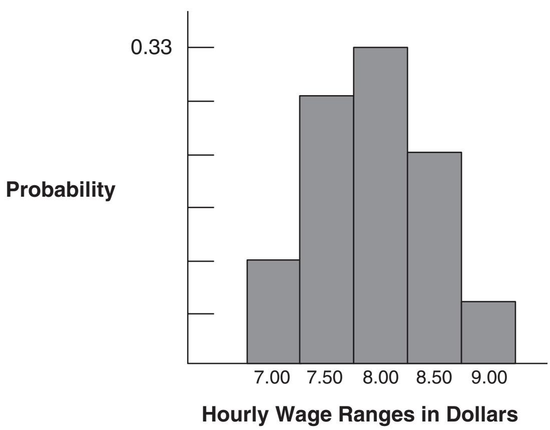


FIGURE 9B.2 Frequency Histogram II

Probability distributions are either discrete or continuous. *Discrete probability distributions* describe distinct values, usually integers, with no intermediate values and are shown as a series of vertical bars. A discrete distribution, for example, might describe the number of heads in four flips of a coin as 0, 1, 2, 3, or 4. *Continuous distributions* are actually mathematical abstractions because they assume the existence of every possible intermediate value between two numbers. That is, a continuous distribution assumes there is an infinite number of values between any two points in the distribution. However, in many situations, you can effectively use a continuous distribution to approximate a discrete distribution even though the continuous model does not necessarily describe the situation exactly.

SELECTING A PROBABILITY DISTRIBUTION

Plotting data is one guide to selecting a probability distribution. The following steps provide another process for selecting probability distributions that best describe the uncertain variables in your spreadsheets.

To select the correct probability distribution, use the following steps:

1. Look at the variable in question. List everything you know about the conditions surrounding this variable. You might be able to gather valuable information about the uncertain variable from historical data. If historical data are not available, use your own judgment, based on experience, listing everything you know about the uncertain variable.
2. Review the descriptions of the probability distributions.
3. Select the distribution that characterizes this variable. A distribution characterizes a variable when the conditions of the distribution match those of the variable.

MONTE CARLO SIMULATION

Monte Carlo simulation in its simplest form is a random number generator that is useful for forecasting, estimation, and risk analysis. A simulation calculates numerous scenarios of a model by repeatedly picking values from a user-predefined *probability distribution* for the uncertain variables and using those values for the model. As all those scenarios produce associated results in a model, each scenario can have a *forecast*. Forecasts are events (usually with formulas or functions) that you define as important outputs of the model, usually such events as totals, net profit, or gross expenses.

Simplistically, think of the Monte Carlo simulation approach as picking golf balls out of a large basket repeatedly with replacement, as seen in the example presented next. The size and shape of the basket depend on the distributional *assumptions* (e.g., a normal distribution with a mean of 100 and a standard deviation of 10, versus a uniform distribution or a triangular distribution) where some baskets are deeper or more symmetrical than others, allowing certain balls to be pulled out more frequently than others. The number of balls pulled repeatedly depends on the number of *trials* simulated. For a large model with multiple related assumptions, imagine the large model as a very large basket, where many baby baskets reside. Each baby basket has its own set of golf balls that are bouncing around. Sometimes these baby baskets are holding hands with each other (if there is a *correlation* between the variables) and the golf balls are bouncing in tandem while others are bouncing independent of one another. The balls that are picked each time from these interactions within the model (the large, central basket) are tabulated and recorded, providing a *forecast* result of the simulation.

With Monte Carlo simulation, Risk Simulator generates random values for each assumption's probability distribution that are totally independent. In other words, the random value selected for one trial has no effect on the next random value generated. Use Monte Carlo sampling when you want to simulate real-world what-if scenarios for your spreadsheet model.

PROBABILITY DENSITY FUNCTIONS, CUMULATIVE DISTRIBUTION FUNCTIONS, AND PROBABILITY MASS FUNCTIONS

In mathematics and Monte Carlo simulation, a probability density function (PDF) represents a *continuous* probability distribution in terms of integrals. If a probability distribution has a density of $f(x)$, then intuitively the infinitesimal interval of $[x, x + dx]$ has a probability of $f(x)\,dx$. The PDF therefore can be seen as a smoothed version of a probability histogram. That is, by providing an empirically large sample of a continuous random variable repeatedly, the histogram using very narrow ranges will resemble the random variable's PDF. The probability of the interval between $[a, b]$ is given by

$$\int_a^b f(x)dx$$

which means that the total integral of the function f must be 1.0. *It is a common mistake to think of f(a) as the probability of a*. This is incorrect. In fact, $f(a)$ can sometimes be larger than 1—consider a uniform distribution between 0.0 and 0.5. The random variable x within this distribution will have $f(x)$ greater than 1. The probability in reality is the function $f(x)dx$ discussed previously, where dx is an infinitesimal amount.

The cumulative distribution function (CDF) is denoted as $F(x) = P(X \leq x)$ indicating the probability of X taking on a less than or equal value to x. Every CDF is monotonically increasing, is continuous from the right, and at the limits, have the following properties: $\lim_{x \to -\infty} F(x) = 0$ and $\lim_{x \to +\infty} F(x) = 1$.

Further, the CDF is related to the PDF by:

$$F(b) - F(a) = P(a \leq X \leq b) = \int_a^b f(x)dx$$

where the PDF function f is the derivative of the CDF function F.

In probability theory, a probability mass function or PMF gives the probability that a *discrete* random variable is exactly equal to some value. The PMF differs from the PDF in that the values of the latter, defined only for continuous random variables, are not probabilities; rather, its integral over a set of possible values of the random variable is a probability. A random variable is discrete if its probability distribution is discrete and can be characterized by a PMF. Therefore, X is a discrete random variable if

$$\sum_u P(X = u) = 1$$

as u runs through all possible values of the random variable X.

DISCRETE DISTRIBUTIONS

Following is a detailed listing of the different types of probability distributions that can be used in Monte Carlo simulation. This listing is included in this appendix for the reader's reference.

Bernoulli or Yes/No Distribution

The Bernoulli distribution is a discrete distribution with two outcomes (e.g., head or tails, success or failure, 0 or 1). The Bernoulli distribution is the binomial distribution with one trial and can be used to simulate Yes/No or

Success/Failure conditions. This distribution is the fundamental building block of other more complex distributions. For instance:

- Binomial distribution: Bernoulli distribution with higher number of n total trials and computes the probability of x successes within this total number of trials.
- Geometric distribution: Bernoulli distribution with higher number of trials and computes the number of failures required before the first success occurs.
- Negative binomial distribution: Bernoulli distribution with higher number of trials and computes the number of failures before the xth success occurs.

The mathematical constructs for the binomial distribution are as follows:

$$P(x) = \begin{cases} 1-p & \text{for } x = 0 \\ p & \text{for } x = 1 \end{cases}$$

$$or$$

$$P(x) = p^{x}(1-p)^{1-x}$$

$$Mean = p$$

$$Standard\ Deviation\ = \sqrt{p(1-p)}$$

$$Skewness = \frac{1-2p}{\sqrt{p(1-p)}}$$

$$Excess\ Kurtosis = \frac{6p^2 - 6p + 1}{p(1-p)}$$

Probability of success (p) is the only distributional parameter. Also, it is important to note that there is only one trial in the Bernoulli distribution, and the resulting simulated value is either 0 or 1.

Input requirements:

Probability of success > 0 and < 1 (that is, $0.0001 \le p \le 0.9999$).

Binomial Distribution

The binomial distribution describes the number of times a particular event occurs in a fixed number of trials, such as the number of heads in 10 flips of a coin or the number of defective items out of 50 items chosen.

The three conditions underlying the binomial distribution are:

1. For each trial, only two outcomes are possible that are mutually exclusive.
2. The trials are independent—what happens in the first trial does not affect the next trial.
3. The probability of an event occurring remains the same from trial to trial.

The mathematical constructs for the binomial distribution are as follows:

$$P(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{(n-x)} \quad \text{for } n > 0;\ x = 0,\ 1,\ 2, \ldots\ n;\ \text{and } 0 < p < 1$$

$$Mean = np$$

$$Standard\ Deviation = \sqrt{np(1-p)}$$

$$Skewness = \frac{1-2p}{\sqrt{np(1-p)}}$$

$$Excess\ Kurtosis = \frac{6p^2 - 6p + 1}{np(1-p)}$$

The probability of success (p) and the integer number of total trials (n) are the distributional parameters. The number of successful trials is denoted x. It is important to note that probability of success (p) of 0 or 1 are trivial conditions and do not require any simulations, and hence, are not allowed in the software.

Input requirements:

Probability of success > 0 and < 1 (that is, $0.0001 \leq p \leq 0.9999$).

Number of trials ≥ 1 or positive integers and ≤ 1000 (for larger trials, use the normal distribution with the relevant computed binomial mean and standard deviation as the normal distribution's parameters).

Discrete Uniform

The discrete uniform distribution is also known as the *equally likely outcomes* distribution, where the distribution has a set of N elements, then each element has the same probability. This distribution is related to the uniform distribution but its elements are discrete and not continuous.

The mathematical constructs for the binomial distribution are as follows:

$$P(x) = \frac{1}{N}$$

$$Mean = \frac{N+1}{2} \text{ ranked value}$$

$$Standard\ Deviation = \sqrt{\frac{(N-1)(N+1)}{12}} \text{ ranked value}$$

$$Skewness = 0 \text{ (that is, the distribution is perfectly symmetrical)}$$

$$Excess\ Kurtosis = \frac{-6(N^2+1)}{5(N-1)(N+1)} \text{ ranked value}$$

Input requirements:

Minimum < Maximum and both must be integers (negative integers and zero are allowed).

Geometric Distribution

The geometric distribution describes the number of trials until the first successful occurrence, such as the number of times you need to spin a roulette wheel before you win.

The three conditions underlying the geometric distribution are:

1. The number of trials is not fixed.
2. The trials continue until the first success.
3. The probability of success is the same from trial to trial.

The mathematical constructs for the geometric distribution are as follows:

$$P(x) = p(1-p)^{x-1} \text{ for } 0 < p < 1 \text{ and } x = 1, 2, \ldots, n$$

$$Mean = \frac{1}{p} - 1$$

$$Standard\ Deviation = \sqrt{\frac{1-p}{p^2}}$$

$$Skewness = \frac{2-p}{\sqrt{1-p}}$$

$$Excess\ Kurtosis = \frac{p^2 - 6p + 6}{1-p}$$

The probability of success (p) is the only distributional parameter. The number of successful trials simulated is denoted x, which can only take on positive integers.

Input requirements:

Probability of success > 0 and < 1 (that is, $0.0001 \le p \le 0.9999$). It is important to note that probability of success (p) of 0 or 1 are trivial conditions and do not require any simulations, and hence, are not allowed in the software.

Hypergeometric Distribution

The hypergeometric distribution is similar to the binomial distribution in that both describe the number of times a particular event occurs in a fixed number of trials. The difference is that binomial distribution trials are independent, whereas hypergeometric distribution trials change the probability for each subsequent trial and are called *trials without replacement*. For example, suppose a box of manufactured parts is known to contain some defective parts. You choose a part from the box, find it is defective, and remove the part from the box. If you choose another part from the box, the probability that it is defective is somewhat lower than for the first part because you have removed a defective part. If you had replaced the defective part, the probabilities would have remained the same, and the process would have satisfied the conditions for a binomial distribution.

The three conditions underlying the hypergeometric distribution are:

1. The total number of items or elements (the population size) is a fixed number, a finite population. The population size must be less than or equal to 1,750.
2. The sample size (the number of trials) represents a portion of the population.
3. The known initial probability of success in the population changes after each trial.

The mathematical constructs for the hypergeometric distribution are as follows:

$$P(x) = \frac{\dfrac{(N_x)!}{x!(N_x - x)!}\dfrac{(N - N_x)!}{(n-x)!(N - N_x - n + x)!}}{\dfrac{N!}{n!(N-n)!}} \text{ for } x = Max(n - (N - N_x), 0), \ldots, Min(n, N_x)$$

$$Mean = \frac{N_x n}{N}$$

$$Standard\ Deviation = \sqrt{\frac{(N-N_x)N_x n(N-n)}{N^2(N-1)}}$$

$$Skewness = \frac{(N-2N_x)(N-2n)}{N-2}\sqrt{\frac{N-1}{(N-N_x)N_x n(N-n)}}$$

$$Excess\ Kurtosis = \frac{V(N,N_x,n)}{(N-N_x)\ \ N_x n(-3+N)(-2+N)(-N+n)} \text{ where}$$

$$\begin{aligned}V(N,N_x,n) = {} & (N-N_x)^3-(N-N_x)^5+3(N-N_x)^2N_x-6(N-N_x)^3N_x \\ & +(N-N_x)^4N_x+3(N-N_x)\ \ N_x^2-12(N-N_x)^2N_x^2+8(N-N_x)^3N_x^2+N_x^3 \\ & -6(N-N_x)\ \ N_x^3+8(N-N_x)^2N_x^3+(N-N_x)\ \ N_x^4-N_x^5-6(N-N_x)^3N_x \\ & +6(N-N_x)^4N_x+18(N-N_x)^2N_x n-6(N-N_x)^3N_x n+18(N-N_x)\ \ N_x^2 n \\ & -24(N-N_x)^2N_x^2 n-6(N-N_x)^3 n-6(N-N_x)\ \ N_x^3 n+6N_x^4 n+6(N-N_x)^2 n^2 \\ & -6(N-N_x)^3 n^2-24(N-N_x)\ \ N_x n^2+12(N-N_x)^2N_x n^2+6N_x^2 n^2 \\ & +12(N-N_x)\ \ N_x^2 n^2-6N_x^3 n^2\end{aligned}$$

The number of items in the population (N), trials sampled (n), and number of items in the population that have the successful trait (N_x) are the distributional parameters. The number of successful trials is denoted x.

Input requirements:

Population ≥ 2 and integer.
Trials > 0 and integer.
Successes > 0 and integer.
Population > Successes.
Trials < Population.
Population < 1750.

Negative Binomial Distribution

The negative binomial distribution is useful for modeling the distribution of the number of trials until the rth successful occurrence, such as the number of sales calls you need to make to close a total of 10 orders. It is essentially a *superdistribution* of the geometric distribution. This distribution shows the probabilities of each number of trials in excess of r produce the required success r.

The three conditions underlying the negative binomial distribution are:

1. The number of trials is not fixed.
2. The trials continue until the rth success.
3. The probability of success is the same from trial to trial.

The mathematical constructs for the negative binomial distribution are as follows:

$$P(x) = \frac{(x+r-1)!}{(r-1)!x!} p^r (1-p)^x \quad \text{for } x = r,\ r+1,\ \ldots;\ \text{and } 0 < p < 1$$

$$Mean = \frac{r(1-p)}{p}$$

$$Standard\ Deviation = \sqrt{\frac{r(1-p)}{p^2}}$$

$$Skewness = \frac{2-p}{\sqrt{r(1-p)}}$$

$$Excess\ Kurtosis = \frac{p^2 - 6p + 6}{r(1-p)}$$

The probability of success (p) and required successes (r) are the distributional parameters.

Input requirements:

Successes required must be positive integers > 0 and < 8000.

Probability of success > 0 and < 1 (that is, $0.0001 \leq p \leq 0.9999$). It is important to note that probability of success (p) of 0 or 1 are trivial conditions and do not require any simulations, and hence, are not allowed in the software.

Poisson Distribution

The Poisson distribution describes the number of times an event occurs in a given interval, such as the number of telephone calls per minute or the number of errors per page in a document.

The three conditions underlying the Poisson distribution are:

1. The number of possible occurrences in any interval is unlimited.
2. The occurrences are independent. The number of occurrences in one interval does not affect the number of occurrences in other intervals.
3. The average number of occurrences must remain the same from interval to interval.

The mathematical constructs for the Poisson are as follows:

$$P(x) = \frac{e^{-\lambda}\lambda^x}{x!} \quad \text{for } x \text{ and } \lambda > 0$$

$$Mean = \lambda$$
$$Standard\ \ Deviation\ = \sqrt{\lambda}$$
$$Skewness = \frac{1}{\sqrt{\lambda}}$$
$$Excess\ Kurtosis = \frac{1}{\lambda}$$

Rate (λ) is the only distributional parameter.

Input requirements:

Rate > 0 and ≤ 1000 (that is, 0.0001 ≤ rate ≤ 1000).

CONTINUOUS DISTRIBUTIONS

Beta Distribution

The beta distribution is very flexible and is commonly used to represent variability over a fixed range. One of the more important applications of the beta distribution is its use as a conjugate distribution for the parameter of a Bernoulli distribution. In this application, the beta distribution is used to represent the uncertainty in the probability of occurrence of an event. It is also used to describe empirical data and predict the random behavior of percentages and fractions, as the range of outcomes is typically between 0 and 1.

The value of the beta distribution lies in the wide variety of shapes it can assume when you vary the two parameters, alpha and beta. If the parameters are equal, the distribution is symmetrical. If either parameter is 1 and the other parameter is greater than 1, the distribution is J-shaped. If alpha is less than beta, the distribution is said to be positively skewed (most of the values are near the minimum value). If alpha is greater than beta, the distribution is negatively skewed (most of the values are near the maximum value).

The mathematical constructs for the beta distribution are as follows:

$$f(x) = \frac{\left(x\right)^{(\alpha-1)}\left(1-x\right)^{(\beta-1)}}{\left[\dfrac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}\right]} \quad \text{for } \alpha > 0; \beta > 0; x > 0$$

$$Mean = \frac{\alpha}{\alpha+\beta}$$

$$S\tau\alpha v dard\ Deviation\ = \sqrt{\frac{\alpha\beta}{(\alpha+\beta)^2(1+\alpha+\beta)}}$$

$$Skewness = \frac{2(\beta-\alpha)\sqrt{1+\alpha+\beta}}{(2+\alpha+\beta)\sqrt{\alpha\beta}}$$

$$Excess\ Kurtosis = \frac{3(\alpha+\beta+1)[\alpha\,\beta(\alpha+\beta-6)+2(\alpha+\beta)^2]}{\alpha\,\beta(\alpha+\beta+2)(\alpha+\beta+3)} - 3$$

Alpha (α) and beta (β) are the two distributional shape parameters, and Γ is the gamma function.

The two conditions underlying the beta distribution are:

1. The uncertain variable is a random value between 0 and a positive value.
2. The shape of the distribution can be specified using two positive values.

Input requirements:

Alpha and beta both > 0 and can be any positive value.

Cauchy Distribution or Lorentzian Distribution or Breit-Wigner Distribution

The Cauchy distribution, also called the Lorentzian distribution or Breit-Wigner distribution, is a continuous distribution describing resonance behavior. It also describes the distribution of horizontal distances at which a line segment tilted at a random angle cuts the x-axis.

The mathematical constructs for the Cauchy or Lorentzian distribution are as follows:

$$f(x) = \frac{1}{\pi}\frac{\gamma/2}{(x-m)^2+\gamma^2/4}$$

The Cauchy distribution is a special case where it does not have any theoretical moments (mean, standard deviation, skewness, and kurtosis) as they are all undefined.

Mode location (m) and scale (γ) are the only two parameters in this distribution. The location parameter specifies the peak or mode of the distribution while the scale parameter specifies the half-width at half-maximum of the distribution. In addition, the mean and variance of a Cauchy or Lorentzian distribution are undefined.

In addition, the Cauchy distribution is the Student's t-distribution with only 1 degree of freedom. This distribution is also constructed by taking the ratio of two standard normal distributions (normal distributions with a mean of zero and a variance of one) that are independent of one another.

Input requirements:

Location can be any value.
Scale > 0 and can be any positive value.

Chi-Square Distribution

The chi-square distribution is a probability distribution used predominantly in hypothesis testing, and is related to the gamma distribution and the standard normal distribution. For instance, the sum of independent normal distributions are distributed as a chi-square (χ^2) with k degrees of freedom:

$$Z_1^2 + Z_2^2 + \ldots + Z_k^2 \overset{d}{\sim} \chi_k^2$$

The mathematical constructs for the chi-square distribution are as follows:

$$f(x) = \frac{2^{-k/2}}{\Gamma(k/2)} x^{k/2-1} e^{-x/2} \text{ for all } x > 0$$

$$Mean = k$$

$$Standard\ Deviation = \sqrt{2k}$$

$$Skewness = 2\sqrt{\frac{2}{k}}$$

$$Excess\ Kurtosis = \frac{12}{k}$$

The gamma function is written as Γ. Degrees of freedom k is the only distributional parameter.

The chi-square distribution can also be modeled using a gamma distribution by setting the shape parameter = $k/2$ and scale = $2S^2$ where S is the scale.

Input requirements:

Degrees of freedom > 1 and must be an integer < 1000.

Exponential Distribution

The exponential distribution is widely used to describe events recurring at random points in time, such as the time between failures of electronic equipment or the time between arrivals at a service booth. It is related to the Poisson distribution, which describes the number of occurrences of an event in a given interval of time. An important characteristic of the exponential distribution is the "memoryless" property, which means that the future lifetime of a given object has the same distribution, regardless of the time it existed. In other words, time has no effect on future outcomes.

The mathematical constructs for the exponential distribution are as follows:

$$f(x) = \lambda e^{-\lambda x} \quad \text{for } x \geq 0;\ \lambda > 0$$

$$Mean = \frac{1}{\lambda}$$

$$Standard\ Deviation = \frac{1}{\lambda}$$

$$Skewness = 2 \text{ (this value applies to all success rate } \lambda \text{ inputs)}$$

$$Excess\ Kurtosis = 6 \text{ (this value applies to all success rate } \lambda \text{ inputs)}$$

Success rate (λ) is the only distributional parameter. The number of successful trials is denoted x.

The one condition underlying the exponential distribution is:

1. The exponential distribution describes the amount of time between occurrences.

Input requirements:

Rate > 0 and ≤ 300.

Extreme Value Distribution or Gumbel Distribution

The extreme value distribution (Type 1) is commonly used to describe the largest value of a response over a period of time, for example, in flood flows, rainfall, and earthquakes. Other applications include the breaking strengths of materials, construction design, and aircraft loads and tolerances. The extreme value distribution is also known as the Gumbel distribution.

The mathematical constructs for the extreme value distribution are as follows:

$$f(x) = \frac{1}{\beta} z e^{-Z} \text{ where } z = e^{\frac{x-m}{\beta}} \quad \text{for } \beta > 0;\ \text{and any value of } x \text{ and } m$$

$$Mean = m + 0.577215\beta$$

$$Standard\ Deviation = \sqrt{\frac{1}{6}\pi^2\beta^2}$$

$$Skewness = \frac{12\sqrt{6}(1.2020569)}{\pi^3} = 1.13955$$

$$Excess\ Kurtosis = 5.4$$

Mode (m) and scale (β) are the distributional parameters.

There are two standard parameters for the extreme value distribution: mode and scale. The mode parameter is the most likely value for the variable (the highest point on the probability distribution). After you select the mode parameter, you can estimate the scale parameter. The scale parameter is a number greater than 0. The larger the scale parameter, the greater the variance.

Input requirements:

Mode can be any value.
Scale > 0.

F Distribution or Fisher-Snedecor Distribution

The F distribution, also known as the Fisher-Snedecor distribution, is also another continuous distribution used most frequently for hypothesis testing. Specifically, it is used to test the statistical difference between two variances in analysis of variance tests and likelihood ratio tests. The F distribution with the numerator degree of freedom n and denominator degree of freedom m is related to the chi-square distribution in that:

$$\frac{\chi_n^2 / n}{\chi_m^2 / m} \overset{d}{\sim} F_{n,m} \text{ or } f(x) = \frac{\Gamma\left(\frac{n+m}{2}\right)\left(\frac{n}{m}\right)^{n/2} x^{n/2-1}}{\Gamma\left(\frac{n}{2}\right)\Gamma\left(\frac{m}{2}\right)\left[x\left(\frac{n}{m}\right)+1\right]^{(n+m)/2}}$$

$$Mean = \frac{m}{m-2}$$

$$Standard\ Deviation = \frac{2m^2(m+n-2)}{n(m-2)^2(m-4)} \text{ for all } m > 4$$

$$Skewness = \frac{2(m+2n-2)}{m-6}\sqrt{\frac{2(m-4)}{n(m+n-2)}}$$

$$Excess\ Kurtosis = \frac{12(-16+20m-8m^2+m^3+44n-32mn+5m^2n-22n^2+5mn^2}{n(m-6)(m-8)(n+m-2)}$$

The numerator degree of freedom n and denominator degree of freedom m are the only distributional parameters.

Input requirements:

Degrees of freedom numerator and degrees of freedom denominator both > 0 integers.

Gamma Distribution (Erlang Distribution)

The gamma distribution applies to a wide range of physical quantities and is related to other distributions: lognormal, exponential, Pascal, Erlang, Poisson, and chi-square. It is used in meteorological processes to represent pollutant concentrations and precipitation quantities. The gamma distribution is also used to measure the time between the occurrence of events when the event process is not completely random. Other applications of the gamma distribution include inventory control, economic theory, and insurance risk theory.

The gamma distribution is most often used as the distribution of the amount of time until the *r*th occurrence of an event in a Poisson process. When used in this fashion, the three conditions underlying the gamma distribution are:

1. The number of possible occurrences in any unit of measurement is not limited to a fixed number.
2. The occurrences are independent. The number of occurrences in one unit of measurement does not affect the number of occurrences in other units.
3. The average number of occurrences must remain the same from unit to unit.

The mathematical constructs for the gamma distribution are as follows:

$$f(x) = \frac{\left(\frac{x}{\beta}\right)^{\alpha-1} e^{-\frac{x}{\beta}}}{\Gamma(\alpha)\beta} \quad \text{with any value of } \alpha > 0 \text{ and } \beta > 0$$

$$Mean = \alpha\beta$$

$$Standard\ Deviation = \sqrt{\alpha\beta^2}$$

$$Skewness = \frac{2}{\sqrt{\alpha}}$$

$$Excess\ Kurtosis = \frac{6}{\alpha}$$

Shape parameter alpha (α) and scale parameter beta (β) are the distributional parameters, and Γ is the gamma function.

When the alpha parameter is a positive integer, the gamma distribution is called the Erlang distribution, used to predict waiting times in queuing systems, where the Erlang distribution is the sum of independent and identically distributed random variables each having a memoryless exponential distribution. Setting n as the number of these random variables, the mathematical construct of the Erlang distribution is:

$$f(x) = \frac{x^{n-1}e^{-x}}{(n-1)!} \text{ for all } x > 0 \text{ and all positive integers of } n$$

Input requirements:

Scale beta > 0 and can be any positive value.
Shape alpha ≥ 0.05 and can be any positive value.
Location can be any value.

Logistic Distribution

The logistic distribution is commonly used to describe growth, that is, the size of a population expressed as a function of a time variable. It also can be used to describe chemical reactions and the course of growth for a population or individual.

The mathematical constructs for the logistic distribution are as follows:

$$f(x) = \frac{e^{\frac{\mu - x}{\alpha}}}{\alpha\left[1 + e^{\frac{\mu - x}{\alpha}}\right]^2} \quad \text{for any value of } \alpha \text{ and } \mu$$

$$Mean = \mu$$

$$Standard\ Deviation = \sqrt{\frac{1}{3}\pi^2\alpha^2}$$

$$Skewness = 0 \text{ (this applies to all mean and scale inputs)}$$

$$Excess\ Kurtosis = 1.2 \text{ (this applies to all mean and scale inputs)}$$

Mean (μ) and scale (α) are the distributional parameters.

There are two standard parameters for the logistic distribution: mean and scale. The mean parameter is the average value, which for this distribution is the same as the mode, because this distribution is symmetrical. After you select the mean parameter, you can estimate the scale parameter. The scale parameter is a number greater than 0. The larger the scale parameter, the greater the variance.

Input requirements:

Scale > 0 and can be any positive value.
Mean can be any value.

Lognormal Distribution

The lognormal distribution is widely used in situations where values are positively skewed, for example, in financial analysis for security valuation or in real estate for property valuation, and where values cannot fall below zero.

Stock prices are usually positively skewed rather than normally (symmetrically) distributed. Stock prices exhibit this trend because they cannot fall below the lower limit of zero but might increase to any price without limit. Similarly, real estate prices illustrate positive skewness as property values cannot become negative.

The three conditions underlying the lognormal distribution are:

1. The uncertain variable can increase without limits but cannot fall below zero.
2. The uncertain variable is positively skewed, with most of the values near the lower limit.
3. The natural logarithm of the uncertain variable yields a normal distribution.

Generally, if the coefficient of variability is greater than 30 percent, use a lognormal distribution. Otherwise, use the normal distribution.

The mathematical constructs for the lognormal distribution are as follows:

$$f(x) = \frac{1}{x\sqrt{2\pi}\ln(\sigma)} e^{-\frac{[\ln(x)-\ln(\mu)]^2}{2[\ln(\sigma)]^2}} \quad \text{for } x > 0;\ \mu > 0 \text{ and } \sigma > 0$$

$$Mean = \exp\left(\mu + \frac{\sigma^2}{2}\right)$$

$$Standard\ Deviation = \sqrt{\exp\left(\sigma^2 + 2\mu\right)\left[\exp\left(\sigma^2\right) - 1\right]}$$

$$Skewness = \left[\sqrt{\exp\left(\sigma^2\right) - 1}\right](2 + \exp(\sigma^2))$$

$$Excess\ Kurtosis = \exp\left(4\sigma^2\right) + 2\exp\left(3\sigma^2\right) + 3\exp\left(2\sigma^2\right) - 6$$

Mean (μ) and standard deviation (σ) are the distributional parameters. Input requirements:

Mean and standard deviation both > 0 and can be any positive value.

Lognormal Parameter Sets By default, the lognormal distribution uses the arithmetic mean and standard deviation. For applications for which historical data are available, it is more appropriate to use either the logarithmic mean and standard deviation, or the geometric mean and standard deviation.

Normal Distribution

The normal distribution is the most important distribution in probability theory because it describes many natural phenomena, such as people's IQs or heights. Decision makers can use the normal distribution to describe uncertain variables such as the inflation rate or the future price of gasoline.

The three conditions underlying the normal distribution are:

1. Some value of the uncertain variable is the most likely (the mean of the distribution).
2. The uncertain variable could as likely be above the mean as it could be below the mean (symmetrical about the mean).
3. The uncertain variable is more likely to be in the vicinity of the mean than further away.

The mathematical constructs for the normal distribution are as follows:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad \text{for all values of } x \text{ and } \mu; \text{ while } \sigma > 0$$

$$Mean = \mu$$

$$Standard\ Deviation\ = \sigma$$

$$Skewness = 0$$

$$Excess\ Kurtosis = 0$$

Mean (μ) and standard deviation (σ) are the distributional parameters. Input requirements:

Standard deviation > 0 and can be any positive value.
Mean can take on any value.

Pareto Distribution

The Pareto distribution is widely used for the investigation of distributions associated with such empirical phenomena as city population sizes, the occurrence of natural resources, the size of companies, personal incomes, stock price fluctuations, and error clustering in communication circuits.

The mathematical constructs for the Pareto are as follows:

$$f(x) = \frac{\beta L^{\beta}}{x^{(1+\beta)}} \quad \text{for } x > L$$

$$Mean = \frac{\beta L}{\beta - 1}$$

$$Standard\ Deviation = \sqrt{\frac{\beta L^2}{(\beta-1)^2(\beta-2)}}$$

$$Skewness = \sqrt{\frac{\beta-2}{\beta}}\left[\frac{2(\beta+1)}{\beta-3}\right]$$

$$Excess\ Kurtosis = \frac{6(\beta^3+\beta^2-6\beta-2)}{\beta(\beta-3)(\beta-4)}$$

Location (L) and shape (β) are the distributional parameters.

There are two standard parameters for the Pareto distribution: location and shape. The location parameter is the lower bound for the variable. After you select the location parameter, you can estimate the shape parameter. The shape parameter is a number greater than 0, and usually greater than 1. The larger the shape parameter, the smaller the variance and the thicker the right tail of the distribution.

Input requirements:

Location > 0 and can be any positive value.
Shape ≥ 0.05.

Student's t-Distribution

The Student's t-distribution is the most widely used distribution in hypothesis test. This distribution is used to estimate the mean of a normally distributed population when the sample size is small, and is used to test the statistical significance of the difference between two sample means or confidence intervals for small sample sizes.

The mathematical constructs for the t-distribution are as follows:

$$f(t) = \frac{\Gamma[(r+1)/2]}{\sqrt{r\pi}\,\Gamma[r/2]}(1+t^2/r)^{-(r+1)/2}$$

$Mean = 0$ (this applies to all degrees of freedom r except if the distribution is shifted to another nonzero central location)

$$Standard\ Deviation = \sqrt{\frac{r}{r-2}}$$

$Skewness = 0$ (this applies to all degrees of freedom r)

$$Excess\ Kurtosis = \frac{6}{r-4} \text{ for all } r > 4$$

where $t = \dfrac{x - \bar{x}}{s}$ and Γ is the gamma function.

Degree of freedom r is the only distributional parameter.

The t-distribution is related to the F-distribution as follows: the square of a value of t with r degrees of freedom is distributed as F with 1 and r degrees of freedom. The overall shape of the probability density function of the t-distribution also resembles the bell shape of a normally distributed variable with mean 0 and variance 1, except that it is a bit lower and wider or is leptokurtic (fat tails at the ends and peaked center). As the number of degrees of freedom grows (say, above 30), the t-distribution approaches the normal distribution with mean 0 and variance 1.

Input requirements:

Degrees of freedom ≥ 1 and must be an integer.

Triangular Distribution

The triangular distribution describes a situation where you know the minimum, maximum, and most likely values to occur. For example, you could describe the number of cars sold per week when past sales show the minimum, maximum, and usual number of cars sold.

The three conditions underlying the triangular distribution are:

1. The minimum number of items is fixed.
2. The maximum number of items is fixed.
3. The most likely number of items falls between the minimum and maximum values, forming a triangular-shaped distribution, which shows that values near the minimum and maximum are less likely to occur than those near the most likely value.

The mathematical constructs for the triangular distribution are as follows:

$$f(x) = \begin{cases} \dfrac{2(x - Min)}{(Max - Min)(Likely - Min)} & \text{for } Min < x < Likely \\[2ex] \dfrac{2(Max - x)}{(Max - Min)(Max - Likely)} & \text{for } Likely < x < Max \end{cases}$$

$$Mean = \frac{1}{3}(Min + Likely + Max)$$

$$Standard\ Deviation = \sqrt{\frac{1}{18}(Min^2 + Likely^2 + Max^2 - Min\,Max - Min\,Likely - Max\,Likely)}$$

$$Skewness = \frac{\sqrt{2}(Min + Max - 2Likely)(2Min - Max - Likely)(Min - 2Max + Likely)}{5(Min^2 + Max^2 + Likely^2 - MinMax - MinLikely - MaxLikely)^{3/2}}$$

$$Excess\ Kurtosis = -0.6 \text{ (this applies to all inputs of } Min,\ Max, \text{ and } Likely)$$

Minimum value (*Min*), most likely value (*Likely*), and maximum value (*Max*) are the distributional parameters.

Input requirements:

$Min \le Most\ Likely \le Max$ and can take any value.
However, $Min < Max$ and can take any value.

Uniform Distribution

With the uniform distribution, all values fall between the minimum and maximum and occur with equal likelihood.

The three conditions underlying the uniform distribution are:

1. The minimum value is fixed.
2. The maximum value is fixed.
3. All values between the minimum and maximum occur with equal likelihood.

The mathematical constructs for the uniform distribution are as follows:

$$f(x) = \frac{1}{Max - Min} \quad \text{for all values such that } Min < Max$$

$$Mean = \frac{Min + Max}{2}$$

$$Standard\ Deviation = \sqrt{\frac{(Max - Min)^2}{12}}$$

$$Skewness = 0 \text{ (this applies to all inputs of } Min \text{ and } Max)$$

$$Excess\ Kurtosis = -1.2 \text{ (this applies to all inputs of } Min \text{ and } Max)$$

Maximum (*Max*) and minimum (*Min*) are the distributional parameters.
Input requirements:

$Min < Max$ and can take any value.

Weibull Distribution (Rayleigh Distribution)

The Weibull distribution describes data resulting from life and fatigue tests. It is commonly used to describe failure time in reliability studies as well as the breaking strengths of materials in reliability and quality control tests. Weibull distributions are also used to represent various physical quantities, such as wind speed.

The Weibull distribution is a family of distributions that can assume the properties of several other distributions. For example, depending on the shape parameter you define, the Weibull distribution can be used to model the exponential and Rayleigh distributions, among others. The Weibull distribution is very flexible. When the Weibull shape parameter is equal to 1.0, the Weibull distribution is identical to the exponential distribution. The Weibull location parameter lets you set up an exponential distribution to start at a location other than 0.0. When the shape parameter is less than 1.0, the Weibull distribution becomes a steeply declining curve. A manufacturer might find this effect useful in describing part failures during a burn-in period.

The mathematical constructs for the Weibull distribution are as follows:

$$f(x) = \frac{\alpha}{\beta}\left[\frac{x}{\beta}\right]^{\alpha-1} e^{-\left(\frac{x}{\beta}\right)^{\alpha}}$$

$$Mean = \beta\,\Gamma(1+\alpha^{-1})$$

$$Standard\ Deviation\ = \beta^2\left[\Gamma(1+2\alpha^{-1}) - \Gamma^2(1+\alpha^{-1})\right]$$

$$Skewness = \frac{2\Gamma^3(1+\beta^{-1}) - 3\Gamma(1+\beta^{-1})\Gamma(1+2\beta^{-1}) + \Gamma(1+3\beta^{-1})}{\left[\Gamma(1+2\beta^{-1}) - \Gamma^2(1+\beta^{-1})\right]^{3/2}}$$

$$Excess\ Kurtosis = \frac{-6\Gamma^4(1+\beta^{-1}) + 12\Gamma^2(1+\beta^{-1})\Gamma(1+2\beta^{-1}) - 3\Gamma^2(1+2\beta^{-1}) - 4\Gamma(1+\beta^{-1})\Gamma(1+3\beta^{-1}) + \Gamma(1+4\beta^{-1})}{\left[\Gamma(1+2\beta^{-1}) - \Gamma^2(1+\beta^{-1})\right]^2}$$

Location (L), shape (α) and scale (β) are the distributional parameters, and Γ is the gamma function.

Input requirements:

Scale > 0 and can be any positive value.
Shape ≥ 0.05.
Location can take on any value.

APPENDIX 9C

Forecasting

In the broadest sense, forecasting refers to the act of predicting the future, usually for purposes of planning and managing resources. There are many scientific approaches to forecasting. You can perform "what-if" forecasting by creating and simulating a model, such as with Risk Simulator's forecasting tool, or by collecting data over a period of time and analyzing the trends and patterns. Forecasting uses this latter concept, that is, the patterns of a time-series, to forecast future data.

These scientific approaches usually fall into one of several categories of forecasting:

Time-series	Performs time-series analysis on past patterns of data to forecast results. This works best for stable situations where conditions are expected to remain the same.
Regression	Forecasts results using past relationships between a variable of interest and several other variables that might influence it. This works best for situations where you need to identify the different effects of different variables. This category includes multiple linear regression.
Simulation	Randomly generates many different scenarios for a model to forecast the possible outcomes. This method works best where you might not have historical data but you can build the model of your situation to analyze its behavior.
Qualitative	Uses subjective judgment and expert opinion to forecast results. These methods work best for situations for which there are no historical data or models available.

TIME-SERIES FORECASTING

Time-series forecasting is a category of forecasting that assumes that the historical data is a combination of a pattern and some random error. Its goal is to isolate the pattern from the error by understanding the pattern's level, trend, and seasonality. You can then measure the error using a statistical measurement both to describe how well a pattern reproduces historical data and to estimate how accurately it forecasts the data into the future.

MULTIPLE LINEAR REGRESSION

Multiple linear regression is used for data where one data series (the dependent variable) is a function of, or depends on, other data series (the independent variables). For example, the yield of a lettuce crop depends on the amount of water provided, the hours of sunlight each day, and the amount of fertilizer used.

The goal of multiple linear regression is to find an equation that most closely matches the historical data. The word "multiple" indicates that you can use more than one independent variable to define your dependent variable in the regression equation. The word "linear" indicates that the regression equation is a linear equation. The linear equation describes how the independent variables ($x_1, x_2, x_3, \ldots$) combine to define the single dependent variable (y). Multiple linear regression finds the coefficients for the equation:

$$y = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 + \ldots + e$$

where b_1, b_2, and b_3, are the coefficients of the independent variables, b_0 is the y-intercept, and e is the error.

If there is only one independent variable, the equation defines a straight line. This uses a special case of multiple linear regression called simple linear regression, with the equation:

$$y = b_0 + b_1 x + e$$

where b_0 is the place on the graph where the line crosses the y axis, x is the independent variable, and e is the error. When the regression equation has only two independent variables, it defines a plane. When the regression equation has more than two independent variables, it defines a hyperplane. To find the coefficients of these equations, you can use singular value decomposition.[1]

APPENDIX 9D
Optimization

In most simulation models, there are variables over which you have control, such as how much to charge for a product or how much to invest in a project. These controlled variables are called decision variables. Finding the optimal values for decision variables can make the difference between reaching an important goal and missing that goal.

Obtaining optimal values generally requires that you search in an iterative or ad-hoc fashion. This involves running a simulation for an initial set of values, analyzing the results, changing one or more values, rerunning the simulation, and repeating the process until you find a satisfactory solution. This process can be very tedious and time consuming even for small models, and it is often not clear how to adjust the values from one simulation to the next.

A more rigorous method systematically enumerates all possible alternatives. This approach guarantees optimal solutions. Suppose that a simulation model depends on only two decision variables. If each variable has 10 possible values, trying each combination requires 100 simulations (10^2 alternatives). If each simulation is very short (e.g., two seconds), then the entire process could be done in approximately three minutes of computer time.

However, instead of two decision variables, consider six, then consider that trying all combinations requires 1,000,000 simulations (10^6 alternatives). It is easily possible for complete enumeration to take weeks, months, or even years to carry out.

WHAT IS AN OPTIMIZATION MODEL?

In today's competitive global economy, companies are faced with many difficult decisions. These decisions include allocating financial resources, building or expanding facilities, managing inventories, and determining product-mix strategies. Such decisions might involve thousands or millions of potential alternatives. Considering and evaluating each of them would be impractical or even impossible. A model can provide valuable assistance in incorporating relevant variables when analyzing decisions, and finding the best solutions for making decisions. Models capture the most important features of a problem and present them in a form that is easy to interpret. Models often provide insights that intuition alone cannot. An optimization model has three major elements: decision variables, constraints, and an objective.

- ***Decision variables*** are quantities over which you have control; for example, the amount of a product to make, the number of dollars to allocate among different investments, or which projects to select from among a limited set. In real options, portfolio optimization analysis includes a go or no-go decision on particular projects. In addition, the dollar or percentage budget allocation across multiple projects can also be structured as decision variables.
- ***Constraints*** describe relationships among decision variables that restrict the values of the decision variables. For example, a constraint might ensure that the total amount of money allocated among various investments cannot exceed a specified amount, or at most one project from a certain group can be selected. In real options analysis, this could include budget constraints, timing restrictions, minimum returns, or risk tolerance levels.
- ***Objective*** gives a mathematical representation of the model's objective, such as maximizing profit or minimizing cost, in terms of the decision variables. In real options, the objective may be to maximize returns while minimizing risks (maximizing the returns-to-risk ratio).

Conceptually, an optimization model might look like Figure 9D.1.

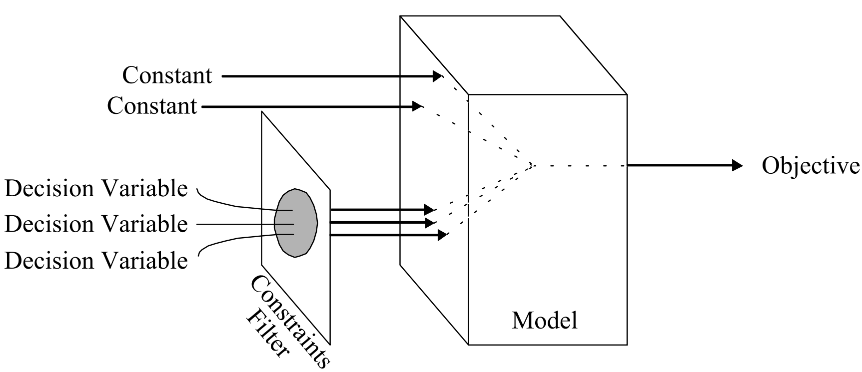


FIGURE 9D.1 Deterministic Optimization Model

The solution to an optimization model provides a set of values for the decision variables that optimizes (maximizes or minimizes) the associated objective. If the world were simple and the future were predictable, all data in an optimization model would be constant, making the model deterministic.

In many cases, however, a deterministic optimization model cannot capture all the relevant intricacies of a practical decision environment. When model data are uncertain and can only be described probabilistically, the objective will have some probability distribution for any chosen set of decision variables. You can find this probability distribution by simulating the model using Risk Simulator.

An optimization model with uncertainty has several additional elements:

- ***Assumptions*** capture the uncertainty of model data using probability distributions.
- ***Forecasts*** are frequency distributions of possible results for the model.
- ***Forecast statistics*** are summary values of a forecast distribution, such as the mean, standard deviation, and variance. You control the optimization by maximizing, minimizing, or restricting forecast statistics.
- ***Requirements*** are additional restrictions on forecast statistics. You can set upper and lower limits for any statistic of a forecast distribution. You can also define a range of requirement values by defining a variable requirement (see Figure 9D.2).

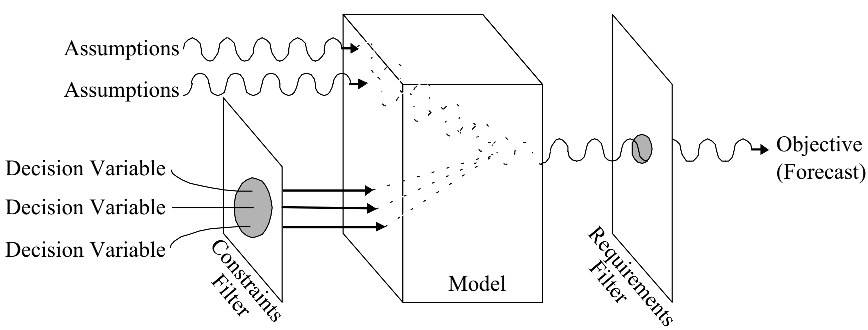


FIGURE 9D.2 Optimization with Uncertainty Model

DECISION VARIABLES

Decision variables are variables in your model that you have control over, such as how much to charge for a product or how much money to invest in a project. In real options, decision variables are the dollar or percentage budget allocation across multiple projects that make up the portfolio.

When you define a decision variable, you define its:

- ***Bounds,*** which define the upper and lower limits for the variable.
- ***Type,*** which defines whether the variable is discrete or continuous. A discrete variable can assume integer or noninteger values and must have a defined step size that is greater than 0 (integer or noninteger). A continuous variable requires no step size, and any given range contains an infinite number of possible values. In real options, a discrete variable includes a go or no-go decision on each project. A continuous variable implies the dollar or percentage allocation in each project can take on any continuous value.
- ***Step size,*** which defines the difference between successive values of a discrete decision variable in the defined range. For example, a discrete decision variable with a range of 1 to 5 and a step size of 1 can only take on the values 1, 2, 3, 4, or 5; a discrete decision variable with a range of 0 to 2 with a step size of 0.25 can only take on the values 0, 0.25, 0.5, 0.75, 1.0, 1.25, 1.5, 1.75, and 2.0.

In an optimization model, you select which decision variables to optimize from a list of all the defined decision variables. The values of the decision variables you select will change with each simulation until the best value for each decision variable is found within the available time limit.

CONSTRAINTS

Constraints restrict the decision variables by defining relationships among them. For example, if the total amount of money invested in two projects must be $50,000, you can define this as

$$\text{Project } X + \text{Project } Y = \$50{,}000$$

Or if your budget restricts your spending on both projects to $2,500, you can define this as

$$\text{Project } X + \text{Project } Y \leq \$2{,}500$$

Feasibility

A feasible solution is one that satisfies all constraints. Infeasibility occurs when no combination of values of the decision variables can satisfy a set of constraints. Note that a solution (i.e., a single set of values for the decision variables) can be infeasible, by failing to satisfy the problem constraints, and this doesn't imply that the problem or model itself is infeasible.

For example, suppose that an investor insists on finding an optimal investment portfolio with the following constraints:

$$\text{Project } X + \text{Project } Y \leq \$10{,}000$$

$$\text{Project } X + \text{Project } Y \geq \$12{,}000$$

Clearly, there is no combination of investments that will make the sum of the projects no more than $10,000 and at the same time greater than or equal to $12,000.

Or, for this same example, suppose the bounds for a decision variable were

$$\$15{,}000 \leq \text{portfolio expenses} \leq \$25{,}000$$

And a constraint was

$$\text{portfolio expenses} \leq \$5{,}000$$

This also results in an infeasible problem.

You can make infeasible problems feasible by fixing the inconsistencies of the relationships modeled by the constraints.

OBJECTIVE

Each optimization model has one objective, a forecast variable, that mathematically represents the model's objective in terms of the assumption and decision variables. Optimization's job is to find the optimal value of the objective by selecting and improving different values for the decision variables.

When model data are uncertain and can only be described using probability distributions, the objective itself will have some probability distribution for any set of decision variables.

Forecast Statistics

You can't use an entire forecast distribution as the objective but rather must characterize the distribution using a single summary measure for comparing and choosing one distribution over another. The statistic you choose depends on your goals for the objective. For maximizing or minimizing some quantity, the mean or median are often used as measures of central tendency, with the mean being the more common of the two. For highly skewed distributions, however, the mean might become the less stable (have a higher standard error) of the two, and so the median becomes a better measure of central tendency. For minimizing overall risk, the standard deviation and the variance of the objective are the two best statistics to use. For maximizing or minimizing the extreme values of the objective, a low or high percentile might be the appropriate statistic. For controlling the shape or range of the objective, the skewness, kurtosis, or certainty statistics might be used.

Minimizing or Maximizing

Whether you want to maximize or minimize the objective depends on which statistic you select to optimize. For example, if your forecast is returns and you select the mean as the statistic, you would want to maximize the mean of the returns. However, if you select the standard deviation as the statistic, you might want to minimize it to limit the uncertainty of the forecast. In real options portfolio optimization, the objective to maximize is usually a returns-to-risk ratio. Maximizing this objective will automatically select the optimal allocation across projects that will maximize returns with the minimum amount of risk (obtaining an efficient frontier).

REQUIREMENTS

Requirements restrict forecast statistics. These differ from constraints, as constraints restrict decision variables (or relationships among decision variables). Requirements are sometimes called “probabilistic constraints,” “chance constraints,” or “goals” in other literature.

When you define a requirement, you first select a forecast (either the objective forecast or another forecast). As with the objective, you then select a statistic for that forecast, but instead of maximizing or minimizing it, you give it an upper bound, a lower bound, or both (a range). In real options, requirements may be to set the maximum and minimum allowable allocation of capital and resources on each project.

Feasibility

Like constraints, requirements must be satisfied for a solution to be considered feasible. When an optimization model includes requirements, a solution that is constraint-feasible might be infeasible with respect to one or more requirements.

Variable Requirements

Variable requirements let you define a range for a requirement bound (instead of a single point) and a number of points to check within the range. When you define a variable requirement, you first select a forecast (either the objective forecast or another forecast). Like the objective or the requirement, you then select a statistic for that forecast, but instead of maximizing or minimizing it, you select to restrict the upper bound or the lower bound. You then define the upper or lower bound with a range.

TYPES OF OPTIMIZATION MODELS

Optimization models can be classified as

Model	Has:
Discrete	Only discrete decision variables
Continuous	Only continuous decision variables
Mixed	Both discrete and continuous decision variables

Linear or Nonlinear

An optimization model can be linear or nonlinear, depending on the form of the mathematical relationships used to model the objective and constraints. In a linear relationship, all terms in the formulas only contain a single variable multiplied by a constant.

For example, $3X - 1.2Y$ is a linear relationship, because the first and second terms only involve a constant multiplied by a variable. Terms such as X^2, XY, $1/X$, or 3.1^X make nonlinear relationships. Any models that contain such terms in either the objective or a constraint are classified as nonlinear.

Deterministic or Stochastic

Optimization models might also be classified as deterministic or stochastic, depending on the nature of the model data. In a deterministic model, all input data are constant or assumed to be known with certainty. In a stochastic model, some of the model data are uncertain and are described with probability distributions. Stochastic models are much more difficult to optimize because they require simulation to compute the objective.

CHAPTER 10

Real Options Valuation Application Cases

This chapter goes into more detail in terms of the applications and computations of real options problems using the Real Options Valuation's Super Lattice Solver (SLS) software and Risk Simulator software developed by the author. Specifically, the types of options covered in this chapter include:

- American, European, Bermudan, and Customized Abandonment Options.
- American, European, Bermudan, and Customized Contraction Options.
- American, European, Bermudan, and Customized Expansion Options.
- Contraction, Expansion, and Abandonment Options.
- Basic American, European, and Bermudan Call Options.
- Basic American, European, and Bermudan Put Options.
- Exotic Chooser Options.
- Sequential Compound Options.
- Multiple-Phased Sequential Compound Options.
- Customized Sequential Compound Options.
- Path Dependent, Path Independent, Mutually Exclusive, Nonmutually Exclusive, and Complex Combinatorial Nested Options.
- Simultaneous Compound Options.
- American and European Options Using Trinomial Lattices.
- American and European Mean-Reversion Option Using Trinomial Lattices, and Jump-Diffusion Option Using Pentanomial Lattices.
- Dual Variable Rainbow Options Using Pentanomial Lattices.
- American and European Lower Barrier Options.
- American and European Upper Barrier Options.
- American and European Double Barrier Options and Exotic Barriers.
- American ESO with Vesting Period.
- Changing Volatilities and Risk-free Rates Options.
- American ESO with Suboptimal Exercise Behavior.

- American ESO with Vesting and Suboptimal Exercise Behavior.
- American ESO with Vesting, Suboptimal Exercise Behavior, Blackout Periods, and Forfeiture Rate.

Each option is discussed in detail followed by simple case studies and solutions using the SLS software. Additional study questions and hands-on exercises are also included at the end of each case study. Chapter 11's case studies are more detailed and protracted, and are focused on understanding how the real options analysis process works, whereas this chapter's short cases are focused on how the real options computations and software work.

AMERICAN, EUROPEAN, BERMUDAN, AND CUSTOMIZED ABANDONMENT OPTION

The *Abandonment Option* looks at the value of a project's or asset's flexibility in being abandoned over the life of the option. As an example, suppose that a firm owns a project or asset and that based on traditional discounted cash flow (DCF) models, it estimates the present value of the asset (*PV Underlying Asset*) to be $120M (for the abandonment option this is the net present value of the project or asset). Monte Carlo simulation using the Risk Simulator software indicates that the *Volatility* of this asset value is significant, estimated at 25 percent. There is a lot of uncertainty as to the success or failure of this project (the volatility calculated models the different sources of uncertainty and computes the risks in the DCF model including price uncertainty, probability of success, competition, cannibalization, etc.), and the value of the project might be significantly higher or significantly lower than the expected value of $120M. Suppose an abandonment option is created whereby a counterparty is found and a contract is signed that lasts five years (*Maturity*) such that for some monetary consideration now, the firm has the ability to sell the asset or project to the counterparty at any time within these five years (indicative of an American option) for a specified *Salvage* of $90M. The counterparty agrees to this $30M discount and signs the contract.

What has just occurred is that the firm bought itself a $90M insurance policy. That is, if the asset or project value increases above its current value, the firm may decide to continue funding the project, or sell it off in the market at the prevailing fair market value. Alternatively, if the value of the asset or project falls below the $90M threshold, the firm has the right to execute the option and sell off the asset to the counterparty at $90M. In other words, a safety net of sorts has been erected to prevent the value of the asset from falling below this salvage level. Thus, how much is this safety net or insurance policy worth? One can create competitive advantage in negotiation if the counterparty does not have the answer and you do. Further assume that the five-year Treasury note *Risk-Free Rate* (zero coupon) is 5 percent from the U.S. Department of Treasury (http://www.treas.gov/offices/domestic-finance/debt-management/interest-rate/yield-hist.html). The *American Abandonment Option* results in Figure 10.1 show a value of $125.48M, indicating that the option value is $5.48M as the present value of the asset is $120M. Hence, the *maximum* value one should be willing to pay for the contract on *average* is $5.48M. This resulting expected value weights the continuous probabilities that the asset value exceeds $90M versus when it does not (where the abandonment option is valuable). Also, it weights when the timing of executing the abandonment is optimal such that the expected value is $5.48M.

In addition, some experimentation can be conducted. Changing the salvage value to $30M (this means a $90M discount from the starting asset value) yields a result of $120M, or $0M for the option. This result means that the option or contract is worthless because the safety net is set so low that it will never be utilized. Conversely, setting the salvage level to thrice the prevailing asset value or $360M would yield a result of $360M, and the options

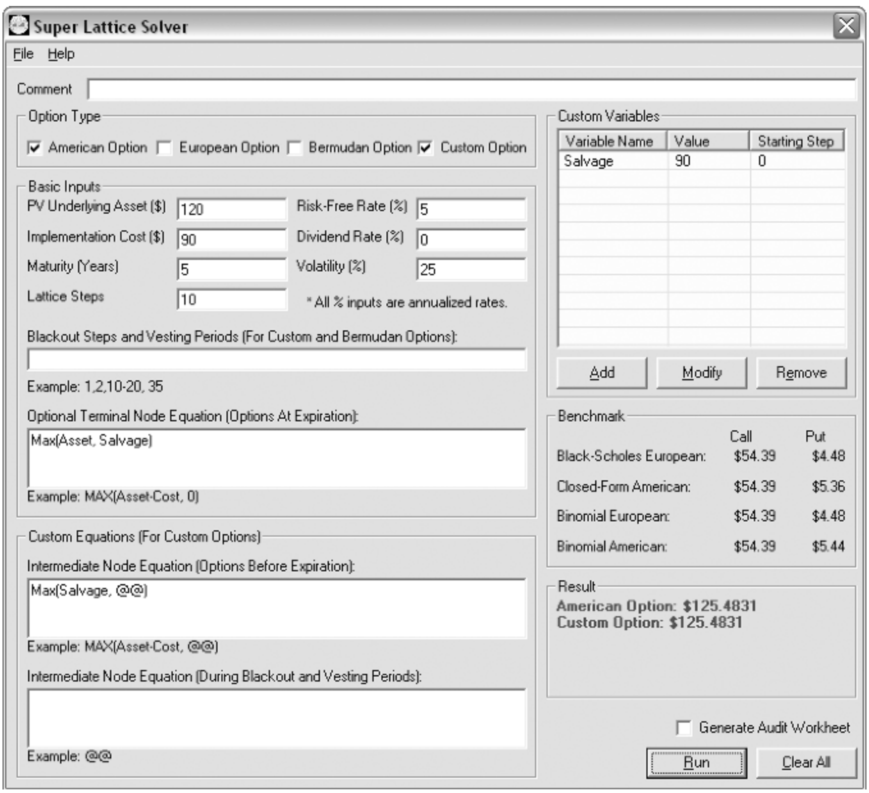


FIGURE 10.1 Simple American Abandonment Option

valuation results indicate \$360M, which means that there is no option value, there is no value in waiting and having this option, or simply, execute the option immediately and sell the asset if someone is willing to pay three times the value of the project right now. Thus, you can keep changing the salvage value until the option value disappears, indicating the *optimal trigger value* has been reached. For instance, if you enter \$166.80 as the salvage value, the abandonment option analysis yields a result of \$166.80, indicating that at this price and above, the optimal decision is to sell the asset immediately, given the assumed volatility and the other input parameters. At any lower salvage value, there is option value, and at any higher salvage value, there will be no option value. This break-even salvage point is the *optimal trigger value*. Once the market price of this asset exceeds this value, it is optimal to abandon. Finally, adding a *Dividend Rate*, the *cost of waiting before abandoning the asset* (e.g., the annualized taxes and maintenance fees that have to be paid if you keep the asset and do not sell it off, measured as a percentage of the present value of the asset) will decrease the option value. Hence, the break-even trigger point, where the option becomes worthless, can be calculated by successively choosing higher dividend levels. This break-even point again illustrates the trigger value at which the option should be optimally executed immediately, but this time with respect to a dividend yield. That is, if the *cost of carry* or holding on to the option, or the option's *leakage value* is high, that is, if the *cost of waiting* is too high, don't wait and execute the option immediately.

Other applications of the abandonment option include buyback lease provisions in a contract (guaranteeing a specified asset value); asset preservation flexibility; insurance policies; walking away from a project and selling off its intellectual property; purchase price of an acquisition; and so forth. To illustrate, following are some additional quick examples of the abandonment option and sample exercises.

- An aircraft manufacturer sells its planes of a particular model in the primary market for say \$30M each to various airline companies. Airlines are usually risk-averse and may find it hard to justify buying an additional plane with all the uncertainties in the economy, demand, price competition, and fuel costs. When uncertainties become resolved over time, airline carriers may have to reallocate and reroute their existing portfolio of planes globally, and an excess plane on the tarmac is very costly. The airline can sell the excess plane in the secondary market where smaller regional carriers buy used planes, but the price uncertainty is very high and is subject to significant volatility, of say, 45 percent, and may fluctuate wildly between \$10M and \$25M for this class of aircraft. The aircraft manufacturer can reduce the airline's risk by providing a *buyback provision* or abandonment option, where at anytime within the next five years, the manufacturer agrees to buy back the plane at a guaranteed residual salvage price of \$20M, at the request of the airline.

The corresponding risk-free rate for the next five years is 5 percent. This reduces the downside risk of the airline, and hence reduces its risk, chopping off the left tail of the price fluctuation distribution, and shifting the expected value to the right. This abandonment option provides risk reduction and value enhancement to the airline. *Applying the abandonment option in SLS using a 100-step binomial lattice, we find that this option is worth $3.52M. If the airline is the smarter counterparty and calculates this value and gets this buyback provision for free as part of the deal, the aircraft manufacturer has just lost over 10 percent of its aircraft value that it left on the negotiation table. Information and knowledge is highly valuable in this case.*

- A high-tech disk-drive manufacturer is thinking of acquiring a small startup firm with a new micro drive technology (a super-fast and high-capacity pocket hard drive) that may revolutionize the industry. The startup is for sale and its asking price is $50M based on an NPV fair market value analysis some third-party valuation consultants have performed. The manufacturer can either develop the technology themselves or acquire this technology through the purchase of the firm. The question is, how much is this firm worth to the manufacturer, and is $50M a good price? Based on internal analysis by the manufacturer, the NPV of this microdrive is expected to be $45M, with a cash flow volatility of 40 percent, and it would take another three years before the microdrive technology is successful and goes to market. Assume that the three-year risk-free rate is 5 percent. In addition, it would cost the manufacturer $45M in present value to develop this drive internally. If using an NPV analysis, the manufacturer should build it themselves. However, if you include an abandonment option analysis whereby if this specific microdrive does not work, the start-up still has an abundance of intellectual property (patents and proprietary technologies) as well as physical assets (buildings and manufacturing facilities) that can be sold in the market at up to $40M. *The abandonment option together with the NPV yields $51.83, making buying the start-up worth more than developing the technology internally, and making the purchase price of $50M worth it.* (See the section on Expansion Option for more examples on how this start-up's technology can be used as a platform to further develop newer technologies that can be worth a lot more than just the abandonment option.)

Figure 10.1 shows the results of a simple abandonment option with a 10-step lattice as discussed previously, while Figure 10.2 shows the audit sheet that is generated from this analysis.

Figure 10.3 shows the same abandonment option but with a 100-step lattice. To follow along, open the example file by clicking on *Start* | *Programs* | *Real Options Valuation* | *Real Options Super Lattice Solver* | *Sample Files* | *Abandonment American Option*. Notice that the 10-step lattice yields

Option Valuation Audit Sheet

Assumptions		***Intermediate Computations***	
PV Asset Value ($)	$120.00	*Stepping Time (dt)*	0.5000
Implementation Cost ($)	$90.00	*Up Step Size (up)*	1.1934
Maturity (Years)	5.00	*Down Step Size (down)*	0.8380
Risk-free Rate (%)	5.00%	*Risk-neutral Probability*	0.5272
Dividends (%)	0.00%		
Volatility (%)	25.00%	***Results***	
Lattice Steps	10	*Auditing Lattice Result (10 steps)*	$125.48
Option Type	Custom	*Super Lattice Result (10 steps)*	$125.48

User-Defined Inputs *Terminal: Max(Asset, Salvage)*
Intermediate: Max(Salvage, @@)

Name	salvage									
Value	90.00									
Starting Step	0									

Underlying Asset Lattice

										702.93
									589.03	
								493.59		493.59
							413.61		413.61	
						346.59		346.59		346.59
					290.43		290.43		290.43	
				243.37		243.37		243.37		243.37
			203.94		203.94		203.94		203.94	
		170.89		170.89		170.89		170.89		170.89
	143.20		143.20		143.20		143.20		143.20	
120.00		120.00		120.00		120.00		120.00		120.00
	100.56		100.56		100.56		100.56		100.56	
		84.26		84.26		84.26		84.26		84.26
			70.61		70.61		70.61		70.61	
				59.17		59.17		59.17		59.17
					49.58		49.58		49.58	
						41.55		41.55		41.55
							34.82		34.82	
								29.17		29.17
									24.45	
										20.49

Option Valuation Lattice

										702.93
									589.03	
								493.59		493.59
							413.61		413.61	
						346.59		346.59		346.59
					290.43		290.43		290.43	
				243.43		243.37		243.37		243.37
			204.30		204.06		203.94		203.94	
		172.07		171.61		171.15		170.89		170.89
	146.01		145.36		144.61		143.77		143.20	
125.48		124.77		123.88		122.77		121.22		120.00
	109.32		108.49		107.41		105.93		103.20	
		97.95		97.13		96.03		94.57		90.00
			91.44		90.88		90.13		90.00	
				90.00		90.00		90.00		90.00
					90.00		90.00		90.00	
						90.00		90.00		90.00
							90.00		90.00	
								90.00		90.00
									90.00	
										90.00

FIGURE 10.2 Audit Sheet for the Abandonment Option

$125.48 while the 100-step lattice yields $125.45, indicating that the lattice results have achieved convergence. The Terminal Node Equation is *Max (Asset,Salvage)* which means the decision at maturity is to decide if the option should be executed, selling the asset and receiving the salvage value, or not to execute, holding on to the asset. The Intermediate Node Equation used is *Max(Salvage,@@)* indicating that before maturity, the decision is either to execute early in this American option to abandon and receive the salvage value, or to hold on to the asset, and hence, hold on to and keeping the

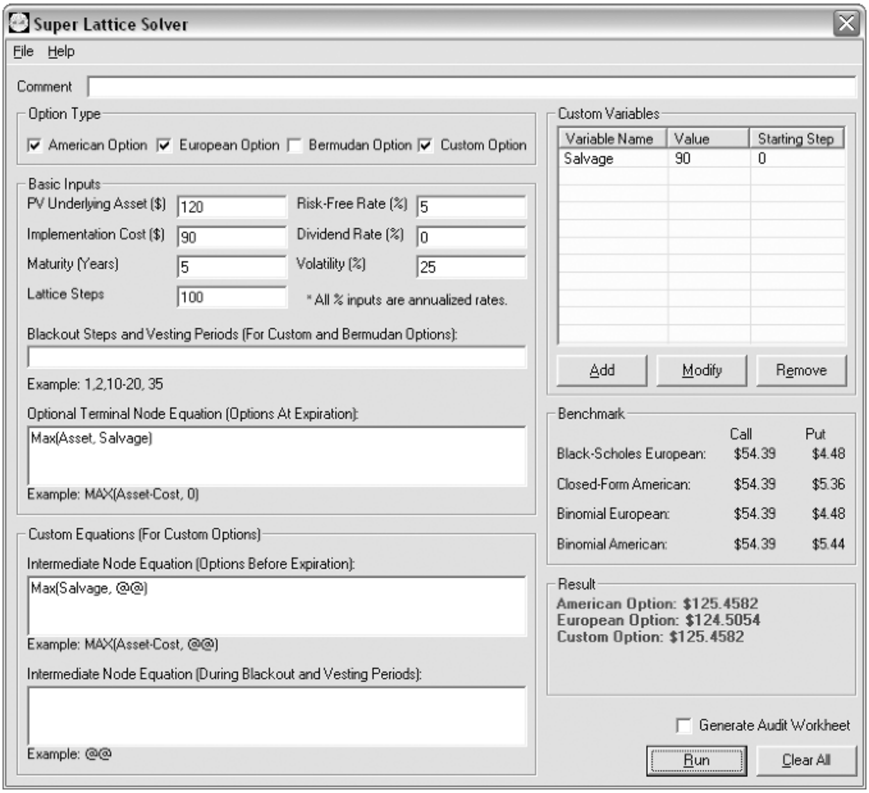


FIGURE 10.3 American Abandonment Option with 100-Step Lattice

option open for potential future execution, denoted simply as @@. Figure 10.4 shows the European version of the abandonment option, where the Intermediate Node Equation is simply @@, as early execution is prohibited before maturity. Of course, being only able to execute the option at maturity is worth less ($124.5054 compared to $125.4582) than being able to exercise earlier. The example files used are: *Abandonment American Option* and *Abandonment European Option*. For example, the airline manufacturer in the previous case example can agree to a buyback provision that can be exercised at any time by the airline customer versus only at a specific date at the end of five years—the former American option will clearly be worth more than the latter European option.

Sometimes, a Bermudan option is appropriate, where there might be a vesting period or blackout period when the option cannot be executed. For instance, if the contract stipulates that for the five-year abandonment buyback contract, the airline customer cannot execute the abandonment option

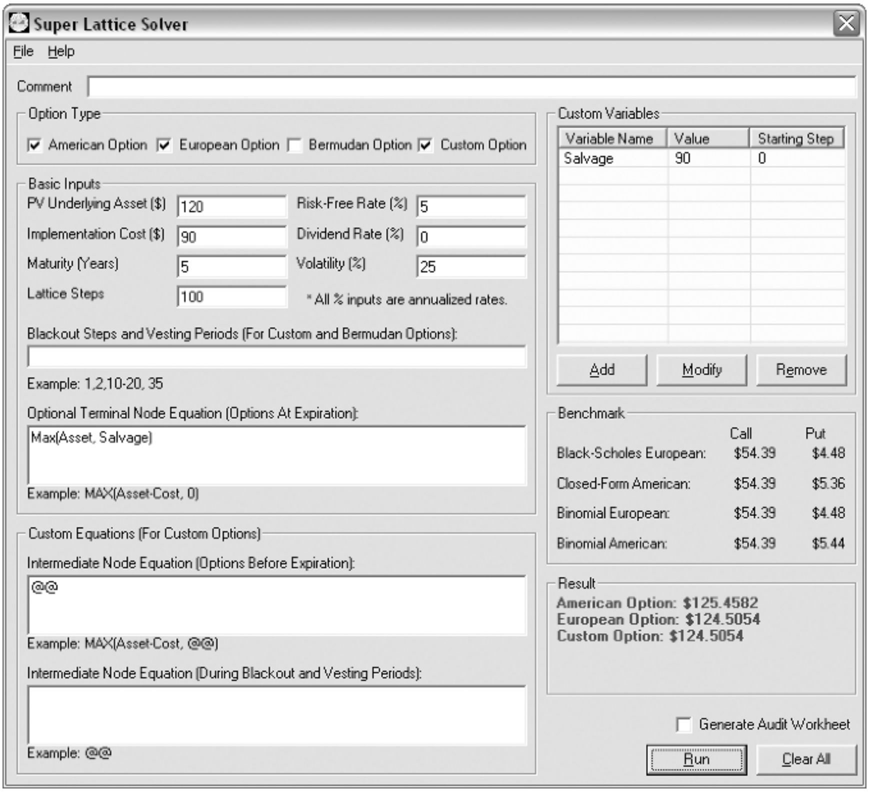


FIGURE 10.4 European Abandonment Option with 100-Step lattice

within the first 2.5 years. This is shown in Figure 10.5 using a Bermudan option with a 100-step lattice on five years, where the blackout steps are from 0 to 50. This means that during the first 50 steps (as well as right now or step 0), the option cannot be executed. This is modeled by inserting @@ into the Intermediate Node Equation (During Blackout and Vesting Periods). This forces the option holder to only keep the option open during the vesting period, preventing execution during this blackout period.

You can see that the American option is worth more than the Bermudan option, which is worth more than the European option in Figure 10.5, by virtue of each option type's ability to execute early and the frequency of execution possibilities.

Sometimes, the salvage value of the abandonment option may change over time. To illustrate, in the previous example of an acquisition of a start-up firm, the intellectual property will most probably increase over time because

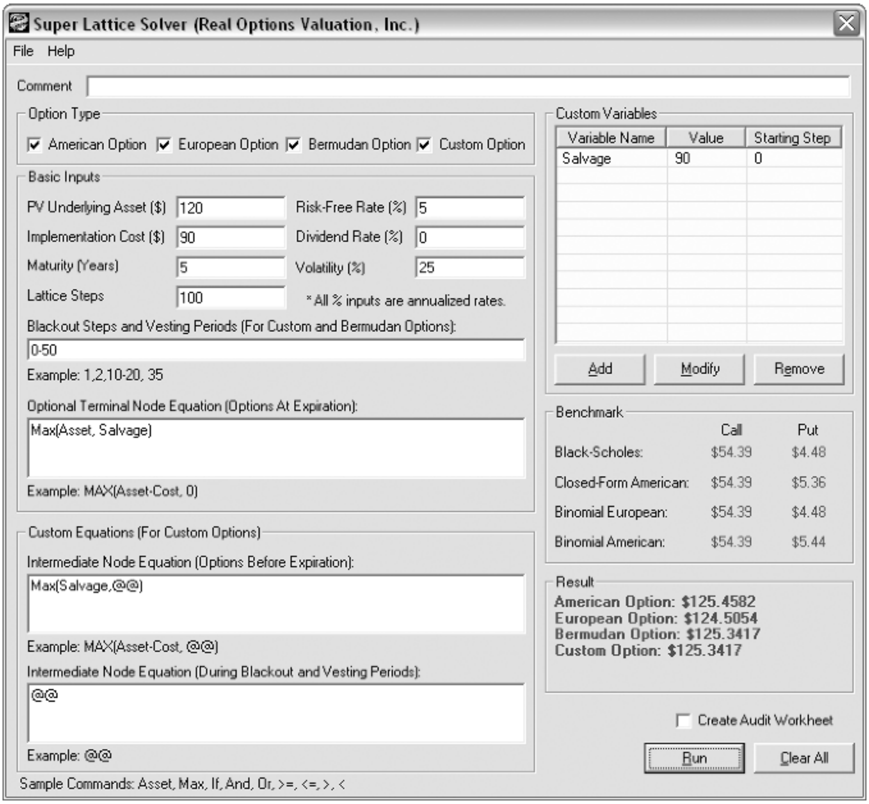


FIGURE 10.5 Bermudan Abandonment Option with 100-Step Lattice

of continued R&D activities, thereby changing the salvage values over time. An example is seen in Figure 10.6, where there are five salvage values over the five-year abandonment option. This can be modeled by using the Custom Variables. Click on *ADD* and input the variables one at a time as seen in Figure 10.6's Custom Variables list. Notice that the same variable name (*Salvage*) is used but the values change over time, and the starting steps represent when these different values become effective. For instance, the salvage value $90 applies at step 0 until the next salvage value of $95 takes over at step 21. This means that for a five-year option with a 100-step lattice, the first year including the current period (steps 0 to 20) will have a salvage value of $90, which then increases to $95 in the second year (steps 21 to 40), and so forth. Notice that as the value of the firm's intellectual property increases over time, the option valuation results also increase, which makes logical sense. You can also model in blackout vesting periods for the first six months (steps 0–10 in

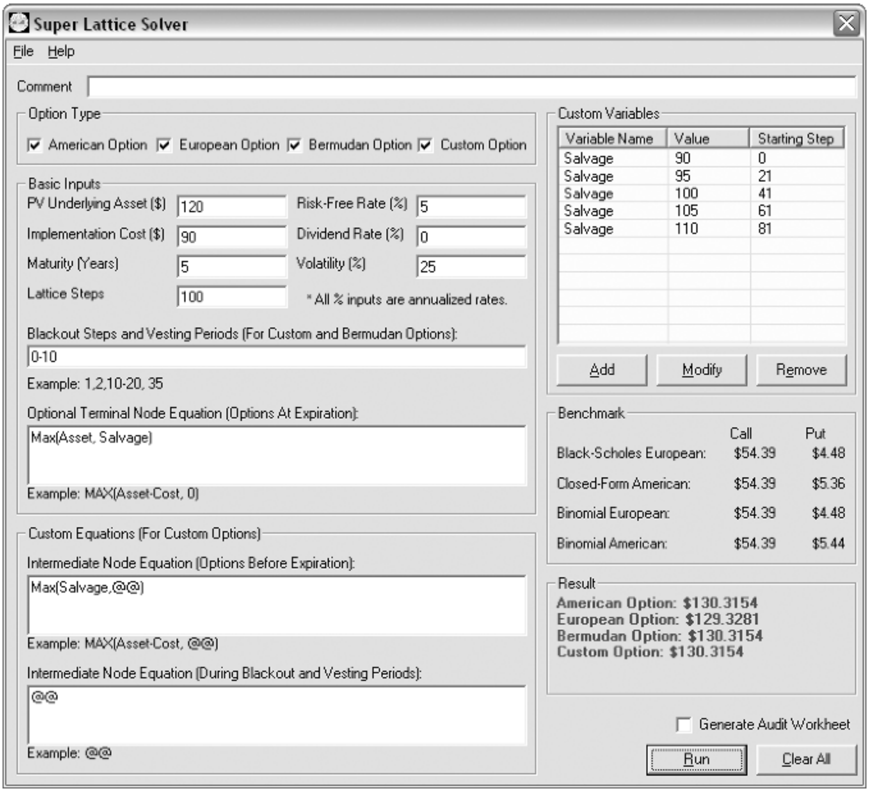


FIGURE 10.6 Customized Abandonment Option

the blackout area). The blackout period is very typical of contractual obligations of abandonment options where during specified periods, the option cannot be executed (a cooling-off period).

Exercise: Option to Abandon

Suppose a pharmaceutical company is developing a particular drug. However, due to the uncertain nature of the drug's development progress, market demand, success in human and animal testing, and FDA approval, management has decided that it will create a strategic abandonment option. That is, at any time period within the next five years of development, management can review the progress of the R&D effort and decide whether to terminate the drug development program. After five years, the firm would have either succeeded or completely failed in its drug development initiative, and there exists no option value after that time period. If the program is terminated, the firm can potentially sell off its intellectual property rights of the drug in question to another pharmaceutical firm with which it has a contractual agreement. This contract with the other firm is exercisable at any time within this time period, at the whim of the firm owning the patents.

Using a traditional DCF model, you find the present value of the expected future cash flows discounted at an appropriate market risk-adjusted discount rate to be \$150 million. Using Monte Carlo simulation, you find the implied volatility of the logarithmic returns on future cash flows to be 30 percent. The risk-free rate on a riskless asset for the same time frame is 5 percent, and you understand from the intellectual property officer of the firm that the value of the drug's patent is \$100 million contractually, if sold within the next five years. For simplicity, you assume that this \$100 million salvage value is fixed for the next five years. You attempt to calculate how much this abandonment option is worth and how much this drug development effort on the whole is worth to the firm. By virtue of having this safety net of being able to abandon drug development, the value of the project is worth more than its net present value. You decide to use a closed-form approximation of an American put option because the option to abandon drug development can be exercised at any time up to and including the expiration date. You also decide to confirm the value of the closed-form analysis with a binomial lattice calculation. With these assumptions, do the following exercises, answering the questions that are posed:

1. Solve the abandonment option problem manually using a 10-step lattice and confirm the results by generating an audit sheet using the SLS software.
2. Select the right choice for each of the following:
   a. Increases in maturity (increase/decrease) an abandonment option value.
   b. Increases in volatility (increase/decrease) an abandonment option value.
   c. Increases in asset value (increase/decrease) an abandonment option value.
   d. Increases in risk-free rate (increase/decrease) an abandonment option value.
   e. Increases in dividend (increase/decrease) an abandonment option value.
   f. Increases in salvage value (increase/decrease) an abandonment option value.
3. Apply 100 steps using the software's binomial lattice.
   a. How different are the results as compared to the five-step lattice?
   b. How close are the closed-form results compared to the 100-step lattice?
4. Apply a 3 percent continuous dividend yield to the 100-step lattice.

a. What happens to the results?
b. Does a dividend yield increase or decrease the value of an abandonment option? Why?

5. Assume that the salvage value increases at a 10 percent annual rate. Show how this can be modeled using the software's *Custom Variables List*.
6. Explain the differences in results when using the *Black-Scholes* and *American Put Option Approximation* in the benchmark section of the Single Asset SLS software.

AMERICAN, EUROPEAN, BERMUDAN, AND CUSTOMIZED CONTRACTION OPTION

A *Contraction Option* evaluates the flexibility value of being able to reduce production output or to contract the scale and scope of a project when conditions are not as amenable, thereby reducing the value of the asset or project by a *Contraction Factor*, but at the same time creating some cost *Savings*. As an example, suppose you work for a large aeronautical manufacturing firm that is unsure of the technological efficacy and market demand for its new fleet of long-range supersonic jets. The firm decides to hedge itself through the use of strategic options, specifically an option to contract 10 percent of its manufacturing facilities at any time within the next five years (i.e., the *Contraction Factor* is 0.9).

Suppose that the firm has a current operating structure whose static valuation of future profitability using a DCF model (in other words, the present value of the expected future cash flows discounted at an appropriate market risk-adjusted discount rate) is found to be \$1,000M (*PV Asset*). Using Monte Carlo simulation, you calculate the implied volatility of the logarithmic returns of the asset value of the projected future cash flows to be 30 percent. The risk-free rate on a riskless asset (five-year U.S. Treasury Note with zero coupons) is found to be yielding 5 percent.

Further, suppose the firm has the option to contract 10 percent of its current operations at any time over the next five years, thereby creating an additional \$50 million in savings after this contraction. These terms are arranged through a legal contractual agreement with one of its vendors, who had agreed to take up the excess capacity and space of the firm. At the same time, the firm can scale back and lay off part of its existing workforce to obtain this level of savings (in present values).

The results indicate that the strategic value of the project is \$1,001.71M (using a 10-step lattice as seen in Figure 10.7), which means that the NPV currently is \$1,000M and the additional \$1.71M comes from this contraction option. This result is obtained because contracting now yields 90 percent of \$1,000M + \$50M, or \$950M, which is less than staying in business and not

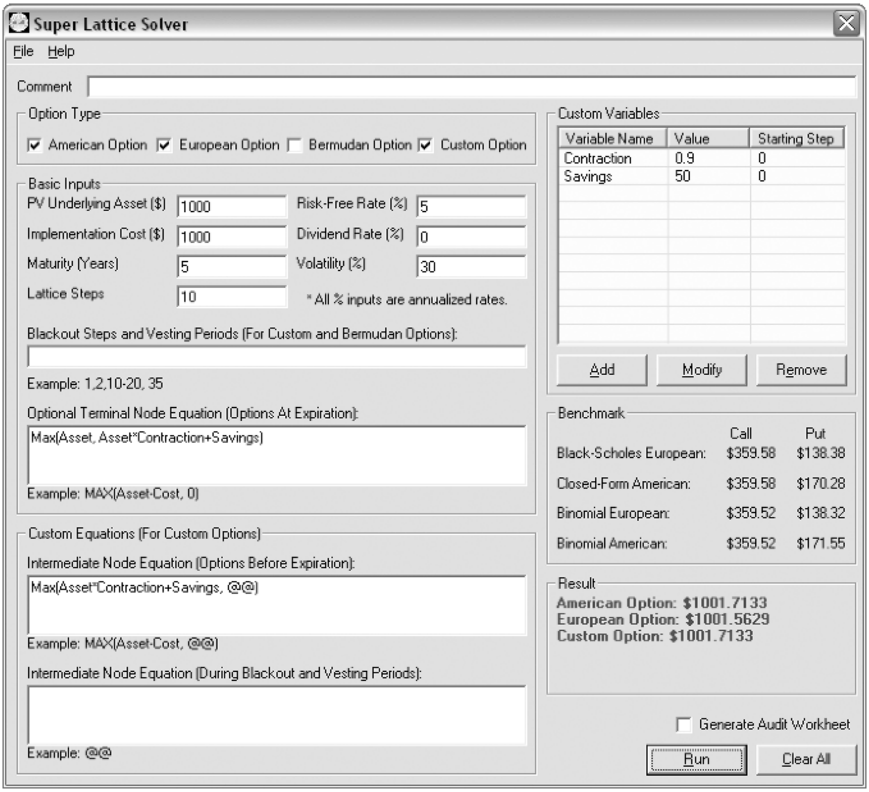


FIGURE 10.7 Simple American and European Options to Contract with a 10-Step Lattice

contracting and obtaining $1,000M. Therefore, the optimal decision is to not contract immediately but keep the ability to do so open for the future. Hence, in comparing this optimal decision of $1,000M to $1,001.71M of being able to contract, the option to contract is worth $1.71M. This should be the maximum amount the firm is willing to spend to obtain this option (contractual fees and payments to the vendor counterparty).

In contrast, if *Savings* were $200M instead, then the strategic project value becomes $1,100M, which means that starting at $1,000M and contracting 10 percent to $900M and keeping the $200 in savings, yields $1,100M in total value. Hence, the additional option value is $0M which means that it is optimal to execute the contraction option immediately as there is no option value and no value to wait to contract. So, the value of executing now is $1,100M as compared to the strategic project value of $1,100M; there is no additional option value, and the contraction should be executed immediately. That is, instead of asking the vendor to wait, the firm is better off executing the contraction option now and capturing the savings.

Other applications include shelving an R&D project by spending a little to keep it going but reserving the right to come back to it should conditions improve; the value of synergy in a merger and acquisition where some management personnel are let go to create the additional savings; reducing the scope and size of a production facility; reducing production rates; a joint venture or alliance, and so forth.

To illustrate, following are some additional quick examples of the contraction option (as before, providing some additional sample exercises).

- A large oil and gas company is embarking on a deep-sea drilling platform that will cost the company billions to implement. A DCF analysis is run and the NPV is found to be $500M over the next 10 years of economic life of the offshore rig. The 10-year risk-free rate is 5 percent, and the volatility of the project is found to be at an annualized 45 percent using historical oil prices as a proxy. If the expedition is highly successful (oil prices are high and production rates are soaring), then the company will continue its operations. However, if things are not looking too good (oil prices are low or moderate and production is only decent), it is very difficult for the company to abandon operations (why lose everything when net income is still positive although not as high as anticipated, not to mention the environmental and legal ramifications of simply abandoning an oil rig in the middle of the ocean). Hence, the oil company decides to hedge its downside risk through an American Contraction Option. The oil company was able to find a smaller oil and gas company (a former partner on other explorations) interested in a joint venture. The joint venture is structured such that the oil company pays this smaller counterparty a lump sum right now for a 10-year contract whereby at any time and at the oil company's request, the smaller counterparty will have to take over all operations of the offshore oil rig (i.e., taking over all operations and hence all relevant expenses) and keep 30 percent of the net revenues generated. The counterparty is in agreement because it does not have to partake in the billions of dollars required to implement the rig in the first place, and it actually obtains some cash up front for this contract to assume the downside risk. The oil company is also in agreement because it reduces its own risks if oil prices are low and production is not up to par, and it ends up saving over $75M in present value of total overhead expenses, which can then be reallocated and invested somewhere else. *In this example, the contraction option using a 100-step lattice is valued to be $14.24M using SLS. This means that the maximum amount that the counterparty should be paid should not exceed this amount. Of course, the option analysis can be further complicated by analyzing the actual savings on a present value basis. For instance, if the option is exercised within the first five years, the savings is $75M but* *if exercising during the last five years then the savings is only \$50M. The revised option value is now \$10.57M.*

- A manufacturing firm is interested in outsourcing its manufacturing of children's toys to a small province in China. By doing so, it will produce overhead savings of over \$20M in present value over the economic life of the toys. However, outsourcing this internationally will mean lower quality control, problems in delayed shipping, added importing costs, and assuming the added risks of unfamiliarity with the local business practices. In addition, the firm will consider outsourcing only if the quality of the workmanship in this Chinese firm is up to the stringent quality standards it requires. The NPV of this particular line of toys is \$100M with a 25 percent volatility. The firm's executives decide to purchase a contraction option by locating a small manufacturing firm in China, spending some resources to try out a *small-scale proof of concept* (thereby reducing the uncertainties of quality, knowledge, import-export issues, and so forth). If successful, the firm will agree to give this small Chinese manufacturer 20 percent of its net income as remuneration for its services, plus some start-up fees. The question is, how much is this option to contract worth, that is, how much should the firm be willing to pay, on average, to cover the initial start-up fees plus the costs of this proof of concept stage? *A contraction option valuation result using SLS shows that the option is worth \$1.59M, assuming a 5 percent risk-free rate for the one-year test period. So, as long as the total costs for a pilot test are less than \$1.59M, it is optimal to obtain this option, especially if it means potentially being able to save more than \$20M.*

Figure 10.7 illustrates a simple 10-step contraction option while Figure 10.8 shows the same option using 100 lattice steps (example file used is *Contraction American and European Option*). Figure 10.9 illustrates a five-year Bermudan Contraction Option with a four-year vesting period (blackout steps of 0 to 80 out of a 5-year, 100-step lattice) where for the first four years, the option holder can only keep the option open and not execute the option (example file used is *Contraction Bermudan Option*). Figure 10.10 shows a customized option where there is a blackout period and the savings from contracting change over time (example file used is *Contraction Customized Option*). These results are for the aeronautical manufacturing firm example.

Exercise: Option to Contract

You work for a large automobile spare parts manufacturing firm that is unsure of the technological efficacy and market demand of its products. The firm decides to hedge itself through the use of strategic options, specifically an option to contract 50 percent of its manufacturing facilities at any time within the next five years.

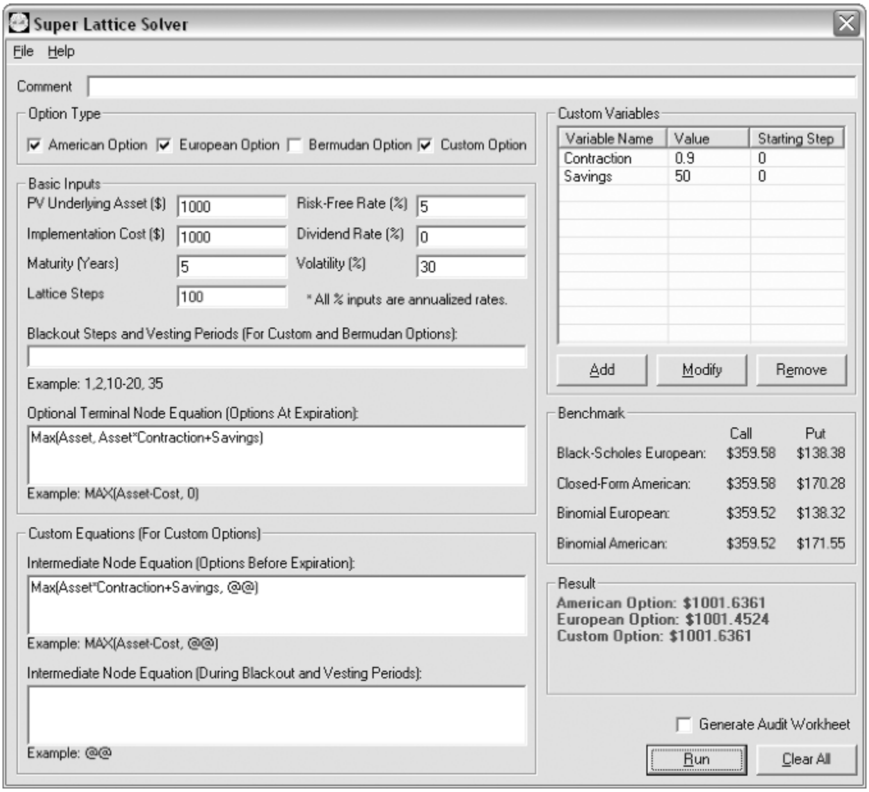


FIGURE 10.8 American and European Options to Contract with a 100-Step Lattice

Suppose the firm has a current operating structure whose static valuation of future profitability using a DCF model (in other words, the present value of the expected future cash flows discounted at an appropriate market risk-adjusted discount rate) is found to be $1 billion. Using Monte Carlo simulation, you calculate the implied volatility of the logarithmic returns on the projected future cash flows to be 50 percent. The risk-free rate on a riskless asset for the next five years is found to be yielding 5 percent. Suppose the firm has the option to contract 50 percent of its current operations at any time over the next five years, thereby creating an additional $400 million in savings after this contraction. This is done through a legal contractual agreement with one of its vendors, who had agreed to take up the excess ca-

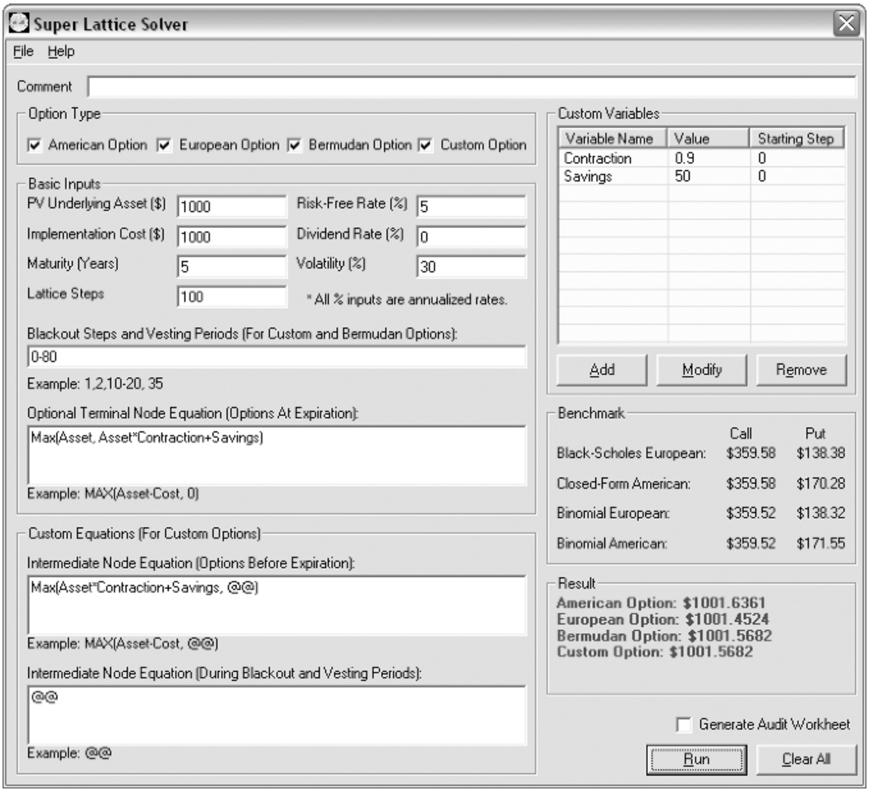


FIGURE 10.9 A Bermudan Option to Contract with Blackout Vesting Periods

pacity and space of the firm. Then the firm can scale back its existing workforce to obtain this level of savings.

A *Closed-Form American Approximation Model* can be used, because the option to contract the firm's operations can be exercised at any time up to the expiration date and can be confirmed with a binomial lattice calculation. Do the following exercises, answering the questions that are posed:

1. Solve the contraction option problem manually using a 10-step lattice and confirm the results by generating an audit sheet using the software.
2. Modify the continuous dividend payout rate until the option breaks even. What observations can you make at this break-even point?

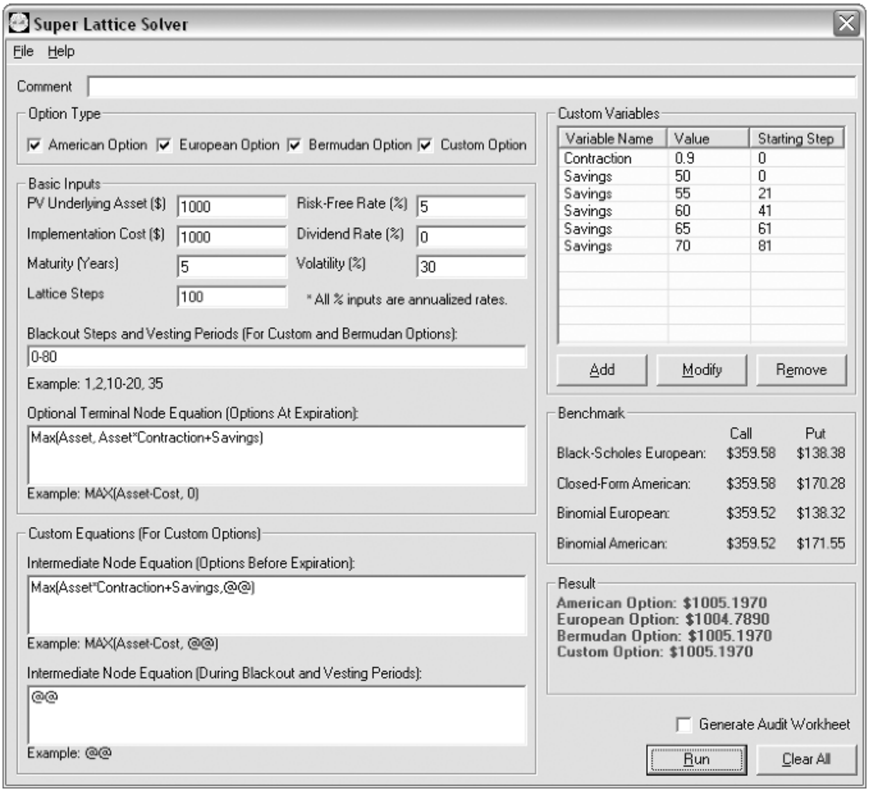


FIGURE 10.10 A Customized Option to Contract with Changing Savings

3. Use the *Closed-Form American Approximation Model* in the benchmark area of the software by using the corresponding put option. In order to do this appropriately, you will need to rerun the model with modified input parameters. What are these required input parameters?
4. How can you use the *American Abandonment Option* as a benchmark to estimate the contraction option? If it is used, are the resulting option values comparable?
5. Change the contraction factor to 0.7, and answer Question 4. Why are the answers different? Suppose the initial estimate of $400 million in savings is applicable only if the contraction option is executed immediately. However, due to opportunity costs and time value of money, assume that the $400 million goes down by $10 million each year. What happens to the value of this option and how much is it worth now?

AMERICAN, EUROPEAN, BERMUDAN, AND CUSTOMIZED EXPANSION OPTION

The *Expansion Option* values the flexibility to expand from a current existing state to a larger or expanded state. Therefore, an existing state or condition must first be present in order to use the expansion option. That is, there must be a base case on which to expand. If there is no base case state, then the simple *Execution Option* (calculated using the simple *Call Option*) is more appropriate, where the issue at hand is whether to execute a project immediately or to defer execution.

As an example, suppose a growth firm has a static valuation of future profitability using a DCF model (in other words, the present value of the expected future cash flows discounted at an appropriate market risk-adjusted discount rate) that is found to be $400 million (*PV Asset*). Using Monte Carlo simulation, you calculate the implied *Volatility* of the logarithmic returns on the assets based on the projected future cash flows to be 35 percent. The *Risk-Free Rate* on a riskless asset (five-year U.S. Treasury note with zero coupons) for the next five years is found to be 7 percent.

Further suppose that the firm has the option to expand and double its operations by acquiring its competitor for a sum of $250 million (*Implementation Cost*) at any time over the next five years (*Maturity*). What is the total value of this firm, assuming that you account for this expansion option? The results in Figure 10.11 indicate that the strategic project value is $638.73M (using a 10-step lattice), which means that the expansion option value is $88.73M. This result is obtained because the NPV of executing immediately is $400M × 2 – $250M, or $550M. Thus, $638.73M less $550M is $88.73M, the value of the ability to defer and to wait and see before executing the expansion option. The example file used is *Expansion American and European Option*.

Increase the dividend rate to, say, 2 percent and notice that both the American and European Expansion Options are now worth less, and that the American Expansion Option is worth more than the European Expansion Option by virtue of the American Option's ability for early execution (Figure 10.12). The dividend rate implies that the cost of waiting to expand, to defer and not execute, the opportunity cost of waiting on executing the option, and the cost of holding the option, is high, then the ability to defer reduces. In addition, increase the *Dividend Rate* to 4.9% and see that the binomial lattice's Custom Option result reverts to $550, (the static, expand-now scenario), indicating that the option is worthless (Figure 10.13). This result means if the cost-of-waiting as a proportion of the asset value (as measured by the dividend rate) is too high, then execute now and stop wasting time deferring the expansion decision! Of course this decision can be reversed if the volatility is significant enough to compensate for the cost of waiting. That is,

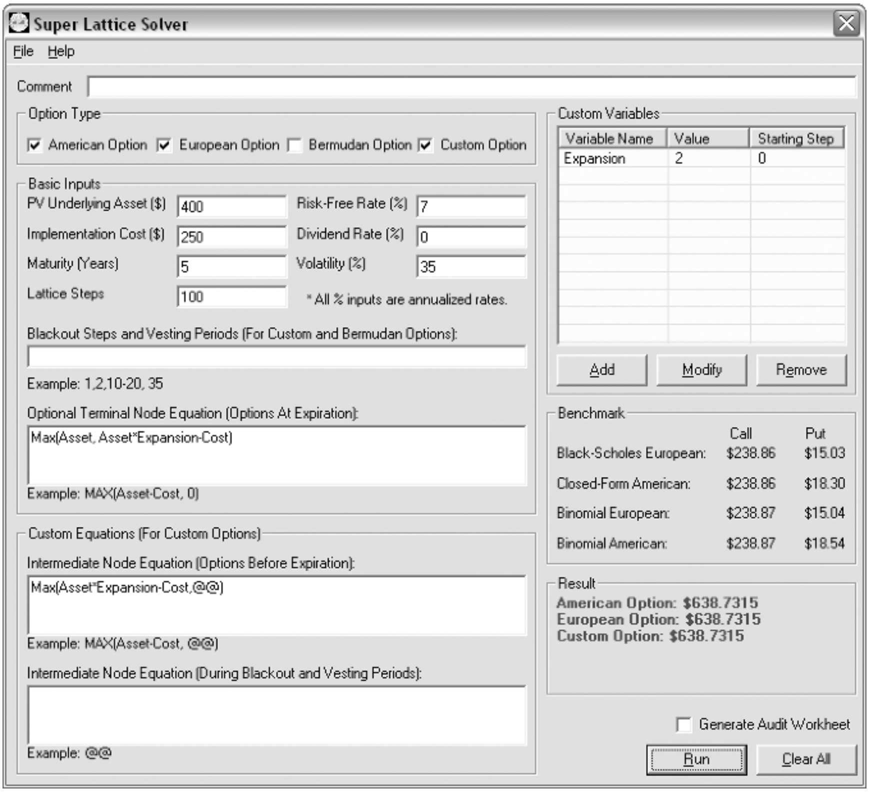


FIGURE 10.11 American and European Options to Expand with a 100-Step Lattice

it might be worth something to wait and see if the uncertainty is too high even if the cost to wait is high.

Other applications of this option simply abound! To illustrate, following are some additional quick examples of the contraction option and some additional sample exercises.

- Suppose a pharmaceutical firm is thinking of developing a new type of insulin that can be inhaled and the drug will be absorbed directly into the blood stream. A novel and honorable idea. Imagine what this means to diabetics who no longer need painful and frequent injections. The problem is, this new type of insulin requires a brand new development effort but if the uncertainties of the market, competition, drug development, and FDA approval are high, perhaps a base insulin drug that can be ingested is first developed. The ingestible version is a required precursor to the inhaled version. The pharmaceutical firm can decide to either take the risk and fast track development

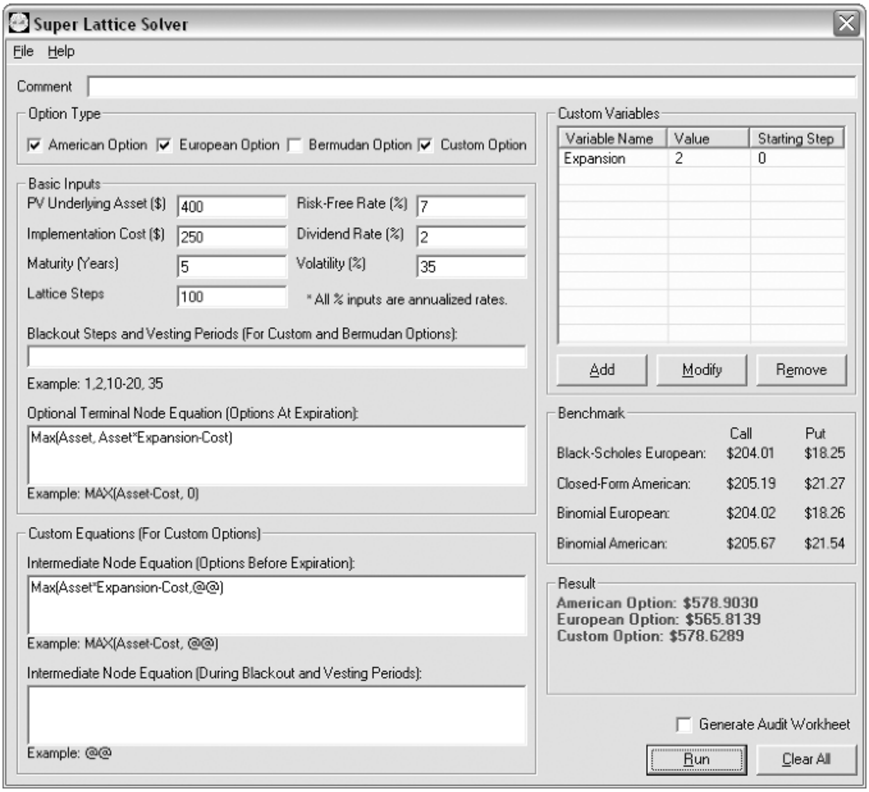


FIGURE 10.12 American and European Options to Expand with a Dividend Rate

into the inhaled version or buy an option to defer, to first wait and see if the ingestible version works. If this precursor works, then the firm has the option to expand into the inhaled version. How much should the firm be willing to spend on performing additional tests on the precursor and under what circumstances should the inhaled version be implemented directly? Suppose the intermediate precursor development work yields an NPV of $100M, but at any time within the next two years, an additional $50M can be further invested into the precursor to develop it into the inhaled version, which will triple the NPV. However, after modeling the risk of technical success and uncertainties in the market (competitive threats, sales, and pricing structure), the annualized volatility of the cash flows using the logarithmic present value returns approach comes to 45 percent. Suppose the risk-free rate is 5 percent for the two-year period. *Using the SLS, the analysis results yields $254.95M, indicating that the option value to wait and defer is worth more than $4.95M after accounting for the $250M NPV if executing now. In playing with several*

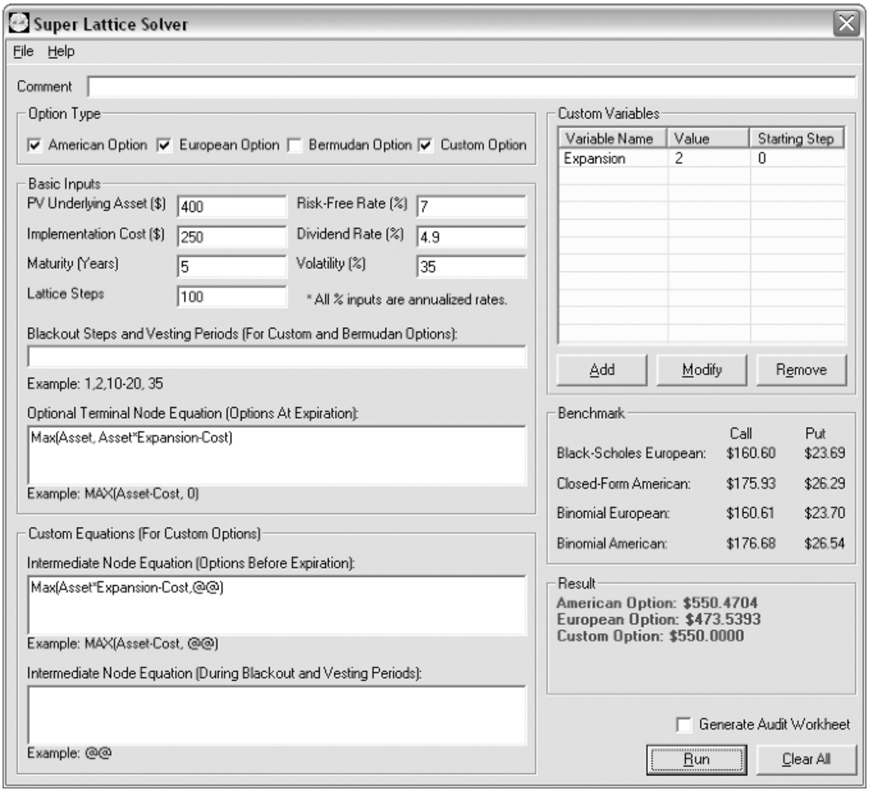


FIGURE 10.13 Dividend Rate Optimal Trigger Value

scenarios, the breakeven point is found when dividend yield is 1.34 percent. This means that if the cost of waiting (lost net revenues in sales by pursuing the smaller market rather than the larger market, and loss of market share by delaying) exceeds $1.34M per year, then it is not optimal to wait and the pharmaceutical firm should engage in the inhaled version immediately. The loss in returns generated each year does not sufficiently cover the risks incurred.

- An oil and gas company is currently deciding on a deep-sea exploration and drilling project. The platform provides an expected NPV of $1,000M. This project is fraught with risks (price of oil and production rate are both uncertain) and the annualized volatility is computed to be 55 percent. The firm is thinking of purchasing an expansion option by spending an additional $10M to build a slightly larger platform that it does not currently need, but if the price of oil is high, or when production rate is low, the firm can execute this expansion option and execute additional drilling to obtain more oil to sell at the higher price, which will cost another $50M, thereby increasing the NPV by 20 percent. The economic life of this platform is 10 years and the risk-free rate for the corresponding term is 5 percent. Is obtaining this slightly larger platform worth it? *Using the SLS, the option value is worth $27.12M when applying a 100-step lattice. Therefore, the option cost of $10M is worth it. However, this expansion option will not be worth it if annual dividends exceed 0.75 percent or $7.5M a year—this is the annual net revenues lost by waiting and not drilling as a percentage of the base case NPV.*

Figure 10.14 shows a Bermudan Expansion Option with certain vesting and blackout steps, while Figure 10.15 shows a Customized Expansion Option to account for the expansion factor changing over time. Of course other flavors of customizing the expansion option exist, including changing the implementation cost to expand, and so forth.

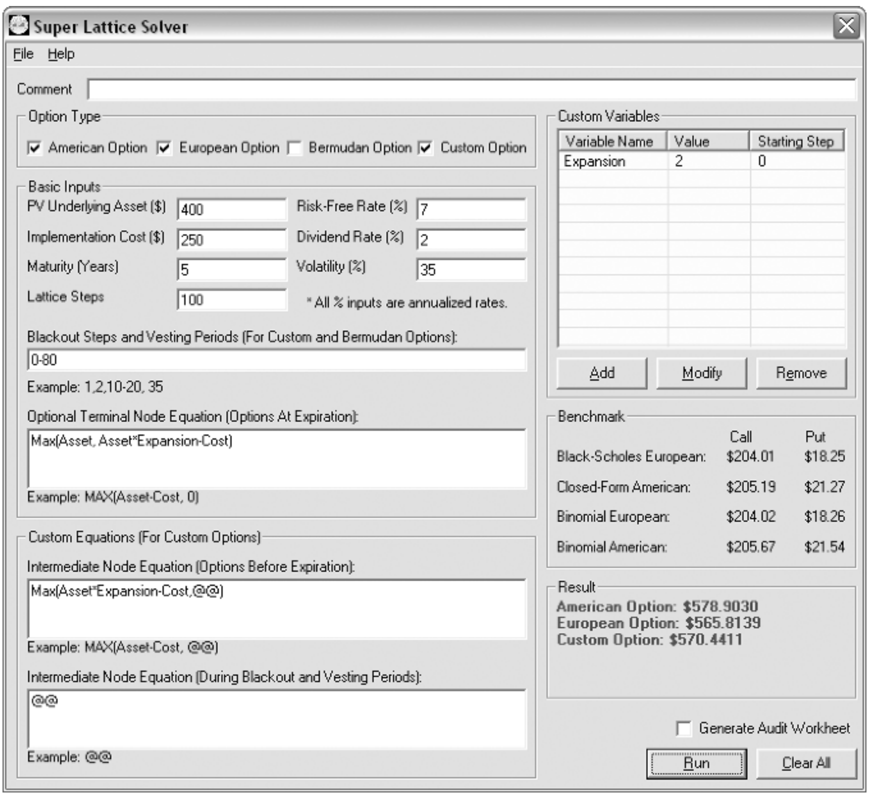


FIGURE 10.14 Bermudan Expansion Option

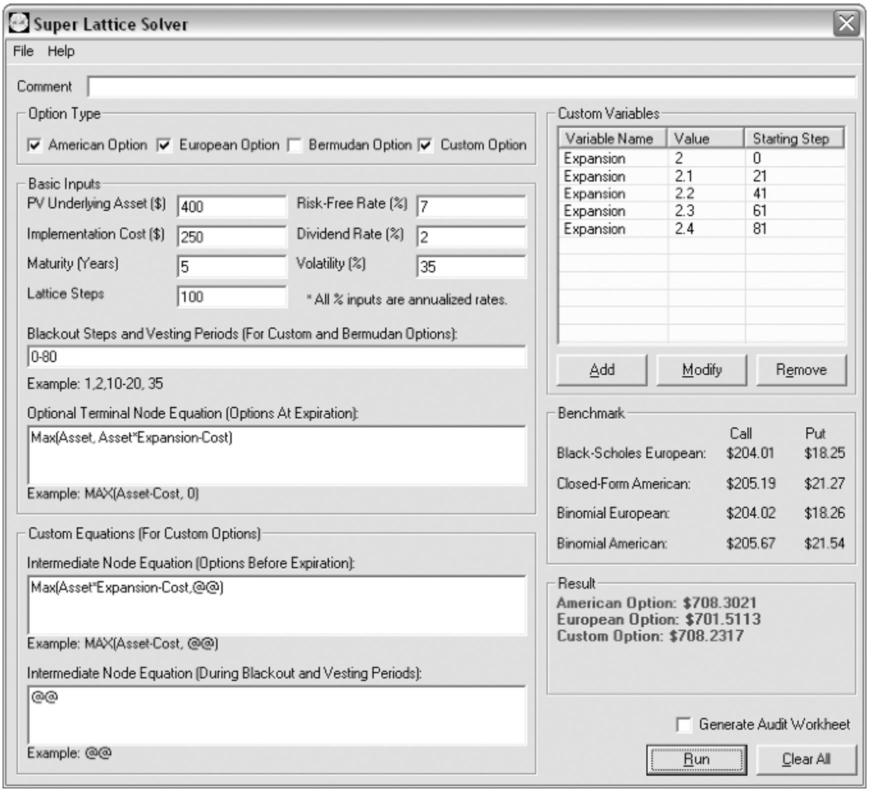


FIGURE 10.15 Customized Expansion Option

Exercise: Option to Expand

Suppose a growth firm has a static valuation of future profitability using a DCF model (in other words, the present value of the expected future cash flows discounted at an appropriate market risk-adjusted discount rate) is found to be $400 million. Using Monte Carlo simulation, you calculate the implied volatility of the logarithmic returns on the projected future cash flows to be 35 percent. The risk-free rate on a riskless asset for the next five years is found to be yielding 7 percent. Suppose that the firm has the option to expand and double its operations by acquiring its competitor for a sum of $250 million at any time over the next five years. What is the total value of this firm assuming you account for this expansion option?

You decide to use a closed-form approximation of an American call option as a benchmark because the option to expand the firm's operations can be exercised at any time up to the expiration date. You also decide to confirm the value of the closed-form analysis with a binomial lattice calculation. Do the following exercises, answering the questions that are posed:

1. Solve the expansion option problem manually using a 10-step lattice and confirm the results by generating an audit sheet using the software.
2. Rerun the expansion option problem using the software for 100 steps, 300 steps, and 1,000 steps. What are your observations?
3. Show how you would use the *Closed-Form American Approximation Model* to estimate and benchmark the results from an expansion option. How comparable are the results?
4. Show the different levels of expansion factors but still yielding the same expanded asset value of $800. Explain your observations of why, when the expansion value changes, the *Black-Scholes* and *Closed-Form American Approximation* models are insufficient to capture the fluctuation in value.
   a. Use an expansion factor of 2.00 and an asset value of $400.00 (yielding an expanded asset value of $800).
   b. Use an expansion factor of 1.25 and an asset value of $640.00 (yielding an expanded asset value of $800).
   c. Use an expansion factor of 1.50 and an asset value of $533.34 (yielding an expanded asset value of $800).
   d. Use an expansion factor of 1.75 and an asset value of $457.14 (yielding an expanded asset value of $800).
5. Add a dividend yield and see what happens. Explain your findings.
   a. What happens when the dividend yield equals or exceeds the risk-free rate?
   b. What happens to the accuracy of closed-form solutions like the *Black-Scholes* and *Closed-Form American Approximation Model* models for use as benchmarks?
6. What happens to the decision to expand if a dividend yield exists? Now suppose that although the firm has an annualized volatility of 35 percent, the competitor has a volatility of 45 percent. This means that the expansion factor of this option changes over time, comparable to the volatilities. In addition, suppose the implementation cost is a constant 120 percent of the existing firm's asset value at any point in time. Show how this problem can be solved using the Multiple Asset SLS. Is there option value in such a situation?

CONTRACTION, EXPANSION, AND ABANDONMENT OPTION

The *Contraction, Expansion, and Abandonment Option* applies when a firm has three competing and mutually exclusive options on a single project to choose from at different times up to the time of expiration. Be aware that this is a mutually exclusive set of options. That is, you cannot execute any combinations of expansion, contraction, or abandonment at the same time. Only one option can be executed at any time. That is, for mutually exclusive options, use a single model to compute the option value as seen in Figure 10.16 (example file used: *Expand Contract Abandon American and European Option*). However, if the options are nonmutually exclusive, calculate them individually in different models and add up the values for the total value of the strategy.

Figure 10.17 illustrates a Bermudan Option with the same parameters but with certain blackout periods (example file used: *Expand Contract Abandon Bermudan Option*), while Figure 10.18 (example file used: *Expand Contract Abandon Customized Option I*) illustrates a more complex Custom Option where during some earlier period of vesting, the option to expand does

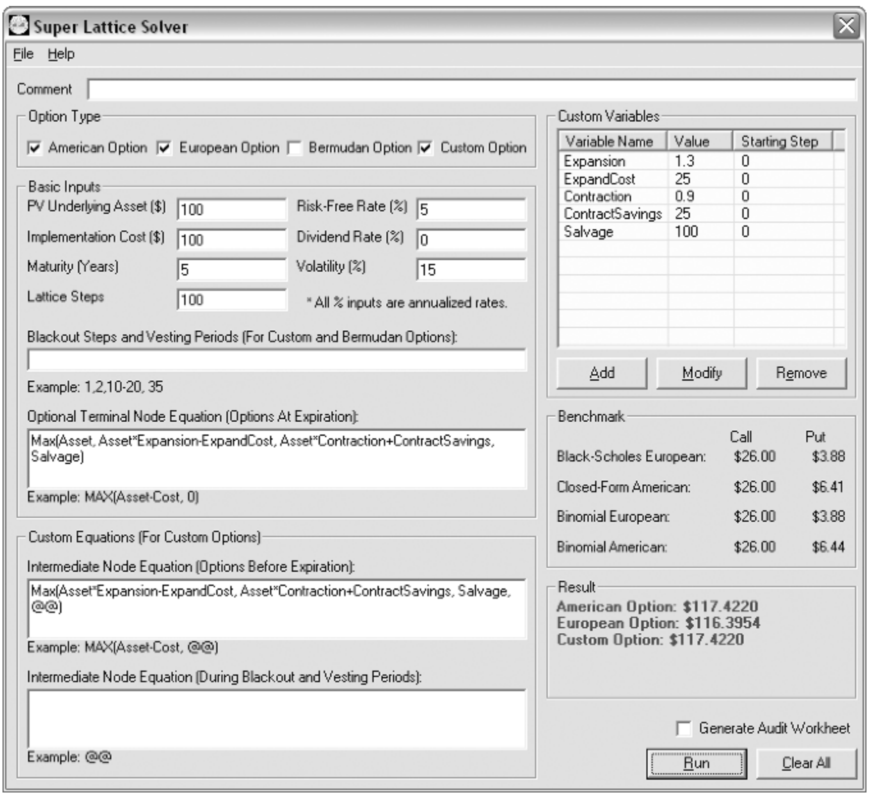


FIGURE 10.16 American, European, and Custom Options to Expand, Contract, and Abandon

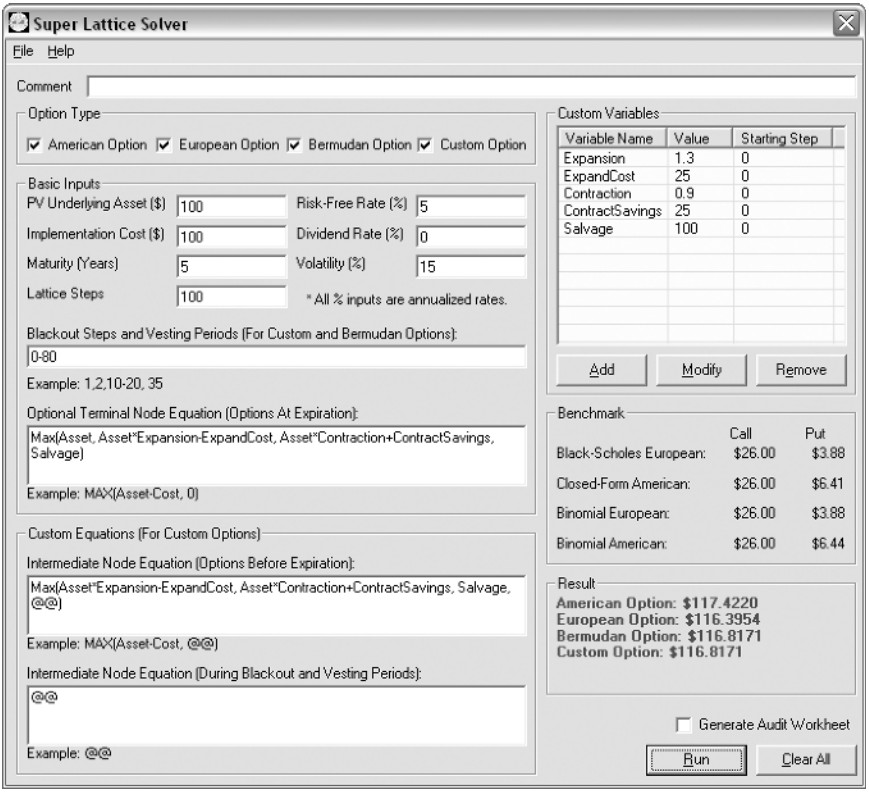


FIGURE 10.17 Bermudan Option to Expand, Contract, and Abandon

not exist yet (perhaps the technology being developed is not yet mature enough in the early stages to be expanded into some spin-off technology). In addition, during the postvesting period but prior to maturity, the option to contract or abandon does not exist (perhaps the technology is now being reviewed for spin-off opportunities), and so forth. Figure 10.19 uses the same example in Figure 10.18 but now the input parameters (salvage value) are allowed to change over time perhaps accounting for the increase in project, asset, or firm value if abandoned at different times (example file used: *Expand Contract Abandon Customized Option II*).

Exercise: Option to Choose—Contraction, Expansion, Abandonment (Dominant Option)

Suppose a large manufacturing firm decides to hedge itself through the use of strategic options. Specifically, it has the option to choose among three

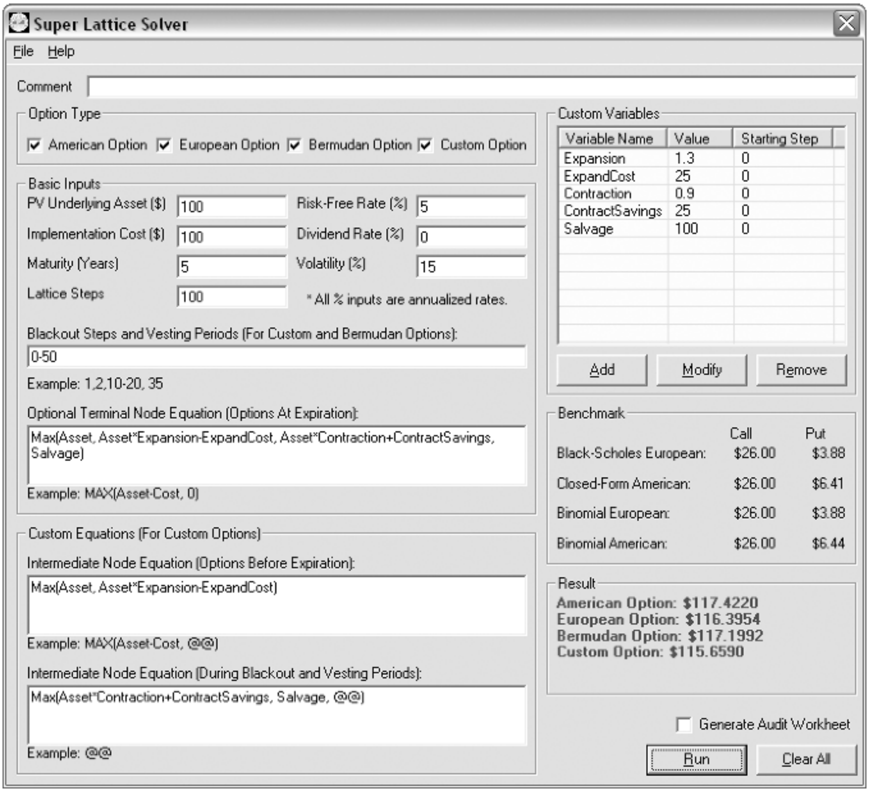


FIGURE 10.18 Custom Options with Mixed Expand, Contract, and Abandon Capabilities

strategies: expanding its current manufacturing operations, contracting its manufacturing operations, or completely abandoning its business unit at any time within the next five years. Suppose the firm has a current operating structure whose static valuation of future profitability using a DCF model (in other words, the present value of the future cash flows discounted at an appropriate market risk-adjusted discount rate) is found to be $100 million.

Using Monte Carlo simulation, you calculate the implied volatility of the logarithmic returns on the projected future cash flows to be 15 percent. The risk-free rate on a riskless asset for the next five years is found to be yielding 5 percent annualized returns. Suppose the firm has the option to contract 10 percent of its current operations at any time over the next five years, thereby creating an additional $25 million in savings after this contraction. The expansion option will increase the firm's operations by 30 percent, with a $20 million implementation cost. Finally, by abandoning its operations,

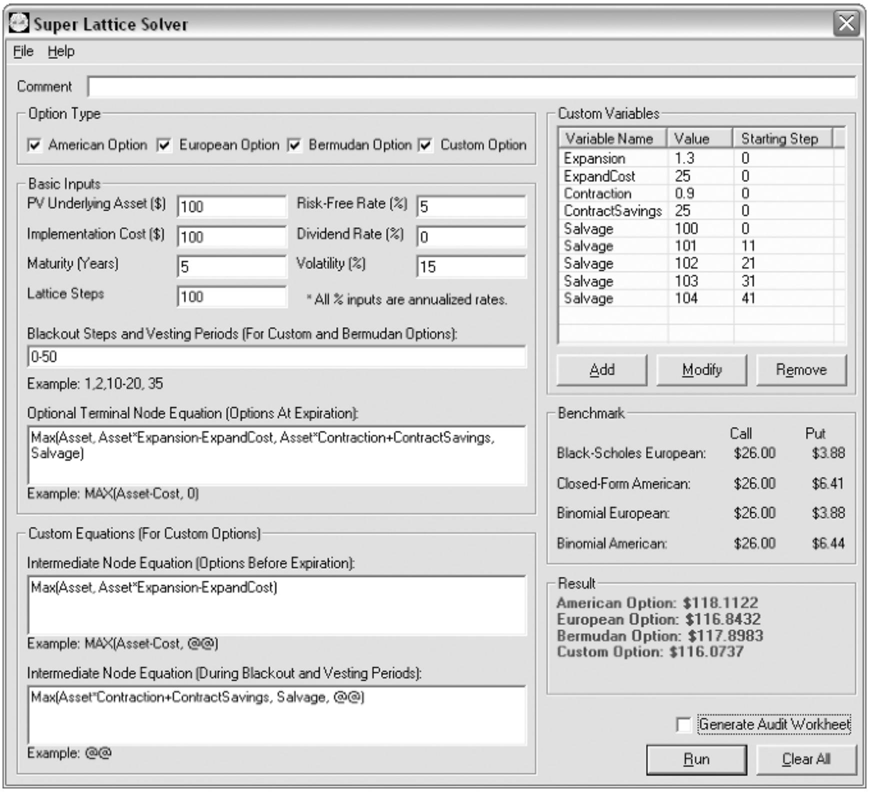


FIGURE 10.19 Custom Options with Mixed Expand, Contract, and Abandon Capabilities with Changing Input Parameters

the firm can sell its intellectual property for $100 million. Do the following exercises, answering the questions that are posed:

1. Solve the chooser option problem manually using a 10-step lattice and confirm the results by generating an audit sheet using the SLS software.
2. Recalculate the option value in question 1 accounting only for an expansion option.
3. Recalculate the option value in question 1 accounting only for a contraction option.
4. Recalculate the option value in question 1 accounting only for an abandonment option.
5. Compare the results of the sum of these three individual options in questions 2 to 4 with the results obtained in question 1 using the chooser option.

   a. Why are the results different?
   b. Which value is correct?
6. Prove that if there are many interacting options, if there is a single dominant strategy, the value of the project's option value approaches this dominant strategy's value. That is, perform the following steps, then compare and explain the results.
   a. Reduce the expansion cost to $1.
   b. Increase the contraction savings to $100.
   c. Increase the salvage value to $150.
   d. What inferences can you make based on these results?
7. Solve the following Contraction and Abandonment option: Asset value of $100, five-year economic life, 5 percent annualized risk-free rate of return, 25 percent annualized volatility, 25 percent contraction with a $25 savings, and a $70 abandonment salvage value.
8. Show and explain what happens when the salvage value of abandonment far exceeds any chances of a contraction. For example, set the salvage value at $200.
9. In contrast, set the salvage value back to $70, and increase the contraction savings to $100. What happens to the value of the project?
10. Solve just the contraction option in isolation. That is, set the contraction savings to $25 and explain what happens. Change the savings to $100 and explain the change in results. What can you infer from dominant option strategies? Solve just the abandonment option in isolation. That is, set the salvage value to $70, and explain what happens. Change the salvage value to $200, and explain the change in results. What can you infer from dominant option strategies?

BASIC AMERICAN, EUROPEAN, AND BERMUDAN CALL OPTIONS

Figure 10.20 shows the computation of basic American, European, and Bermudan Options without dividends (example file used: *Basic American, European, versus Bermudan Call Options*), while Figure 10.21 shows the computation of the same options but with a dividend yield. Of course, European Options can only be executed at termination and not before, while in American Options, early exercise is allowed, versus a Bermudan Option where early exercise is allowed except during blackout or vesting periods. Notice that the results for the three options without dividends are identical for simple call options, but they differ when dividends exist. When dividends are included, the simple call option values for American ≥ Bermudan ≥ European in most basic cases, as seen in Figure 10.21. Of course this generality can be applied only to plain-vanilla call options and do not necessarily apply to other

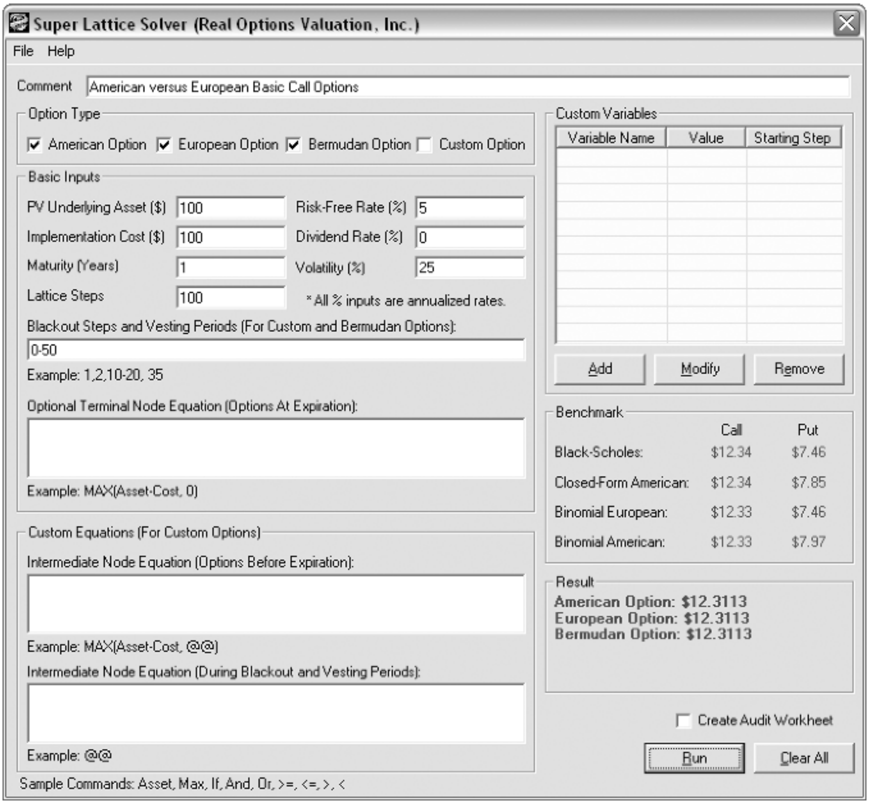


FIGURE 10.20 Simple American, Bermudan, and European Options without Dividends

exotic options (e.g., Bermudan options with vesting and suboptimal exercise behavior multiples tend to sometimes carry a higher value when blackouts and vesting occur than regular American options with the same suboptimal exercise parameters).

BASIC AMERICAN, EUROPEAN, AND BERMUDAN PUT OPTIONS

The *American and European Put Options* without dividends are calculated using the SLS in Figure 10.22. The sample results of this calculation indicate the strategic value of the project's NPV and provide an option to sell the project within the specified *Maturity* in years. There is a chance that the project value can significantly exceed the single-point estimate of *PV Asset Value*

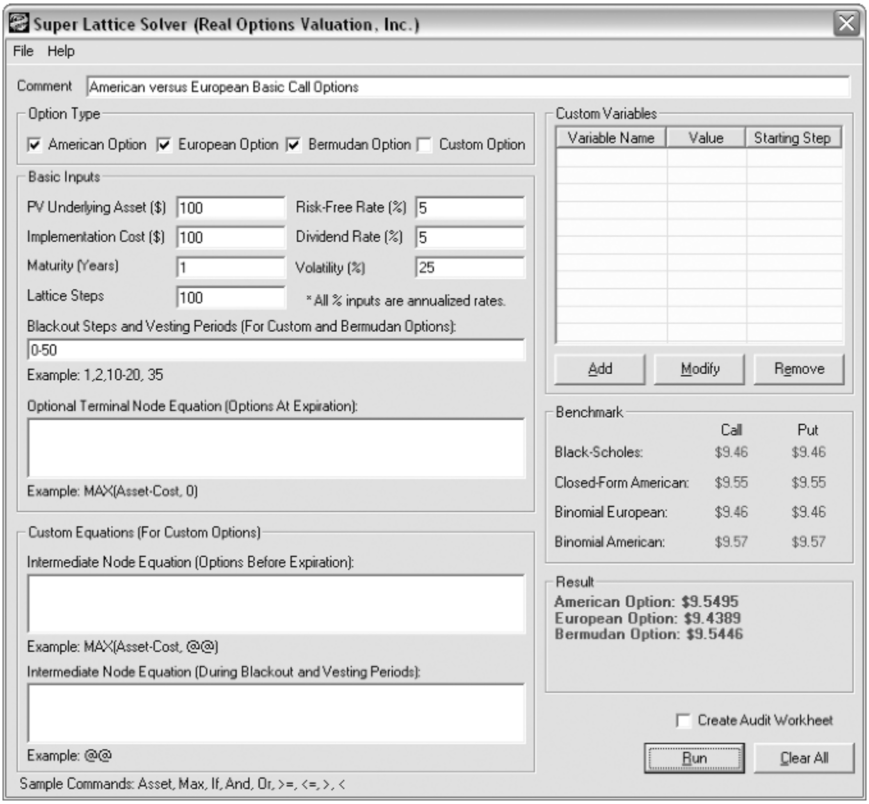


FIGURE 10.21 Simple American, Bermudan, and European Options with Dividends

(measured by the present value of all uncertain future cash flows discounted at the risk-adjusted rate of return) or be significantly below it. Hence, the option to *defer* and *wait* until some of the uncertainty becomes resolved through the passage of time is worth more than executing immediately. The value of being able to wait before executing the option and selling the project at the *Implementation Cost* in present values is the value of the option. The NPV of executing immediately is simply the *Implementation Cost* less the *Asset Value* ($0). The option value of being able to wait and defer selling the asset only if the condition goes bad and becomes optimal is the difference between the calculated result (total strategic value) and the NPV or $24.42 for the American option and $20.68 for the European option. The American put option is worth more than the European put option even when no dividends exist, contrary to the call options seen previously. For simple call options, when no dividends exist, it is never optimal to exercise early. However, it may sometimes be optimal to exercise early for put options, regardless of whether dividend yields exist. In fact, a dividend yield will

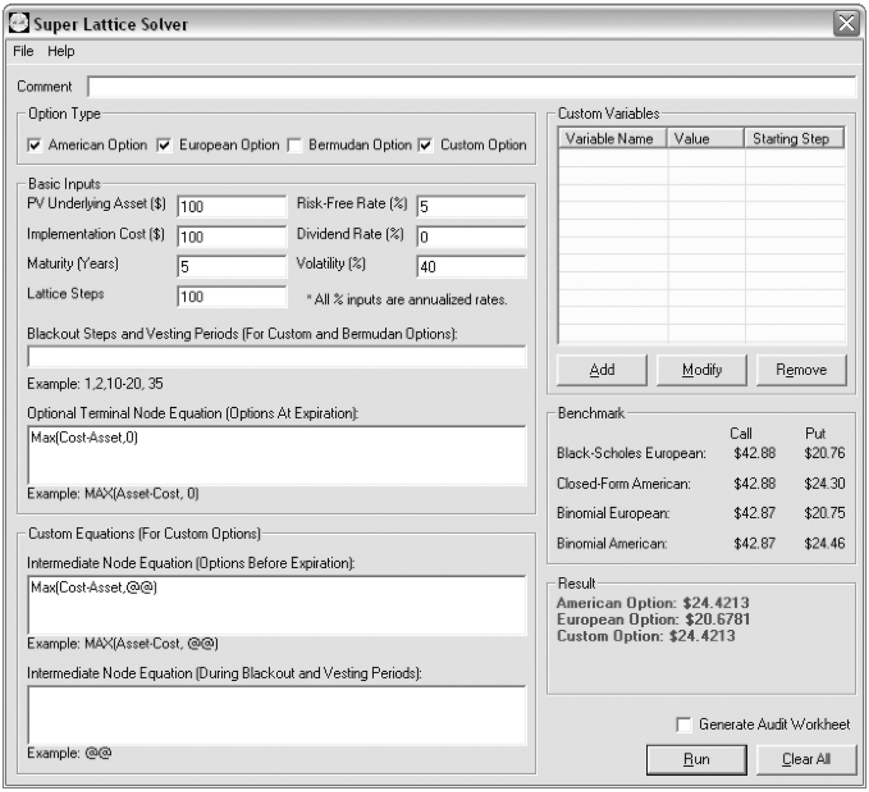


FIGURE 10.22 American and European Put Options

decrease the value of a call option but increase the value of a put option because when dividends are paid out, the value of the asset decreases. Thus, the call option will be worth less and the put option will be worth more. The higher the dividend yield, the earlier the call option should be exercised and the later the put option should be exercised.

The put option can be solved by setting the Terminal Node Equation as *Max(Cost–Asset,0)* as seen in Figure 10.22 (example file used: *Plain Vanilla Put Option*).

Puts have a similar result as calls in that when dividends are included, the basic put option values for American ≥ Bermudan ≥ European in most basic cases. You can confirm this by simply setting the Dividend Rate at 3 percent and Blackout Steps at 0–80 and rerunning the SLS module.

Recall that a higher dividend means a higher put option value but a lower abandonment option value. Thus, a put option is not exactly identical to an abandonment option. A high dividend means the abandonment option's cost of waiting and holding on to the option is high (e.g., not selling a piece of land means having to pay the taxes, insurance, and maintenance on it, reducing the value of waiting before abandoning.) However, for a put option this means that the asset price or value in the market decreases because of this dividend, making the put option (the ability to sell the asset at a predetermined contractual price) worth more. Dividend in the abandonment option affects the holding cost of the option, but dividends in the put option affect the underlying asset value. Therefore, care has to be taken to choose the relevant option model to use.

Exercise: American, Bermudan, and European Options

This set of exercises allows us to compare the results of a European option with American and Bermudan options. In addition, the Black-Scholes model is used to compare the results. The Black-Scholes equation is applicable for analyzing European-type options—that is, options that can be executed only at maturity and not before. The original Black-Scholes model cannot solve an option problem when there are dividend payments. However, extensions of the Black-Scholes model, termed the Generalized Black-Scholes model, can accommodate a continuous dividend payout for a European Option.

Do the following exercises, answering the questions that are posed, assuming that a call option's asset value and strike price are both \$100, subject to 25 percent volatility. The maturity on this option is five years, and the corresponding risk-free rate on a similar asset maturity is 5 percent. Finally, for the Bermudan option, assume a four-year vesting period.

1. Using the Single Asset SLS software, calculate the American, European, and Bermudan call options using a 100-step lattice.
2. Compare your results using 5, 10, 50, 100, 300, 500, and 1,000 steps in the SLS software. Explain what happens when the number of steps gets higher.
3. Now assume that a continuous dividend payout yielding 3 percent exists. What happens to the value of the option?
4. Show that the value of an American call option is identical to the European call option when no dividends are paid. That is, it is never optimal to execute an American call option early when no dividend payouts exist. Now consider the Bermudan option. What generalities can you come up with?
5. Repeat question 4 on the put option. Here you will see a very different set of results. What generalities can you come up with?
6. Show that as a 3 percent dividend yield exists, the value of the American call option exceeds the value of a European option. Why is this so? How does the Bermudan option compare? What happens when the

blackout steps are significant in the Bermudan option (e.g., increase the vesting period from 4 to 4.5 years and then to 5 years)? What happens when the blackout vesting period is identical to the maturity of the option? How do you model this in the software?

Repeat question 6 on the put option. Here you will see a very different set of results. What generalities can you come up with?

EXOTIC CHOOSER OPTIONS

Many types of user-defined and exotic options can be solved using the SLS and MSLS. For instance, Figure 10.23 shows a simple Exotic Chooser Option (example file used: *Exotic Chooser Option*). In this simple analysis, the option holder has two options, a call and a put. Instead of having to purchase

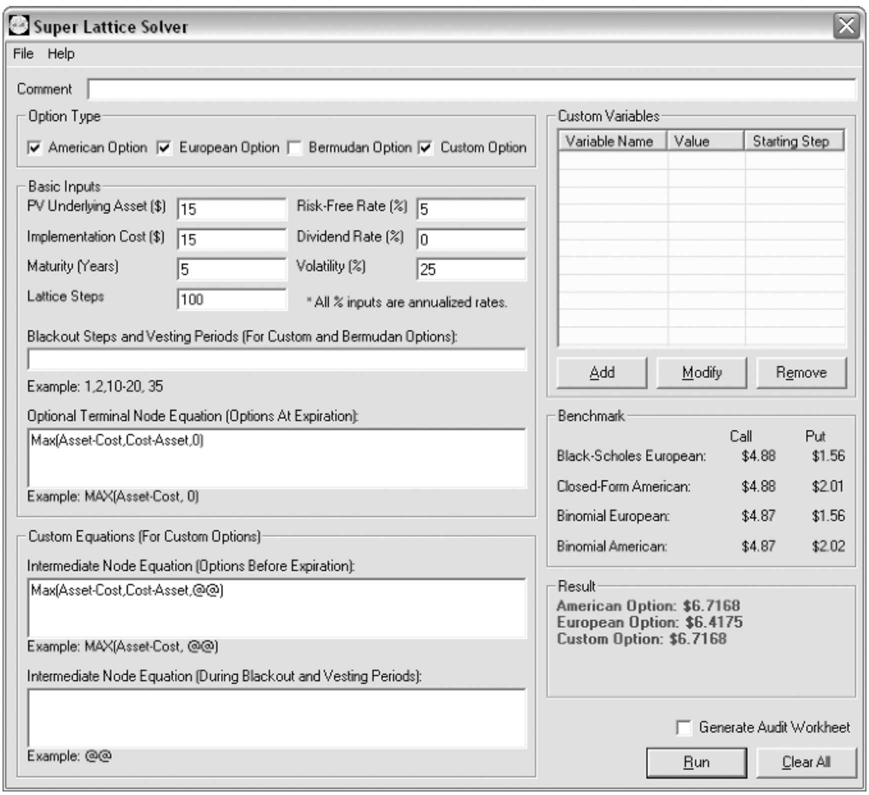


FIGURE 10.23 American and European Exotic Chooser Option Using SLS

or obtain two separate options, one single option is obtained, which allows the option holder to choose whether the option will be a call or a put, thereby reducing the total cost of obtaining two separate options. For instance, with the same input parameters in Figure 10.23, the American Chooser Option is worth $6.7168, as compared to $4.87 for the call and $2.02 for the put ($6.89 total cost for two separate options).

A more complex Chooser Option can be constructed using the MSLS as seen in Figure 10.24 (example file used: *MSLS—Exotic Complex Floating European Chooser*) and Figure 10.25 (example file used: *MSLS—Exotic Complex Floating American Chooser*). In these examples, the execution costs of the call versus put are set at different levels. An interesting example of a Complex Chooser Option is a firm developing a highly uncertain and risky new technology. The firm tries to hedge its downside as well as capitalize its upside by creating a Chooser Option. That is, the firm can decide to build the technology itself once the R&D phase is complete versus selling the intellectual property of the technology, both at different costs. To further complicate matters, you can use the MSLS to easily and quickly solve the situation where building versus selling off the option each has a different volatility and time to choose.

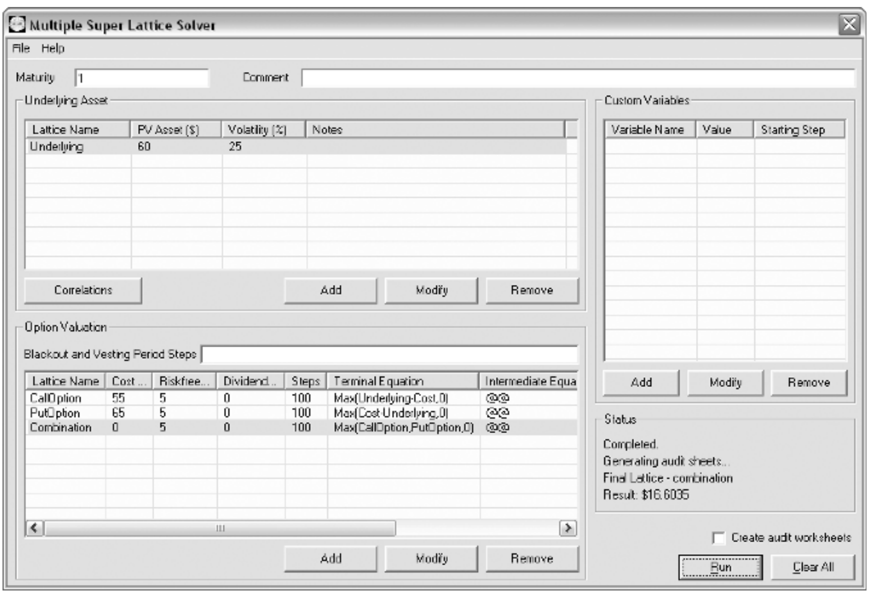


FIGURE 10.24 Complex European Exotic Chooser Option Using MSLS

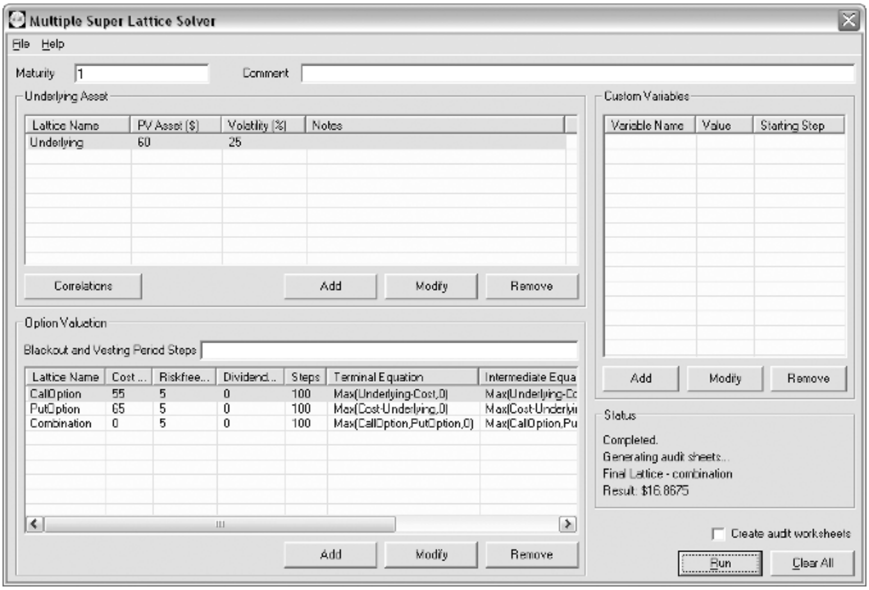


FIGURE 10.25 Complex American Exotic Chooser Option Using MSLS

SEQUENTIAL COMPOUND OPTIONS

Sequential Compound Options are applicable for research and development investments or any other investments that have multiple stages. The MSLS is required for solving Sequential Compound Options. The easiest way to understand this option is to start with a two-phased example as seen in Figure 10.26. In the two-phased example, management has the ability to decide if Phase II (PII) should be implemented after obtaining the results from Phase I (PI). For example, a pilot project or market research in PI indicates that the market is not yet ready for the product, hence PII is not implemented. All that is lost is the PI sunk cost, not the entire investment cost of both PI and PII. The following example illustrates how the option is analyzed.

The illustration in Figure 10.26 is valuable in explaining and communicating to senior management the aspects of an American Sequential Compound Option and its inner workings. In the illustration, the Phase I investment of –\$5M (in present value dollars) in Year 1 is followed by Phase II investment of –\$80M (in present value dollars) in Year 2. Hopefully, positive net free cash flows (CF) will follow in Years 3 to 6, yielding a sum of *PV Asset* of \$100M (CF discounted at, say, a 9.7 percent discount or hurdle rate), and the *Volatility* of these CFs is 30 percent. At a 5 percent risk-free rate, the strategic value is calculated at \$27.67 as seen in Figure 10.27 using a 100-step

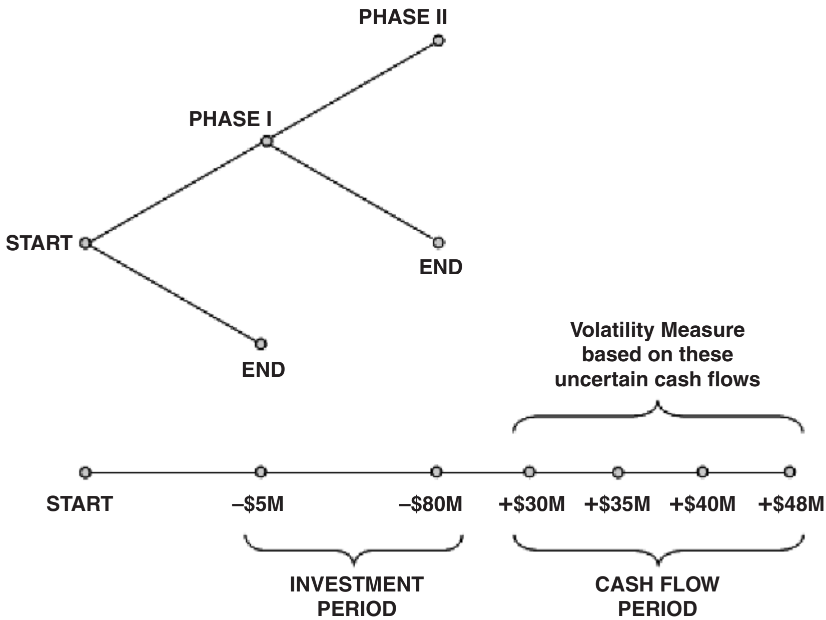


FIGURE 10.26 Graphical Representation of a Two-Phased Sequential Compound Option

lattice, which means that the strategic option value of being able to defer investments and to wait and see until more information becomes available and uncertainties become resolved is worth $12.67M because the NPV is worth $15M ($100M – $5M – $85M). In other words, the Expected Value of Perfect Information is worth $12.67M, which indicates that assuming market research can be used to obtain credible information to decide if this project is a good one, the maximum the firm should be willing to spend in Phase I is *on average no more than* $17.67M (i.e., $12.67M + $5M) if PI is part of the market research initiative, or simply $12.67M otherwise. If the cost to obtain the credible information exceeds this value, then it is optimal to take the risk and execute the entire project immediately at $85M. The example file used is: *MSLS—Simple Two Phased Sequential Compound Option*.

In contrast, if the volatility decreases (uncertainty and risk are lower), the strategic option value decreases. In addition, when the cost of waiting (as described by the *Dividend Rate* as a percentage of the *Asset Value*) increases, it is better not to defer and wait that long. Therefore, the higher the dividend rate, the lower the strategic option value. For instance, at an 8 percent dividend rate and 15 percent volatility, the resulting value reverts to the NPV of $15M, which means that the option value is zero, and that it is better to execute immediately as the cost of waiting far outstrips the value of being able to wait given the level of volatility (uncertainty and risk). Finally, if risks and uncertainty increase significantly, even with a high cost of waiting (e.g., 7 percent dividend rate at 30 percent volatility) it is still valuable to wait.

This model provides the decision maker with a view into the optimal balancing between waiting for more information (Expected Value of Perfect Information) and the cost of waiting. You can analyze this balance by creating strategic options to defer investments through development stages where at every stage the project is reevaluated as to whether it is beneficial to proceed to the next phase. Based on the input assumptions used in this model, the *Sequential Compound Option* results show the strategic value of the project, and the NPV is simply the PV Asset less both phases' Implementation Costs. In other words, the strategic option value is the difference between the calculated strategic value minus the NPV. It is recommended for your consideration that the volatility and dividend inputs are varied to determine their interactions—specifically, where the break-even points are for different combinations of volatilities and dividends. Thus, using this information, you can make better go or no-go decisions (for instance, breakeven volatility points can be traced back into the discounted cash flow model to estimate the probability of crossing over and this ability to wait becomes valuable).

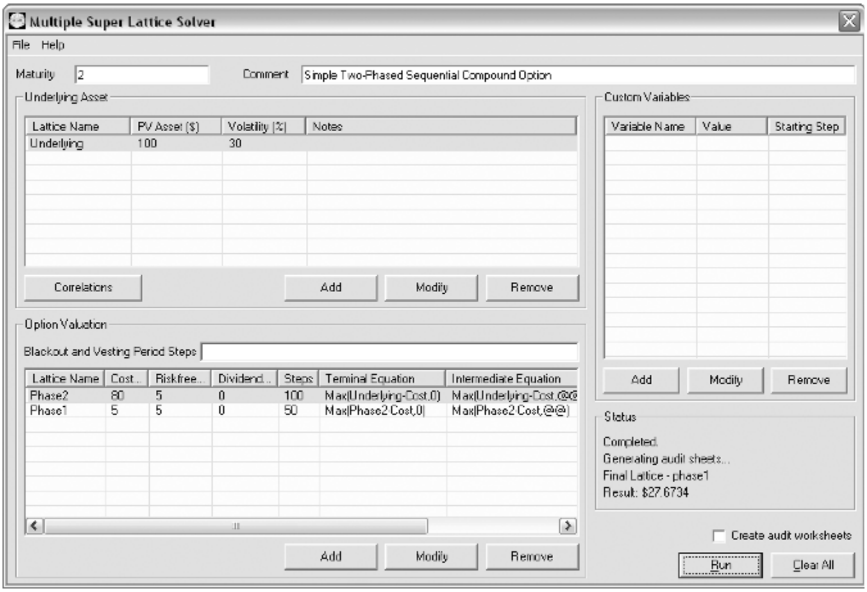


FIGURE 10.27 Solving a Two-Phased Sequential Compound Option Using MSLS

Note that the investment costs of \$80M and \$5M are in present values for several reasons. The first is that only the PV Asset undergoes an underlying asset lattice evolution to future values, while costs do not undergo such an evolution, and are hence in present values. Also, for example, in a five-year option, an implementation that can occur at any time within these five years will have very different actual implementation costs but to make these costs comparable, we use their present values. Thus, an \$80M cost can occur at any time within some specified time period and the present value of the cost is still \$80M regardless of when it hits. Finally, the risk-free rate discounts the expected value of keeping the option open, not the cost. So, there is no double-discounting and using the present value for these costs is hence correct.

MULTIPLE-PHASED SEQUENTIAL COMPOUND OPTION

The Sequential Compound Option can similarly be extended to multiple phases with the use of MSLS. A graphical representation of a multiphased or stage-gate investment is seen in Figure 10.28. The example illustrates a ten-phase project, where, at every phase, management has the option and flexibility to either continue to the next phase if everything goes well, or to terminate the project otherwise. Based on the input assumptions, the results in the MSLS indicate the calculated strategic value of the project, while the

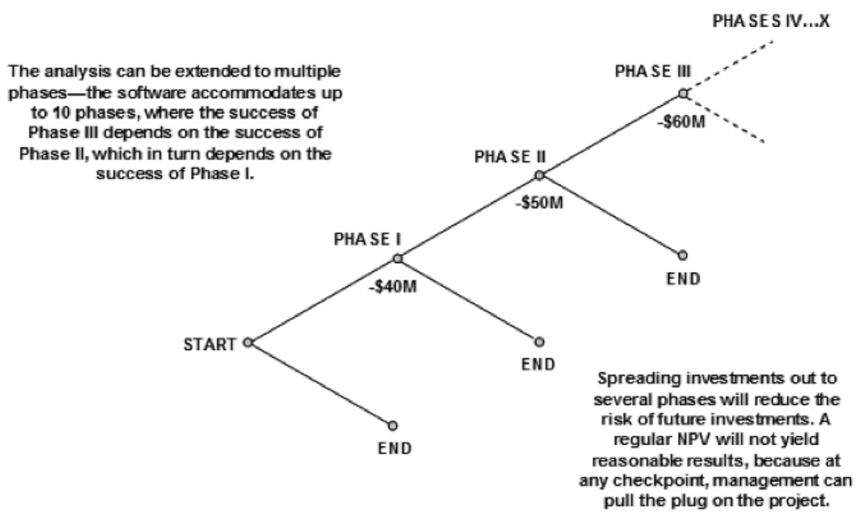


FIGURE 10.28 Graphical Representation of a Multiphased Sequential Compound Option

NPV of the project is simply the *PV Asset* less all *Implementation Costs* (in present values) if implementing all phases immediately. Therefore, with the strategic option value of being able to defer and wait before implementing future phases because due to the volatility, there is a possibility that the asset value will be significantly higher. Hence, the ability to wait before making the investment decisions in the future is the option value or the strategic value of the project less the NPV.

Figure 10.29 shows the results using the MSLS. Notice that due to the backward induction process used, the analytical convention is to start with the last phase and go all the way back to the first phase (example file used: *MSLS—Multiple Phased Sequential Compound Option*). In NPV terms the project is worth –\$500. However, the total strategic value of the stage-gate investment option is worth \$41.78. This means that although on an NPV basis the investment looks bad, in reality, by hedging the risks and uncertainties through sequential investments, the option holder can pull out at any time and not have to keep investing unless things look promising. If after the first phase things look bad, pull out and stop investing and the maximum loss will be \$100 (Figure 10.29) and not the entire \$1,500 investment. If, however, things look promising, the option holder can continue to invest in stages. The expected value of the investments in present values after accounting for the probabilities that things will look bad (and hence stop investing) versus

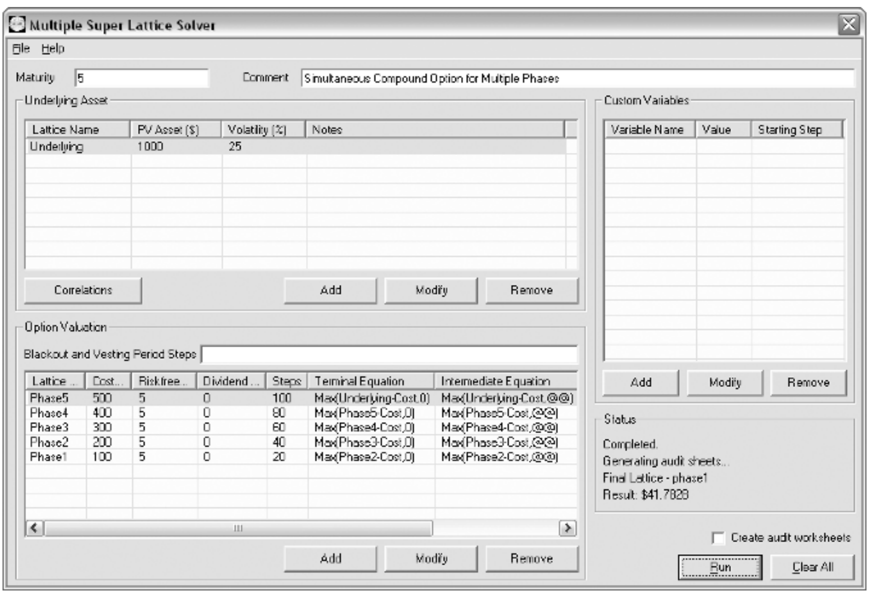


FIGURE 10.29 Solving a Multiphased Sequential Compound Option Using MSLS

things looking great (and hence continuing to invest) is worth an average of \$41.78M.

Notice that the option valuation result will always be greater than or equal to zero (e.g., try reducing the volatility to 5 percent and increasing the dividend yield to 8 percent for all phases). When the option value is very low or zero, this means that it is not optimal to defer investments and that this stage-gate investment process is not optimal here. The cost of waiting is too high (high dividend) or the uncertainties in the cash flows are low (low volatility), hence, invest if the NPV is positive. In such a case, although you obtain a zero value for the option, the analytical interpretation is significant! A zero or very low value is indicative of an optimal decision not to wait.

CUSTOMIZED SEQUENTIAL COMPOUND OPTIONS

The Sequential Compound Option can be further complicated by adding customized options at each phase as illustrated in Figure 10.30, where at every phase, there may be different combinations of mutually exclusive options including the flexibility to stop investing, abandon and salvage the project in return for some value, expand the scope of the project into another project (e.g., spin off projects and expand into different geographical locations), contract the scope of the project resulting in some savings, or continue

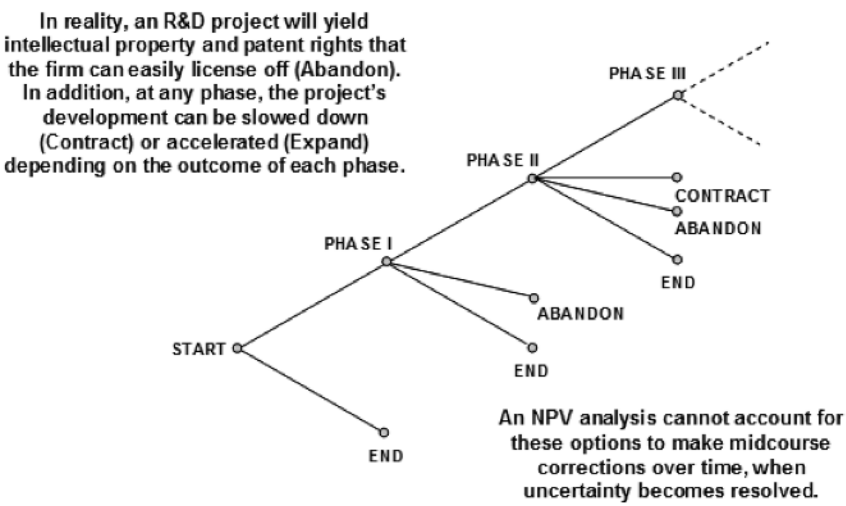


FIGURE 10.30 Graphical Representation of a Complex Multiphased Sequential Compound Option

on to the next phase. The seemingly complicated option can be very easily solved using MSLS as seen in Figure 10.31 (example file used: *MSLS—Multiple Phased Complex Sequential Compound Option*).

To illustrate, Figure 10.31's MSLS path-dependent sequential option uses the following inputs:

Phase 3: Terminal: Max(Underlying*Expansion-Cost,Underlying, Salvage)
Intermediate: Max(Underlying*Expansion-Cost, Salvage,@@)
Steps: 50

Phase 2: Terminal: Max(Phase3,Phase3*Contract+Savings, Salvage,0)
Intermediate: Max(Phase3*Contract+Savings,Salvage,@@)
Steps: 30

Phase 1: Terminal: Max(Phase2,Salvage,0)
Intermediate: Max(Salvage,@@)
Steps: 10

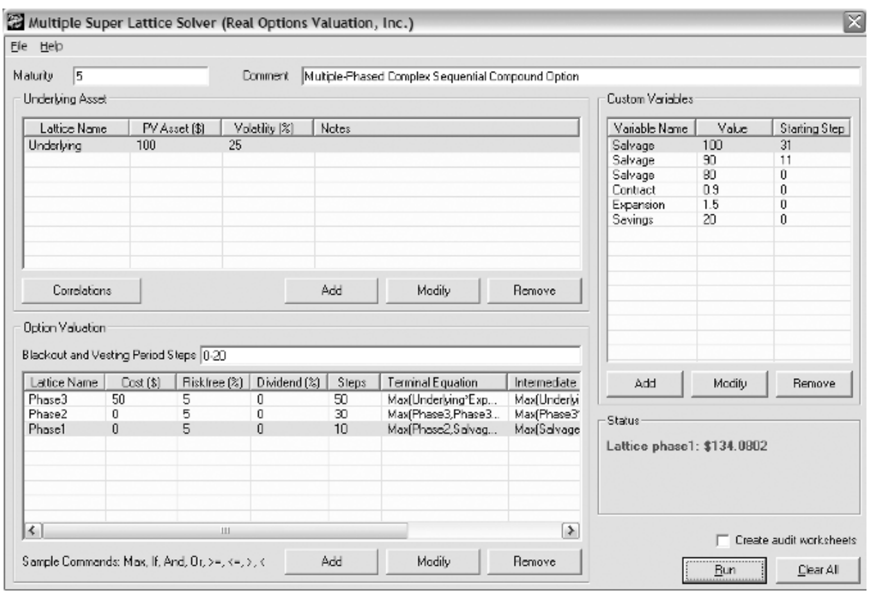


FIGURE 10.31 Solving a Complex Multiphased Sequential Compound Option Using MSLS

Exercise: Sequential Compound Option

A sequential compound option exists when a project has multiple phases and latter phases depend on the success of previous phases. Suppose a project has two phases, of which the first has a one-year expiration that costs $500 million. The second phase's expiration is three years and costs $700 million. Suppose that the implied volatility of the logarithmic returns on the projected future cash flows is calculated to be 20 percent. The risk-free rate on a riskless asset for the next three years is found to be yielding 7.7 percent. The static valuation of future profitability using a discounted cash flow model—in other words, the present value of the future cash flows discounted at an appropriate market risk-adjusted discount rate—is found to be $1,000 million. Do the following exercises, answering the questions that are posed:

1. Solve the sequential compound option problem manually using a 10-step lattice and confirm the results by generating an audit sheet using the software.
2. Change the sequence of the costs. That is, set the first phase's cost to $700 and the second phase's cost to $500 (both in present values). Compare your results. Explain what happens.
3. Now suppose the total cost of $1,200 (in present values) is spread out equally over six phases in six years. How would the analysis results differ?
4. Suppose that the project is divided into three phases with the following options in each phase. Solve the option using MSLS.
   a. Phase 3 occurs in three years, the implementation cost is $300, but the project can be abandoned and salvaged to receive $300 at anytime within the first year, $350 within the second year, and $400 within the third year.
   b. Phase 2 occurs in two years, and the project can be spun off to a partner. In doing this, the partner keeps 15 percent of the NPV and saves the firm $200.
   c. Phase 1 occurs in one year, and the only thing that can be done is to invest the $300 implementation cost and continue to the next phase.

PATH-DEPENDENT, PATH-INDEPENDENT, MUTUALLY EXCLUSIVE, NONMUTUALLY EXCLUSIVE, AND COMPLEX COMBINATORIAL NESTED OPTIONS

Sequential Compound Options are *path-dependent options*, where one phase depends on the success of another, in contrast to *path-independent options* such as those solved using SLS. Figure 10.31 shows that in a complex strategy tree, at certain phases, different combinations of options exist. These options can be *mutually exclusive* or *nonmutually exclusive*. In all these types of options, there might be multiple underlying assets (e.g., Japan has a different risk-return or profitability-volatility profile than the United Kingdom or Australia). You can build multiple underlying asset lattices this way using the MSLS, and combine them in many various ways depending on the options. The following are some examples of path-dependent versus path-independent and mutually exclusive versus nonmutually exclusive options.

- ***Path-Independent and Mutually Exclusive Options:*** Use the SLS to solve these types of options by combining all the options into a single valuation lattice. Examples include the option to expand, contract, and expand. These are mutually exclusive if you cannot both expand into a different country while abandoning and selling the company. These are path independent if there are no restrictions on timing, that is, you can expand, contract, and abandon at any time within the confines of the maturity period.
- ***Path-Independent and Nonmutually Exclusive Options:*** Use the SLS to solve these types of options by running each of the options that are nonmutually exclusive one at a time in SLS. Examples include the option to expand your business into Japan, United Kingdom, and Australia. These are not mutually exclusive if you can choose to expand to any combinations of countries (e.g., Japan only, Japan and United Kingdom, United Kingdom and Australia, and so forth). These are path independent if there are no restrictions on timing, that is, you can expand to any country at any time within the maturity of the option. Add the individual option values and obtain the total option value for expansion.
- ***Path-Dependent and Mutually Exclusive Options:*** Use the MSLS to solve these types of options by combining all the options into one valuation lattice. Examples include the option to expand into the three countries, Japan, United Kingdom, and Australia. However, this time the expansions are mutually exclusive and path dependent. That is, you can only expand into one country at a time, but at certain periods, you can only expand into certain countries (e.g., Japan is only optimal in three years due to current economic conditions, export restrictions, and so forth, as compared to the U.K. expansion, which can be executed right now).
- ***Path-Dependent and Nonmutually Exclusive Options:*** Use MSLS to solve these. These are typically simple Sequential Compound Options with multiple phases. If more than one nonmutually exclusive option exists, rerun the MSLS for each option. Examples include the ability to enter Japan from Years 0 to 3, Australia in Years 3 to 6, and United Kingdom at any time between Years 0 and 10. Each entry strategy is not mutually exclusive if you can enter more than one country and are path dependent as they are time dependent.

- *Nested Combinatorial Options:* These are the most complicated and can take a combination of any of the foregoing four types. In addition, the options are nested within one another in that the expansion into Japan must come only after Australia, and cannot be executed without heading to Australia first. In addition, Australia and United Kingdom are okay but you cannot expand to United Kingdom and Japan (e.g., certain trade restrictions, antitrust issues, competitive considerations, strategic issues, restrictive agreements with alliances). For such options, draw all the scenarios on a strategy tree and use IF, AND, OR, and MAX statements in MSLS to solve the option. That is, if you enter into United Kingdom, that's it, but *if* you enter into Australia, you can still enter into Japan *or* United Kingdom but *not* Japan and United Kingdom.

SIMULTANEOUS COMPOUND OPTION

The Simultaneous Compound Option evaluates a project's strategic value when the value of the project depends on the success of two or more investment initiatives executed simultaneously in time. The Sequential Compound Option evaluates these investments in stages, one after another over time, while the simultaneous option evaluates these options in concurrence. Clearly, the sequential compound is worth more than the simultaneous compound option by virtue of staging the investments. Note that the simultaneous compound option acts like a regular execution call option. Hence, the *American Call Option* is a good benchmark for such an option. Figure 10.32 shows how a Simultaneous Compound Option can be solved using the MSLS (example file used: *MSLS—Simple Two-Phased Simultaneous Compound Option*). Similar to the sequential compound option analysis, the existence of an option value implies that the ability to defer and wait for additional information prior to executing is valuable due to the significant uncertainties and risks as measured by *Volatility*. However, when the cost of waiting as measured by the *Dividend Rate* is high, the option to wait and defer becomes less valuable, until the break-even point where the option value equals zero and the strategic project value equals the NPV of the project. This break-even point provides valuable insights for the decision maker into the interactions between the levels of uncertainty inherent in the project and the cost of waiting to execute. The same analysis can be extended to Multiple Investment Simultaneous Compound Options as seen in Figure 10.33 (example file used: *MSLS—Multiple Phased Simultaneous Compound Option*).

Exercise: Simultaneous Compound Option

In a compound option analysis, the value of the option depends on the value of another option. For instance, a pharmaceutical company currently going

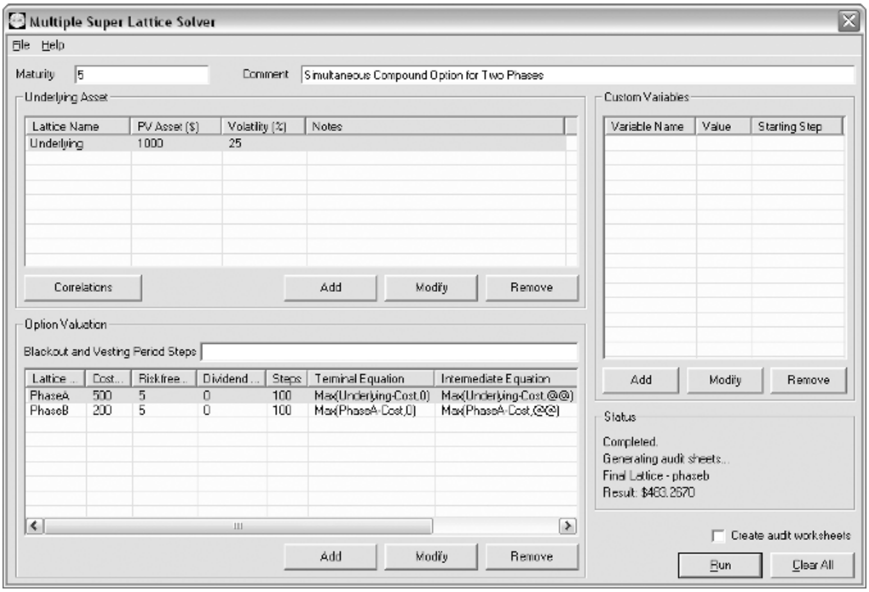


FIGURE 10.32 Solving a Simultaneous Compound Option Using MSLS

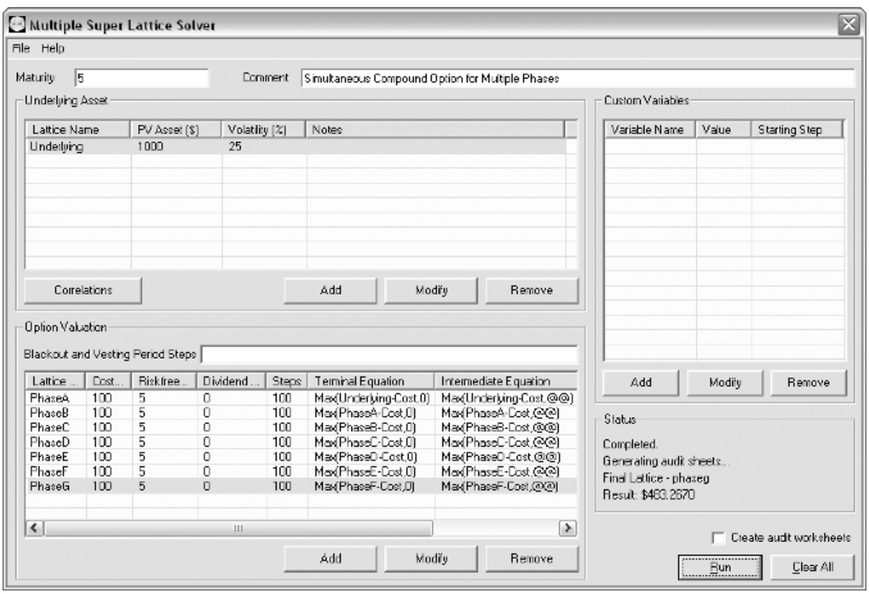


FIGURE 10.33 Solving a Multiple Investment Simultaneous Compound Option Using MSLS

through a particular FDA drug approval process has to go through human trials. The success of the FDA approval depends heavily on the success of human testing, both occurring at the same time. Suppose that the former costs $900 million and the latter $500 million. Further suppose that both phases occur simultaneously and take five years to complete. Using Monte Carlo simulation, you calculate the implied volatility of the logarithmic returns on the projected future cash flows to be 25 percent. The risk-free rate on a riskless asset for the next five years is found to be yielding 7.7 percent. The drug development effort's static valuation of future profitability using a DCF model (in other words, the present value of the future cash flows discounted at an appropriate market risk-adjusted discount rate) is found to be $1 billion. Do the following exercises, answering the questions that are posed:

1. Solve the simultaneous compound option problem manually using a 10-step lattice and confirm the results by generating an audit sheet using the software.
2. Swap the implementation costs such that the first cost is $500 and the second cost is $900. Is the resulting option value similar or different? Why?
3. What happens when part of the cost of the first option is allocated to the second option? For example, make the first cost $450 and the second cost $950. Does the result change? Explain.
4. Show how a *Closed-Form American Approximation Model* can be used to benchmark the results from a simultaneous compound option.
5. Show how a *Sequential Compound Option* can also be used to calculate or at least approximate the simultaneous compound option result.
6. Assume that the total $1,400 implementation cost (in present values) is now distributed equally across seven simultaneous projects. Does the result change? Why or why not?

AMERICAN AND EUROPEAN OPTIONS USING TRINOMIAL LATTICES

Building and solving trinomial lattices is similar to building and solving binomial lattices, complete with the up/down jumps and risk-neutral probabilities, but it is more complicated due to more branches stemming from each node. At the limit, both the binomial and trinomial lattices yield the same result, as seen in the following table. However, the lattice-building complexity is much higher for trinomial or multinomial lattices. The only reason to use a trinomial lattice is that the level of convergence to the correct option value is achieved more quickly than by using a binomial lattice. In the sample table, notice how the trinomial lattice yields the correct option value with fewer steps than it takes for a binomial lattice (1,000 as compared to 5,000). Because both yield identical results at the limit but trinomials are much more difficult to calculate and take a longer computation time, the binomial lattice is usually used instead. However, a trinomial is required only when the underlying asset follows a *mean-reverting process*. An illustration of the convergence of trinomials and binomials can be seen in the following example:

Steps	5	10	100	1,000	5,000
Binomial Lattice	$30.73	$29.22	$29.72	$29.77	$29.78
Trinomial Lattice	$29.22	$29.50	$29.75	$29.78	$29.78

Figure 10.34 shows another example (example file used: *MNLS—Simple Calls and Puts Using Trinomial Lattices*). The computed American Call is $31.99 using a five-step trinomial, and is identical to a 10-step binomial lattice seen in Figure 10.35. Therefore, due to the simpler computation and the speed of computation, the SLS and MSLS use binomial lattices instead of trinomials or other multinomial lattices. The only time a trinomial lattice is truly useful is when the underlying asset of the option follows a mean-reversion tendency. In that case, use the MNLS module instead.

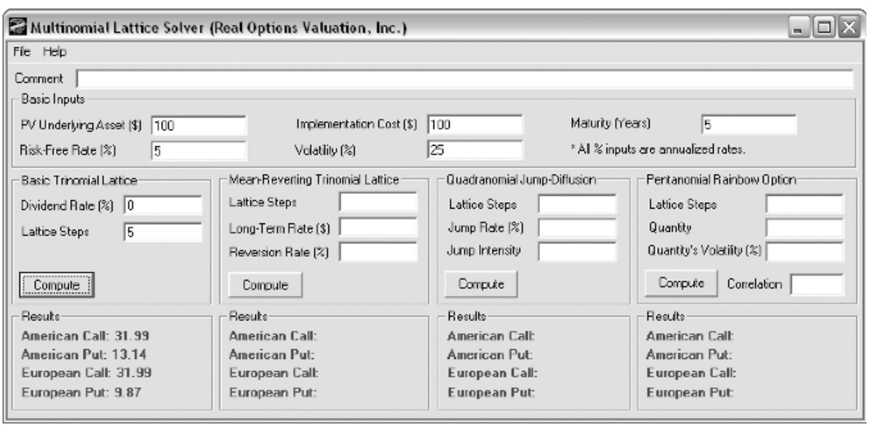


FIGURE 10.34 Simple Trinomial Lattice Solution

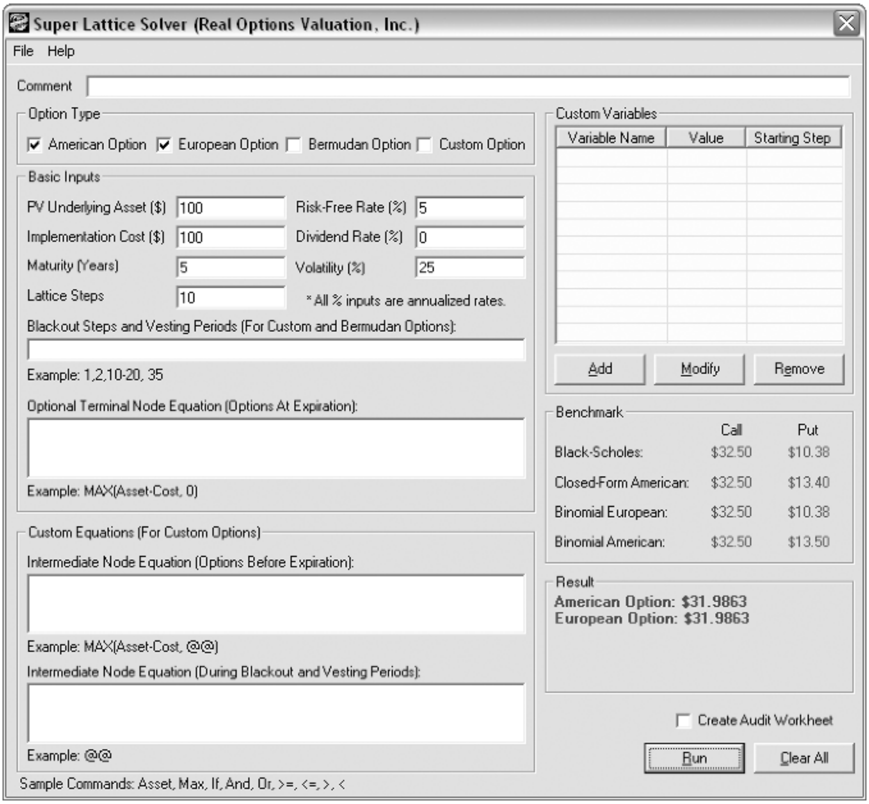


FIGURE 10.35 Ten-Step Binomial Lattice Comparison Result

AMERICAN AND EUROPEAN MEAN-REVERSION OPTION USING TRINOMIAL LATTICES

The *Mean-Reversion Option* in MNLS calculates both the American and European options when the underlying asset value is mean-reverting. A mean-reverting stochastic process reverts back to the long-term mean value (*Long-Term Rate Level*) at a particular speed of reversion (*Reversion Rate*). Examples of variables following a mean-reversion process include inflation rates, interest rates, gross domestic product growth rates, optimal production rates, price of natural gas, and so forth. Certain variables such as these succumb to either natural tendencies or economic/business conditions to revert to a long-term level when the actual values stray too far above or below this level. For instance, monetary and fiscal policy will prevent the economy from significant fluctuations, while policy goals tend to have a specific long-term target rate or level. Figure 10.36 illustrates a regular stochastic process (dotted line) versus a mean-reversion process (solid line). Clearly the mean-reverting process with its dampening effects will have a lower level of uncertainty than the regular process with the same volatility measure.

Figure 10.37 shows the call and put results from a regular option modeled using the Trinomial Lattice versus calls and puts assuming a mean-reverting (MR) tendency of the underlying asset using the Mean-Reverting Trinomial Lattice. Several items are worthy of attention:

- The MR call < regular call because of the dampening effect of the mean-reversion asset. The MR asset value will not increase as high as the regular asset value.
- Conversely, the MR put > regular put because the asset value will not rise as high, indicating that there will be a higher chance that the asset value will hover around the PV Asset, and a higher probability it will be below the PV Asset, making the put option more valuable.
- With the dampening effect, the MR call and MR put ($18.62 and $18.76) are more symmetrical in value than with a regular call and put ($31.99 and $13.14).
- The regular American call = regular European call because without dividends, it is never optimal to execute early. However, because of the mean-reverting tendencies, being able to execute early is valuable, especially before the asset value decreases. So, we see that MR American call > MR European call but of course both are less than the regular call.

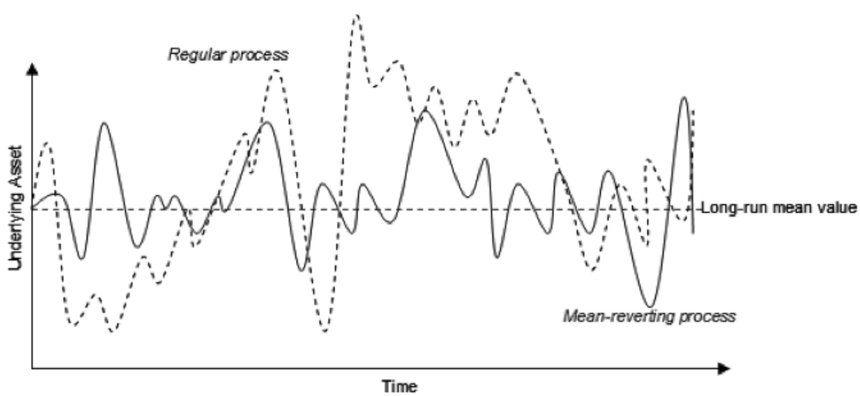


FIGURE 10.36 Mean-Reversion in Action

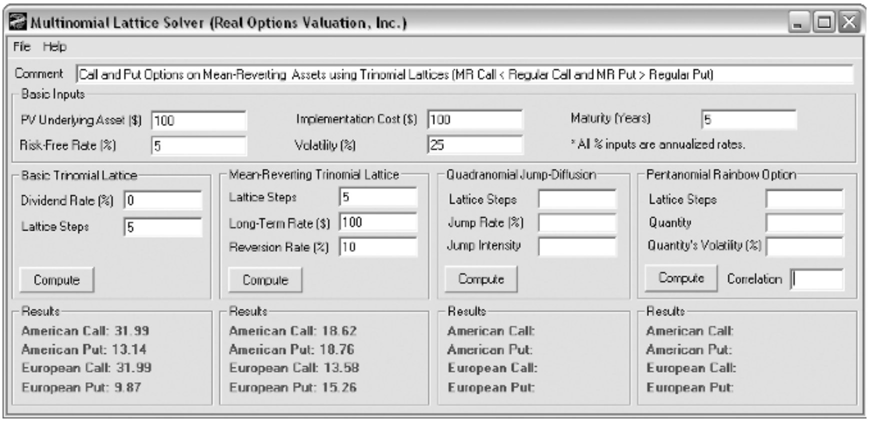


FIGURE 10.37 Comparing Mean-Reverting Calls and Puts to Regular Calls and Puts

Other items of interest in mean-reverting options include:

- The higher (lower) the long-term rate level, the higher (lower) the call options.
- The higher (lower) the long-term rate level, the lower (higher) the put options.

Finally, be careful when modeling mean-reverting options as higher lattice steps are usually required and certain combinations of reversion rates, long-term rate level, and lattice steps may yield unsolvable trinomial lattices. When this occurs, the MNLS will return error messages.

JUMP-DIFFUSION OPTION USING QUADRANOMIAL LATTICES

The *Jump-Diffusion Calls and Puts* for both American and European options applies the *Quadranomial Lattice* approach. This model is appropriate when the underlying variable in the option follows a jump-diffusion stochastic process. Figure 10.38 illustrates an underlying asset modeled using a jump-diffusion process. Jumps are commonplace in certain business variables such as price of oil and price of gas where prices take sudden and unexpected jumps (e.g., during a war). The underlying variable's frequency of jump is denoted as its *Jump Rate*, and the magnitude of each jump is its *Jump Intensity*.

The binomial lattice is only able to capture a stochastic process without jumps (e.g., Brownian Motion and Random Walk processes) but when there

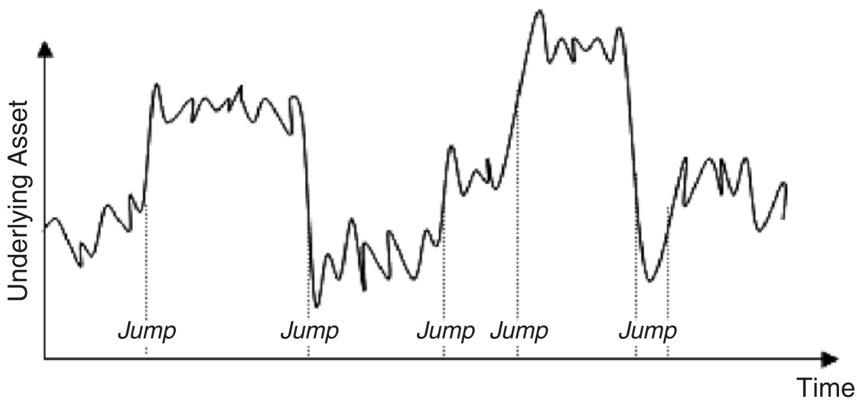


FIGURE 10.38 Jump-Diffusion Process

is a probability of jump (albeit a small probability that follows a Poisson distribution), additional branches are required. The quadranomial lattice (four branches on each node) is used to capture these jumps as seen in Figure 10.39.

Be aware that due to the complexity of the models, some calculations with higher lattice steps may take slightly longer to compute. Furthermore, certain combinations of inputs may yield negative implied risk-neutral probabilities and result in a noncomputable lattice. In that case, make sure the

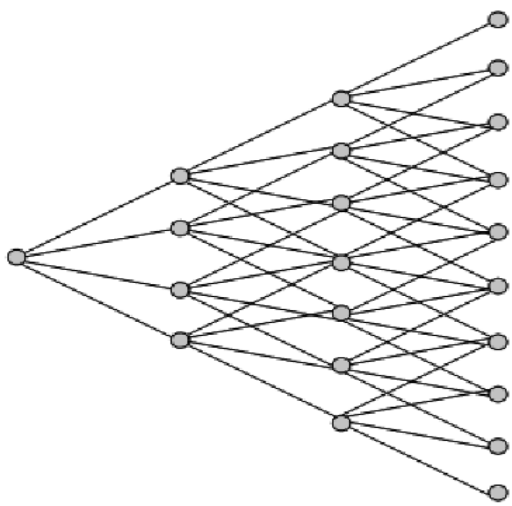


FIGURE 10.39 Quadranomial Lattice

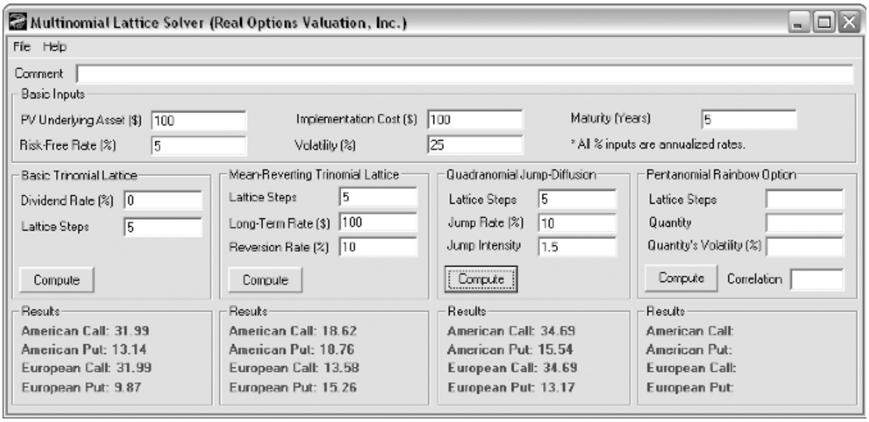


FIGURE 10.40 Quadranomial Lattice Results on Jump-Diffusion Options

inputs are correct (e.g., *Jump Intensity* has to exceed 1, where 1 implies no jumps; check for erroneous combinations of *Jump Rates*, *Jump Sizes*, and *Lattice Steps*). The probability of a jump can be computed as the product of the *Jump Rate* and time-step δt. Figure 10.40 illustrates a sample Quadranomial Jump-Diffusion Option analysis (example file used: *MNLS—Jump-Diffusion Calls and Puts Using Quadranomial Lattices*). Notice that the Jump-Diffusion call and put options are worth more than regular calls and puts, because with the positive jumps (10 percent probability per year with an average jump size of 1.50 times the previous values) of the underlying asset, the call and put options are worth more, even with the same volatility (i.e., \$34.69 compared to \$31.99 and \$15.54 compared to \$13.14).

DUAL-VARIABLE RAINBOW OPTION USING PENTANOMIAL LATTICES

The *Dual-Variable Rainbow Option* for both American and European options requires the *Pentanomial Lattice* approach. Rainbows on the horizon after a rainy day comprise various colors of the light spectrum, and although rainbow options aren't as colorful as their physical counterparts, they get their name from the fact that they have two or more underlying assets rather than one. In contrast to standard options, the value of a rainbow option is determined by the behavior of two or more underlying elements and by the correlation between these underlying elements. That is, the value of a rainbow option is determined by the performance of two or more underlying asset elements. This particular model is appropriate when there are two underlying variables in the option (e.g., *Price of Asset* and *Quantity*) where each fluctuates at different rates of volatilities but at the same time might be correlated (Figure 10.41). These two variables are usually correlated in the real world, and the underlying asset value is the product of price and quantity. Due to the different volatilities, a pentanomial or five-branch lattice is used to capture all possible combinations of products (Figure 10.42). Be aware that certain combinations of inputs may yield an unsolvable lattice with negative implied

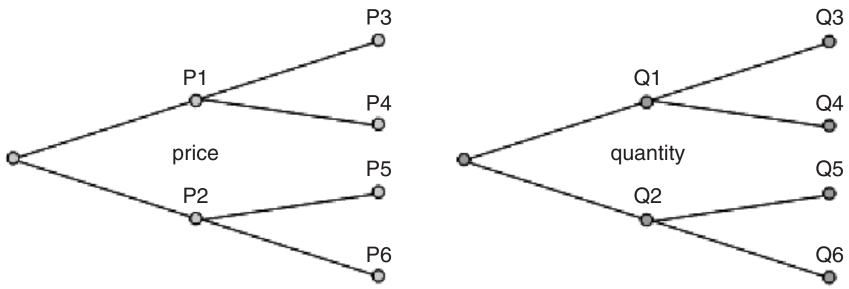


FIGURE 10.41 Two Binomial Lattices (Asset Prices and Quantity)

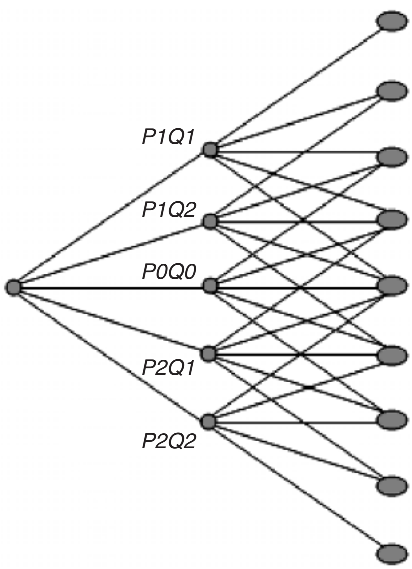


FIGURE 10.42 Pentanomial Lattice (Combining Two Binomial Lattices)

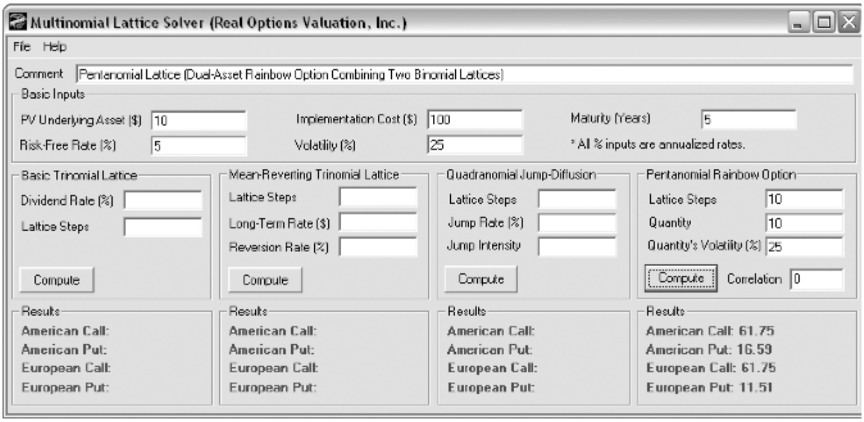


FIGURE 10.43 Pentanomial Lattice Solving a Dual-Asset Rainbow Option

probabilities. If that result occurs, a message will appear. Try a different combination of inputs as well as higher lattice steps to compensate.

Figure 10.43 shows an example Dual-Asset Rainbow Option (example file used: *MNLS—Dual-Asset Rainbow Option Pentanomial Lattice*). Notice that a high positive correlation will increase both the call option and put option values because if both underlying elements move in the same direction, there is a higher overall portfolio volatility (price and quantity can fluctuate at high-high and low-low levels, generating a higher overall underlying asset value). In contrast, negative correlations will reduce both the call option and put option values for the opposite reason due to the portfolio diversification effects of negatively correlated variables. Of course correlation here is bounded between –1 and +1 inclusive. If a real options problem has more than 2 underlying assets, either use the MSLS and/or Risk Simulator to simulate the underlying asset's trajectories and capture their interacting effects in a DCF model.

AMERICAN AND EUROPEAN LOWER BARRIER OPTIONS

The *Lower Barrier Option* measures the strategic value of an option (this applies to both calls and puts) that comes either in-the-money or out-of-the-money when the *Asset Value* hits an artificial *Lower Barrier* that is currently lower than the asset value. Therefore, a *Down-and-In* option (for both calls and puts) indicates that the option becomes live if the asset value hits the lower barrier. Conversely, a *Down-and-Out* option is live only when the lower barrier is not breached.

Examples of this option include contractual agreements whereby if the lower barrier is breached, some event or clause is triggered. The value of a barrier option is lower than standard options, as the barrier option will be valuable only within a smaller price range than the standard option. The holder of a barrier option loses some of the traditional option value and therefore a barrier option should sell at a lower price than a standard option. An example would be a contractual agreement whereby the writer of the contract can get into or out of certain obligations if the asset or project value breaches a barrier.

Figure 10.44 shows a Lower Barrier Option for a Down-and-In-Call. Notice that the value is only $7.3917, much lower than a regular American call option of $42.47, because the barrier is set low, at $90. This means that

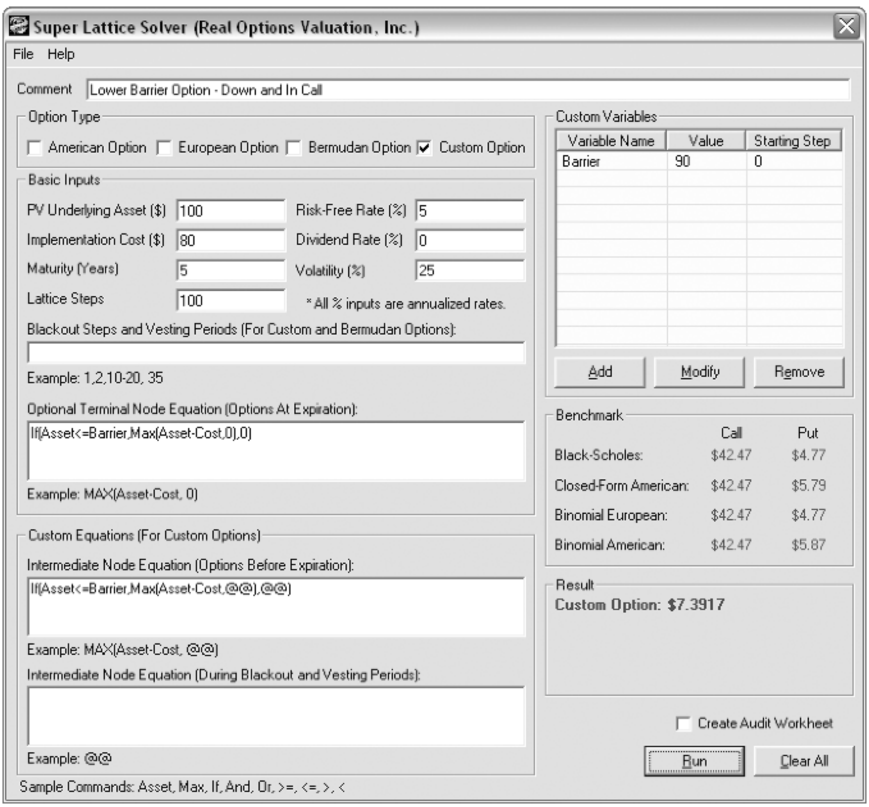


FIGURE 10.44 Down-and-In Lower American Barrier Option

all of the upside potential that the regular call option can have will be reduced significantly, and the option can only be exercised if the asset value falls below this lower barrier of $90 (example file used: *Barrier Option—Down and In Lower Barrier Call*). To make such a Lower Barrier Option binding, the lower barrier level must be below the starting asset value but above the implementation cost. If the barrier level is above the starting asset value, then it becomes an upper barrier option. If the lower barrier is below the implementation cost, then the option will be worthless under all conditions. When the lower barrier level is between the implementation cost and starting asset value, the option is potentially worth something. However, the value of the option is dependent on volatility. Using the same parameters in Figure 10.44 and changing the volatility and risk-free rates, the following examples illustrate what happens:

- At a volatility of 75 percent, the option value is $4.34.
- At a volatility of 25 percent, the option value is $3.14.
- At a volatility of 5 percent, the option value is $0.01.

The lower the volatility, the lower the probability that the asset value will fluctuate enough to breach the lower barrier such that the option will be executed. By balancing volatility with the threshold lower barrier, you can create optimal trigger values for barriers.

In contrast, the Lower Barrier Option for Down-and-Out Call Option is shown in Figure 10.45. Here, if the asset value breaches this lower barrier, the option is worthless, but is only valuable when it does not breach this lower barrier. As call options have higher values when the asset value is high, and lower value when the asset is low, this Lower Barrier Down-and-Out Call Option is hence worth almost the same as the regular American option. The higher the barrier, the lower the value of the lower barrier option will be (example file: *Barrier Option—Down and Out Lower Barrier Call*). For instance:

- At a lower barrier of $90, the option value is $42.19.
- At a lower barrier of $100, the option value is $41.58.

Figures 10.44 and 10.45 illustrate American Barrier Options. To change these into European Barrier Options set the Intermediate Equation Nodes to @@. In addition, for certain types of contractual options, vesting and blackout periods can be imposed. For solving such Bermudan Barrier Options, keep the same Intermediate Equation as the American Barrier Options but set the Intermediate Equation During Blackout and Vesting Periods to @@ and insert the corresponding blackout and vesting period lattice steps. Finally, if the

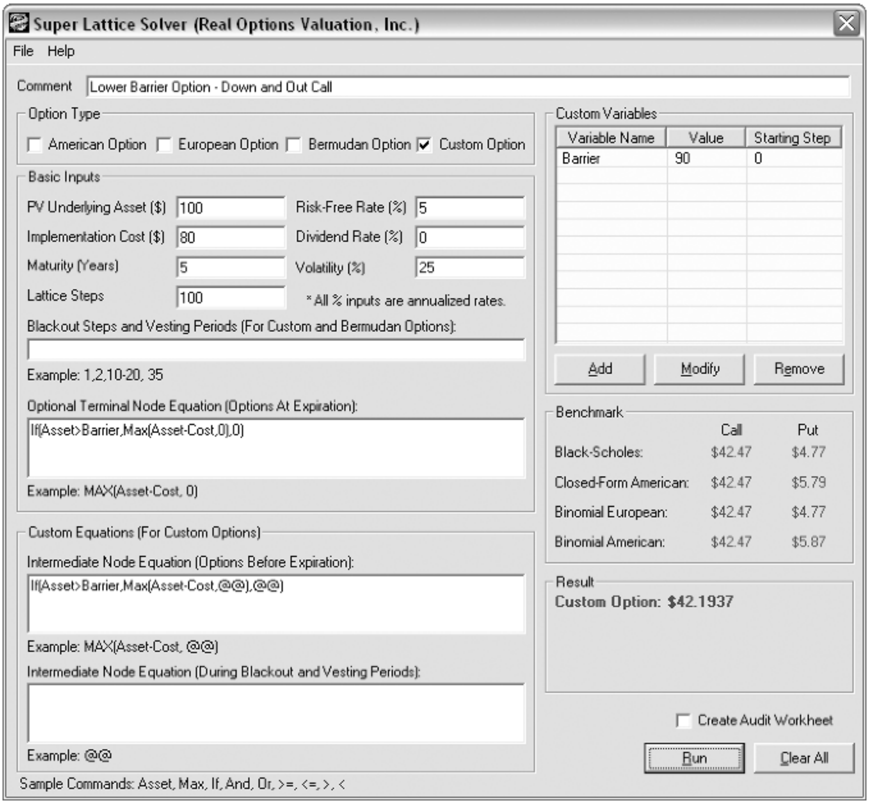


FIGURE 10.45 Down-and-Out Lower American Barrier Option

Barrier is a changing target over time, put in several custom variables named Barrier with the different values and starting lattice steps.

AMERICAN AND EUROPEAN UPPER BARRIER OPTION

The *Upper Barrier Option* measures the strategic value of an option (this applies to both calls and puts) that comes either in-the-money or out-of-the-money when the *Asset Value* hits an artificial *Upper Barrier* that is currently higher than the asset value. Therefore, an *Up-and-In* option (for both calls and puts) indicates that the option becomes live if the asset value hits the upper barrier. Conversely, for the *Up-and-Out* option, the option is live only when the upper barrier is not breached. This is very similar to the Lower

Barrier Option but now the barrier is above the starting asset value, and for a binding barrier option, the implementation cost is typically lower than the upper barrier. That is, the *upper barrier is usually > implementation cost and the upper barrier is also > starting asset value*.

Examples of this option include contractual agreements whereby if the upper barrier is breached some event or clause is triggered. The values of barrier options are typically lower than standard options, as the barrier option will have value within a smaller price range than the standard option. The holder of a barrier option loses some of the traditional option value and therefore a barrier option should sell at a lower price than a standard option. An example would be a contractual agreement whereby the writer of the contract can get into or out of certain obligations if the asset or project value breaches a barrier.

The Up-and-In Upper American Barrier Option has slightly lower value than a regular American call option as seen in Figure 10.46 because some of the option value is lost when the asset is less than the barrier but greater than the implementation cost. Clearly, the *higher the upper barrier, the lower the up-and-in barrier option value* will be as more of the option value is lost due to the inability to execute when the asset value is below the barrier (example file used: *Barrier Option—Up and In Upper Barrier Call*). For instance:

- When the upper barrier is $110, the option value is $41.22.
- When the upper barrier is $120, the option value is $39.89.

In contrast, an Up-and-Out Upper American Barrier Option is worth a lot less because this barrier truncates the option's upside potential. Figure 10.47 shows the computation of such an option. Clearly, the *higher the upper barrier, the higher the option value* will be (example file used: *Barrier Option—Up and Out Upper Barrier Call*). For instance:

- When the upper barrier is $110, the option value is $23.69.
- When the upper barrier is $120, the option value is $29.59.

Finally, note the issues of nonbinding barrier options. Examples of *nonbinding options* are:

- Up-and-Out Upper Barrier Calls when the Upper Barrier ≤ Implementation Cost, then the option will be worthless.
- Up-and-In Upper Barrier Calls when Upper Barrier ≤ Implementation Cost, then the option value reverts to a simple call option.

Examples of Upper Barrier Options are contractual options. Typical examples are:

- A manufacturer contractually agrees not to sell its products at prices higher than a prespecified upper barrier price level.
- A client agrees to pay the market price of a good or product until a certain amount and then the contract becomes void if it exceeds some price ceiling.

Figures 10.46 and 10.47 illustrate American Barrier Options. To change these into European Barrier Options set the Intermediate Equation Nodes to @@. In addition, for certain types of contractual options, vesting and blackout periods can be imposed. For solving such Bermudan Barrier Options, keep the same Intermediate Equation as the American Barrier Options but set the Intermediate Equation During Blackout and Vesting Periods to @@ and insert the corresponding blackout and vesting period lattice steps. Finally, if the Barrier is a changing target over time, put in several custom variables named Barrier with the different values and starting lattice steps.

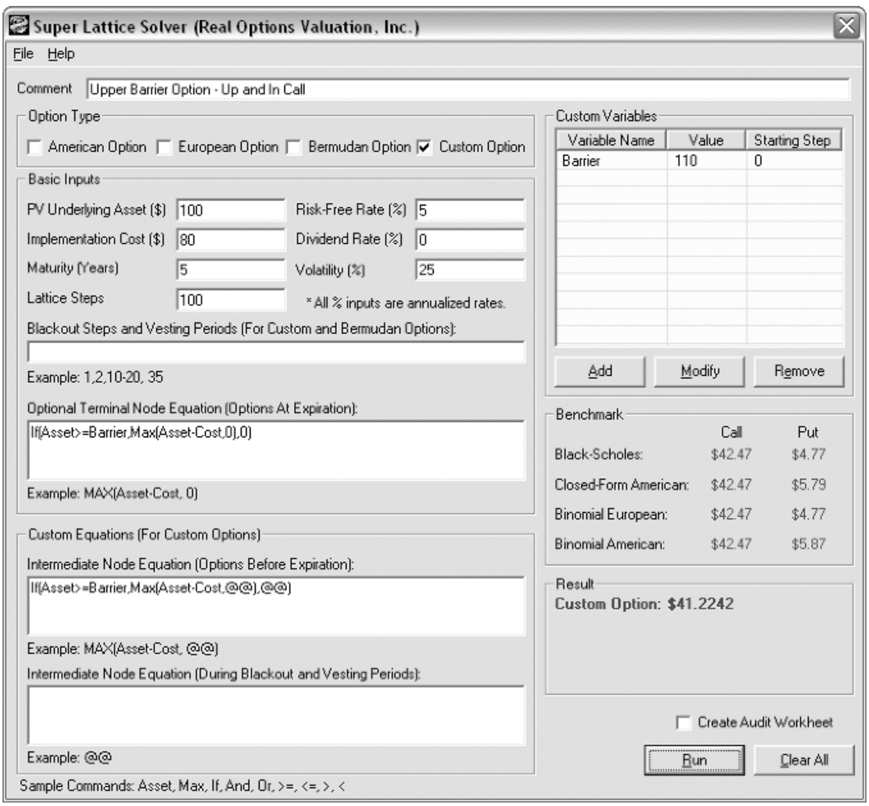


FIGURE 10.46 Up-and-In Upper American Barrier Option

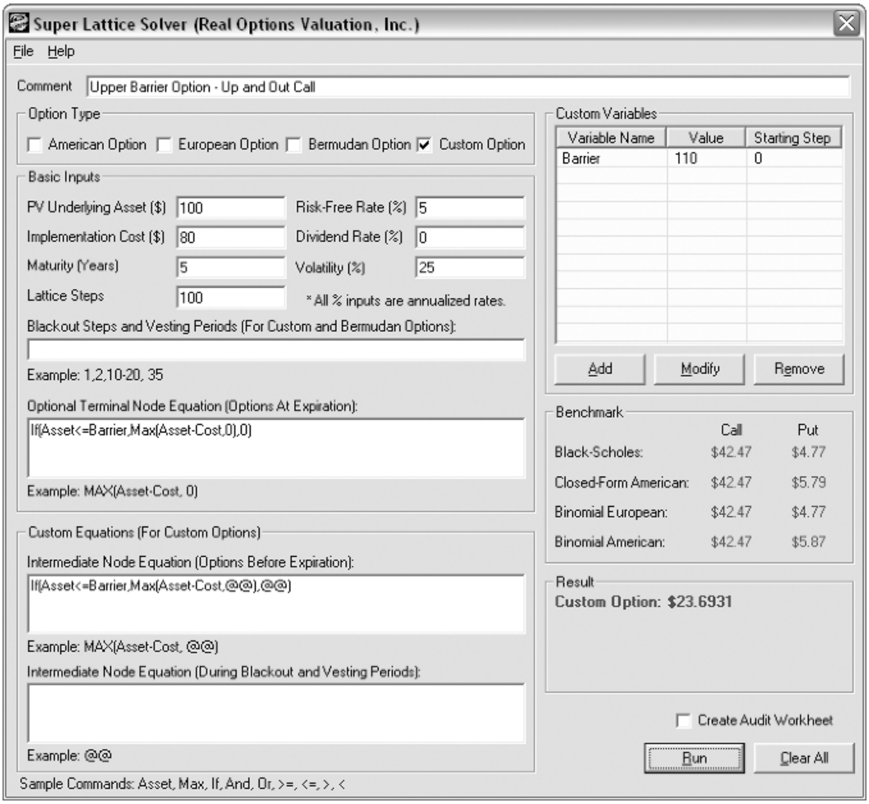


FIGURE 10.47 Up-and-Out Upper American Barrier Option

AMERICAN AND EUROPEAN DOUBLE BARRIER OPTIONS AND EXOTIC BARRIERS

The *Double Barrier Option* is solved using the binomial lattice. This model measures the strategic value of an option (this applies to both calls and puts) that comes either in-the-money or out-of-the-money when the *Asset Value* hits either the artificial *Upper* or *Lower Barriers*. Therefore, an *Up-and-In* and *Down-and-In* option (for both calls and puts) indicates that the option becomes live if the asset value hits either the upper or lower barrier. Conversely, for the *Up-and-Out* and *Down-and-Out* option, the option is live only when neither the upper nor lower barrier is breached. Examples of this option include contractual agreements whereby if the upper barrier is breached, some event or clause is triggered. The value of barrier options is lower than standard options, as the barrier option will have value within a smaller price range than the standard option. The holder of a barrier option loses some of the traditional option value and therefore such options should be worth less than a standard option.

Figure 10.48 illustrates an American Up-and-In, Down-and-In Double Barrier Option, which is a combination of the Upper and Lower Barrier Options shown previously. The same exact logic applies to this Double Barrier Option.

Figure 10.48 illustrates the American Barrier Option solved using the SLS. To change these into a European Barrier Option, set the Intermediate Equation Nodes to @@. In addition, for certain types of contractual options, vesting and blackout periods can be imposed. For solving such Bermudan Barrier Options, keep the same Intermediate Equation as the American Barrier Options but set the Intermediate Equation During Blackout and Vesting Periods to @@ and insert the corresponding blackout and vesting period lattice

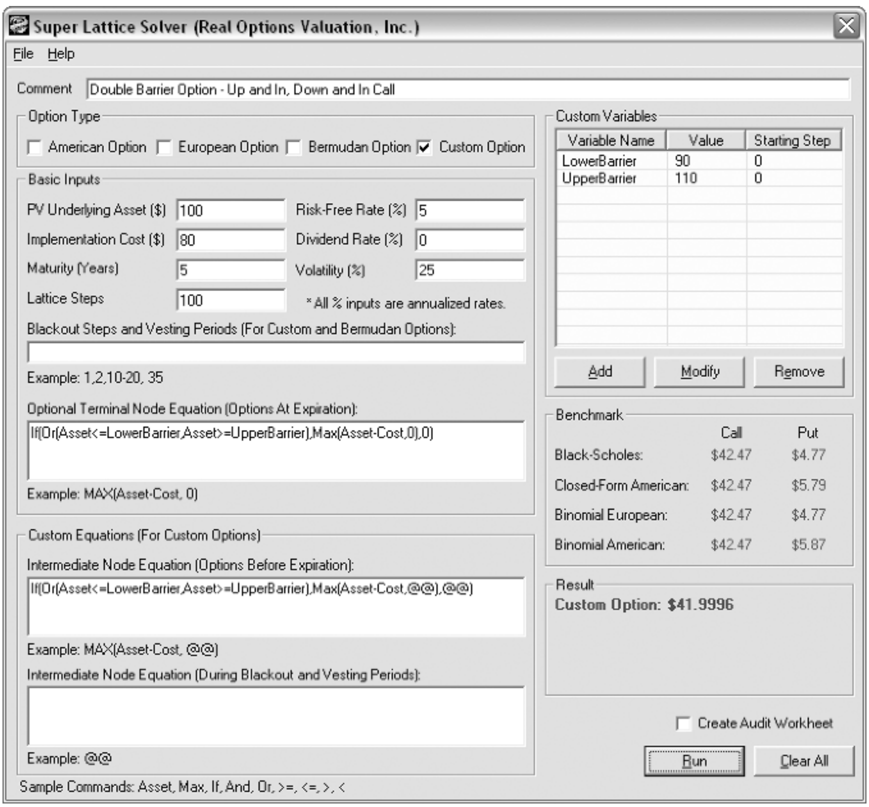


FIGURE 10.48 Up-and-In, Down-and-In Double Barrier Option

steps. Finally, if the Barrier is a changing target over time, put in several custom variables named Barrier with the different values and starting lattice steps.

Exotic Barrier Options exist when other options are combined with barriers. For instance, an option to expand can only be executed if the PV Asset exceeds some threshold, or a contraction option to outsource manufacturing can only be executed when it falls below some breakeven point. Again, such options can be easily modeled using the SLS.

Exercise: Barrier Options

Barrier options are combinations of call and put options such that they become in-the-money or out-of-the-money when the asset value breaches an artificial barrier. Standard single upper barrier options can be call-up-and-in, call-up-and-out, put-up-and-in, and put-up-and-out. Standard single lower barrier options can be call-down-and-in, call-down-and-out, put-down-and-in, and put-down-and-out. Double barrier options are combinations of standard single upper and lower barriers. Assume that the *PV Asset* is \$200, maturity is 5 years, risk-free rate is 5 percent, no dividends, and 25 percent volatility.

1. For each of the barrier option types (upper, lower, and double barrier options), do the following:
   a. What are some examples of nonbinding inputs? That is, show the combinations of cost and barrier levels that make the options nonbinding.
   b. How do you know the option is nonbinding?
   c. Show some binding inputs and explain the differences between binding results and nonbinding results.
2. Running the double barrier option, change each input parameter, and explain the effects on the up-and-in and down-and-in call option, up-and-in and down-and-in put option, up-and-out and down-and-out call option, and up-and-out and down-and-out put option. Explain your observations when the barrier levels change or when volatility increases.
   a. Replicate the analysis using a standard lower barrier option.
   b. Replicate the analysis using a standard upper barrier option.
3. Suppose a chemical manufacturer executes a sales contract with a customer and agrees to charge the customer \$8/pound of a particular reagent if the customer agrees to purchase 1 million pounds per year for the next five years. The market price of the reagent is currently \$10/pound. Thus, the customer receives a \$2 million discount per year. In addition, if the price is always set at \$8/pound, the manufacturer might be losing out on being able to sell his products at the higher market prices. Therefore, an agreement is struck whereby the contract holds as long as the market price is below \$11/pound. If the prevailing price exceeds this threshold,

the product will be sold at this market price. Assume a 25 percent annualized volatility and a 5 percent risk-free rate over the next five years.

a. How much is such a price protection contact worth to the customer if this upper barrier does not exist?
b. How much is such a price protection contact worth to the customer if this upper barrier is implemented?
c. What happens when the upper threshold is increased?
d. What happens when the price volatility of this compound is lower than 25 percent? Are the price differentials between the option with and without barrier constant for various volatility levels? In other words, is the barrier option a linear function of volatility?

AMERICAN ESO WITH VESTING PERIOD

Figure 10.49 illustrates how an employee stock option (ESO) with a vesting period and blackout dates can be modeled. Enter the blackout steps (0–39). Because the blackout dates input box has been used, you will need to enter the Terminal Equation (TE), Intermediate Equation (IE), and Intermediate Equation During Vesting and Blackout Periods (IEV). Enter *Max(Stock-Strike,0)* for the TE; *Max(Stock-Strike,0,@@)* for the IE; and @@ for IEV (example file used: *ESO Vesting*). This means the option is executed or left to expire worthless at termination; execute early or keep the option open during the intermediate nodes; and keep the option open only and no executions are allowed during the intermediate steps when blackouts or vesting occurs. The result is \$49.73 (Figure 10.49) which can be corroborated with the use of the ESO Valuation Toolkit (Figure 10.50). ESO Valuation Toolkit is another software tool developed by the author at Real Options Valuation, Inc., specifically designed to solve ESO problems following the 2004 FAS 123. In fact, this software was used by the Financial Accounting Standards Board to model the valuation example in its final FAS 123 Statement in December 2004. Before starting with ESO valuations, it is suggested that the user read *Valuing Employee Stock Options* (Johnathan Mun, Wiley, Hoboken, NJ, 2004) as a primer.

CHANGING VOLATILITIES AND RISK-FREE RATES OPTIONS

Exercise: Changing Volatilities and Changing Risk-Free Rates

Volatility on cash flow returns may change over time. Assume a two-year option in which volatility is 20 percent in the first year and 30 percent in the

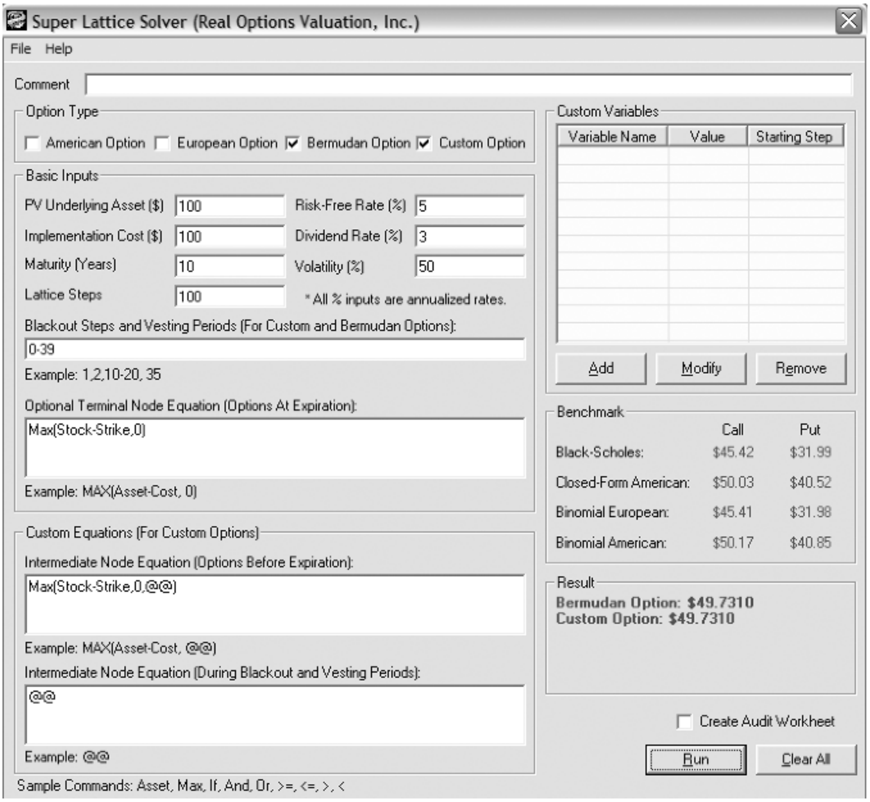


FIGURE 10.49 SLS Results of a Vesting Call Option

second year. In this circumstance, the up and down factors are different over the two time periods. Thus, the binomial lattice will no longer be recombining. Assume an asset value of $100, implementation costs of $110, and a risk-free rate of 10 percent. (Note that changing volatility options can also be solved analytically using nonrecombining lattices—see Appendix 7I on nonrecombining lattices).

1. Solve the problem using the software's SLS Excel Solution using a 10-step lattice.
2. Change the first volatility to 30 percent and the second to 20 percent. What happens?

American Option with Vesting Requirements

Assumptions

Stock Price ($)	$100.00
Strike Price ($)	$100.00
Maturity in Years (.)	10.00
Risk-Free Rate (%)	5.00%
Dividends (%)	3.00%
Volatility (%)	50.00%
Vesting in Years (.)	4.00

Calculate

Main Menu

Analyze

Intermediate Calculations

Stepping-Time (dt)	1.0000
Up Step-Size (up)	1.6487
Down Step-Size (down)	0.6065
Risk-Neutral Probability (prob)	39.69%

Results

10-Step Lattice Results	$48.98
Generalized Black-Scholes	$45.42
American Closed-Form Approx.	$50.03
100-Step Binomial Super Lattice	$49.73
Binomial Super Lattice Steps	100 Steps
10-Step Trinomial Super Lattice	$44.95
Trinomial Super Lattice Steps	10 Steps

Underlying Stock Price Lattice

										14841.32
									9001.71	
								5459.82		5459.82
							3311.55		3311.55	
						2008.55		2008.55		2008.55
					1218.25		1218.25		1218.25	
				738.91		738.91		738.91		738.91
			448.17		448.17		448.17		448.17	
		271.83		271.83		271.83		271.83		271.83
	164.87		164.87		164.87		164.87		164.87	
100.00		100.00		100.00		100.00		100.00		100.00
	60.65		60.65		60.65		60.65		60.65	
		36.79		36.79		36.79		36.79		36.79
			22.31		22.31		22.31		22.31	
				13.53		13.53		13.53		13.53
					8.21		8.21		8.21	
						4.98		4.98		4.98
							3.02		3.02	
								1.83		1.83
									1.11	
										0.67

Vesting Calculation

0	1	2	3	4	5	6	7	8	9	10

Option Valuation Lattice

										14741.32
									8901.71	
								5359.82		5359.82
							3211.55		3211.55	
						1908.55		1908.55		1908.55
					1118.25		1118.25		1118.25	
				638.91		638.91		638.91		638.91
			345.23		348.17		348.17		348.17	
		183.29		181.29		176.74		171.83		171.83
	95.59		92.29		86.87		78.93		64.88	
48.98		45.99		41.55		35.11		24.50		0.00
	22.48		19.43		15.26		9.25		0.00	
		8.91		6.52		3.49		0.00		0.00
			2.75		1.32		0.00		0.00	
				0.50		0.00		0.00		0.00
					0.00		0.00		0.00	
						0.00		0.00		0.00
							0.00		0.00	
								0.00		0.00
									0.00	
										0.00

FIGURE 10.50 ESO Valuation Toolkit Results of a Vesting Call Option

3. Now assume that the option has a 10-year maturity. Rerun the options with the following volatility curve:
   a. Flat curve: 29% for the next 10 years
   b. Upward sloping curve: start at 20% and increase it by 2% each year, with the last year at 38% (averaging at 29%)
   c. Downward sloping curve: start at 38% and decrease it by 2% each year, with the last year at 20% (averaging at 29%)
   d. Frown: 20%, 26%, 29%, 32%, 38%, 38%, 32%, 29%, 26%, and 20% (averaging at 29%)
   e. Smile: 38%, 32%, 29%, 26%, 20%, 20%, 26%, 29%, 32%, and 38% (averaging at 29%)

   Interpret the results. What do you observe? Is an option's value higher or lower when a higher volatility applies earlier rather than later? Why is it that it takes a longer time to run the binomial lattice when volatilities change over time?
4. Now assume that the option has a 10-year maturity with a constant 29 percent volatility. Rerun the options with the following risk-free yield curve:
   a. Flat curve: 2.9% for the next 10 years
   b. Upward sloping curve: start at 2.0% and increase it by 0.2% each year, with the last year at 3.8% (average is 2.9%)
   c. Downward sloping curve: start at 3.8% and decrease it by 0.2% each year, with the last year at 2.0% (average is 2.9%)
   d. Frown: 2.0%, 2.6%, 2.9%, 3.2%, 3.8%, 3.8%, 3.2%, 2.9%, 2.6%, and 2.0% (averaging at 2.9%)
   e. Smile: 3.8%, 3.2%, 2.9%, 2.6%, 2.0%, 2.0%, 2.6%, 2.9%, 3.2%, and 3.8% (averaging at 2.9%)

   Interpret the results. What do you observe? Is an option's value higher or lower when a higher risk-free rate applies earlier rather than later? Why is it faster to run a changing risk-free rate model than a changing volatility model?

AMERICAN ESO WITH SUBOPTIMAL EXERCISE BEHAVIOR

This example shows how suboptimal exercise behavior multiples can be included in the analysis and how the custom variables list can be used as seen in Figure 10.51 (example file used: *ESO Suboptimal Behavior*). The TE is the same as the previous example but the IE assumes that the option will be suboptimally executed if the stock price in some future state exceeds the suboptimal exercise threshold times the strike price. Notice that the IEV is not

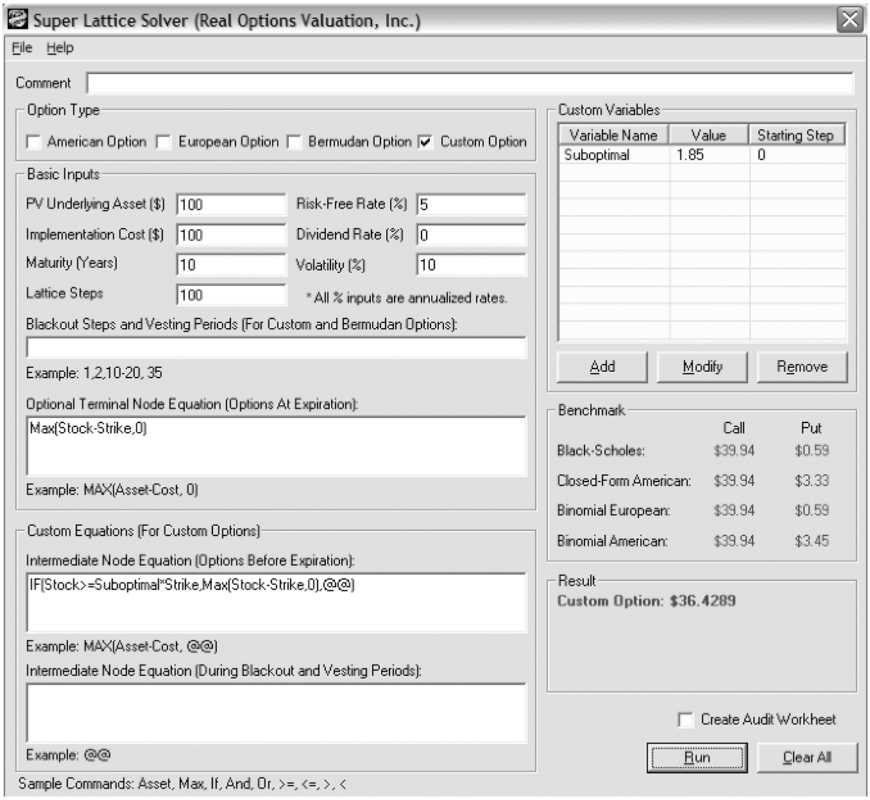


FIGURE 10.51 SLS Results of a Call Option Accounting for Suboptimal Behavior

used because we did not assume any vesting or blackout periods. Also, the *Suboptimal* exercise multiple variable is listed on the customs variable list with the relevant value of 1.85 and a starting step of 0. This means that 1.85 is applicable starting from step 0 in the lattice all the way through to step 100. The results again are verified through the ESO Toolkit (Figure 10.52).

AMERICAN ESO WITH VESTING AND SUBOPTIMAL EXERCISE BEHAVIOR

Next, we have the ESO with vesting and suboptimal exercise behavior. This is simply the extension of the previous two examples. Again, the result of \$9.22 (Figure 10.53) is verified using the ESO Toolkit as seen in Figure 10.54 (example file used: *ESO Vesting with Suboptimal Behavior*).

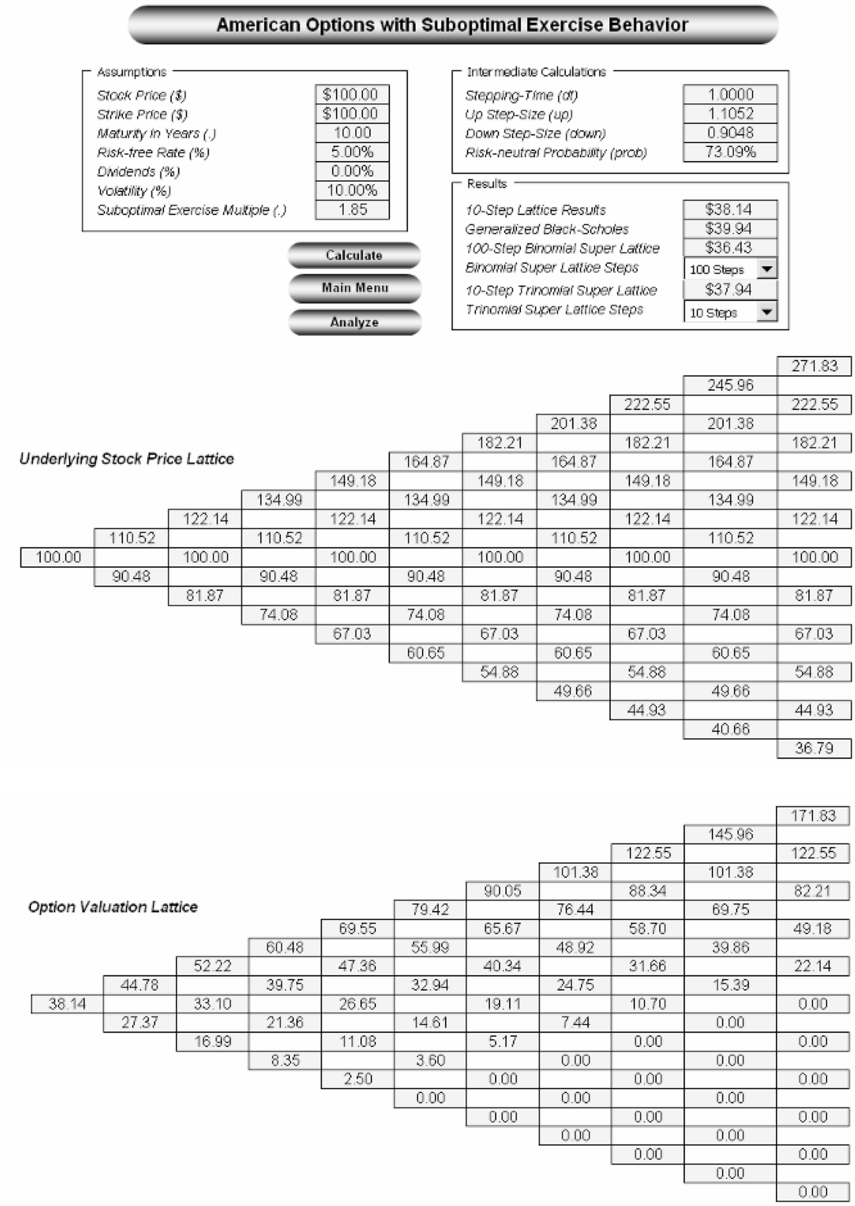


FIGURE 10.52 ESO Toolkit Results of a Call Option Accounting with Suboptimal Behavior

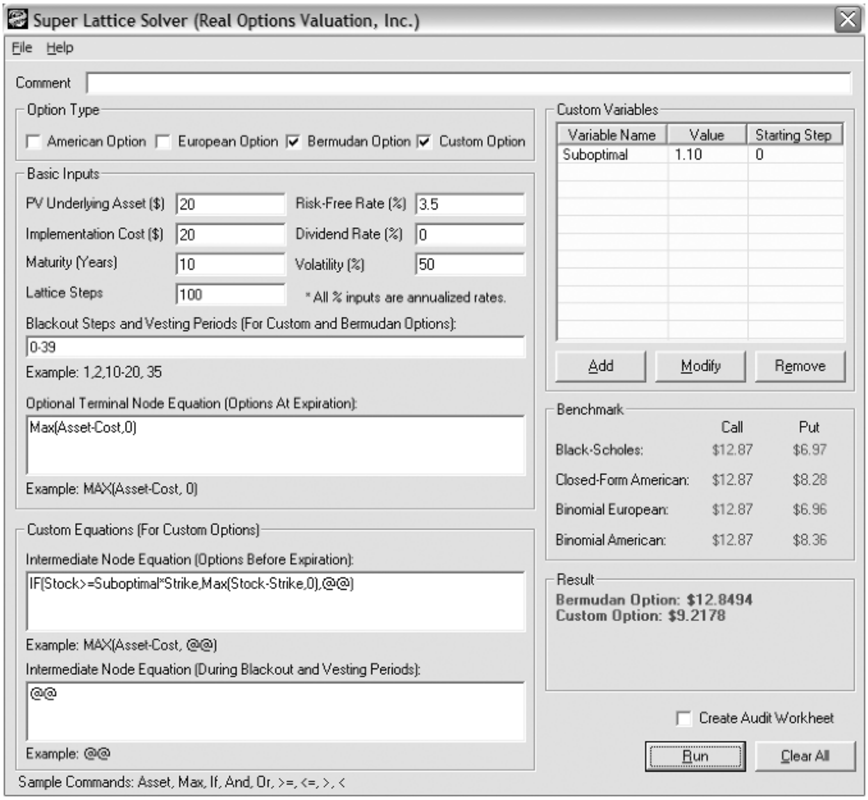


FIGURE 10.53 SLS Results of a Call Option Accounting for Vesting and Suboptimal Behavior

AMERICAN ESO WITH VESTING, SUBOPTIMAL EXERCISE BEHAVIOR, BLACKOUT PERIODS, AND FORFEITURE RATE

This example now incorporates the element of forfeiture into the model as seen in Figure 10.55 (example file used: *ESO Vesting, Blackout, Suboptimal, Forfeiture*). This means that if the option is vested and the prevailing stock price exceeds the suboptimal threshold above the strike price, the option will be summarily and suboptimally executed. If vested but not exceeding the threshold, the option will be executed only if the postvesting forfeiture occurs,

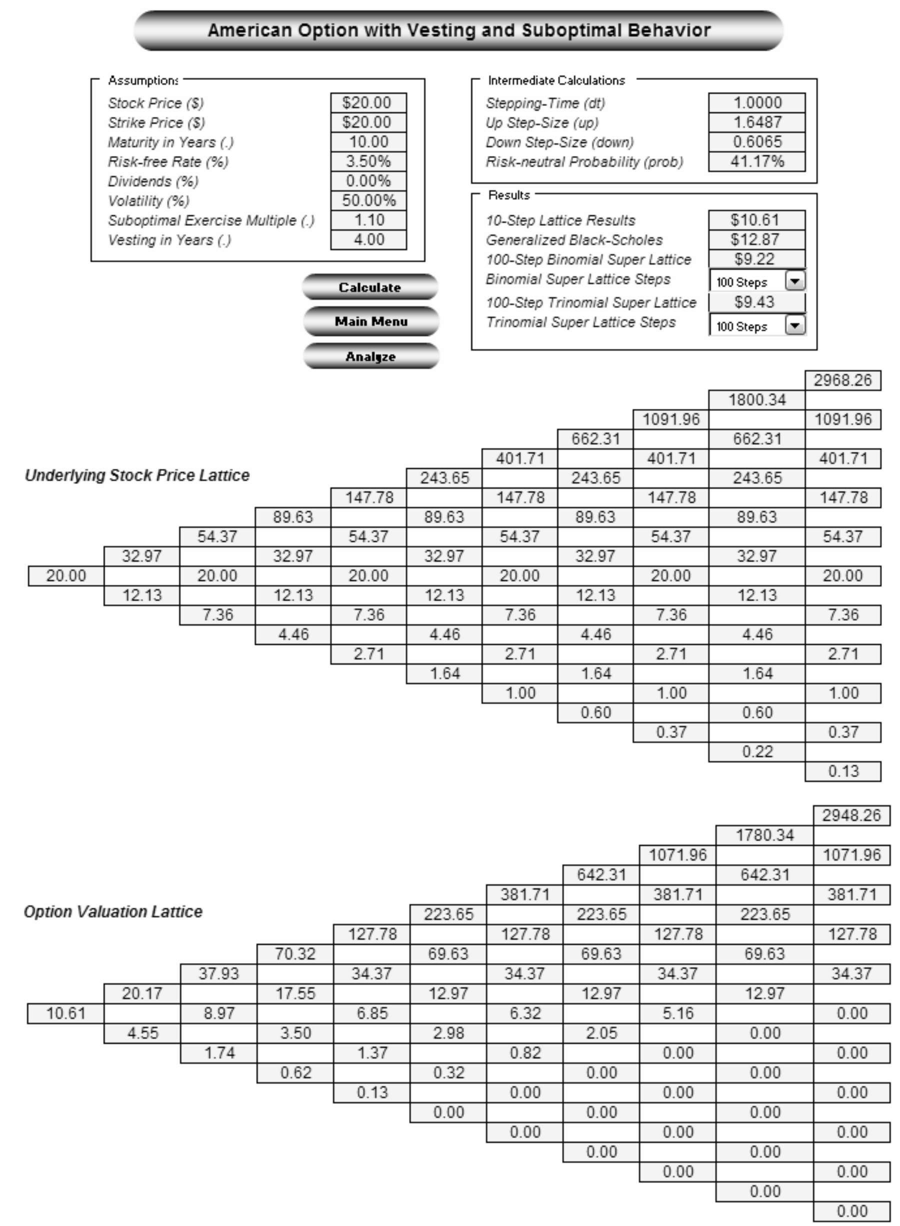


FIGURE 10.54 ESO Toolkit Results of a Call Option Accounting for Vesting and Suboptimal Behavior

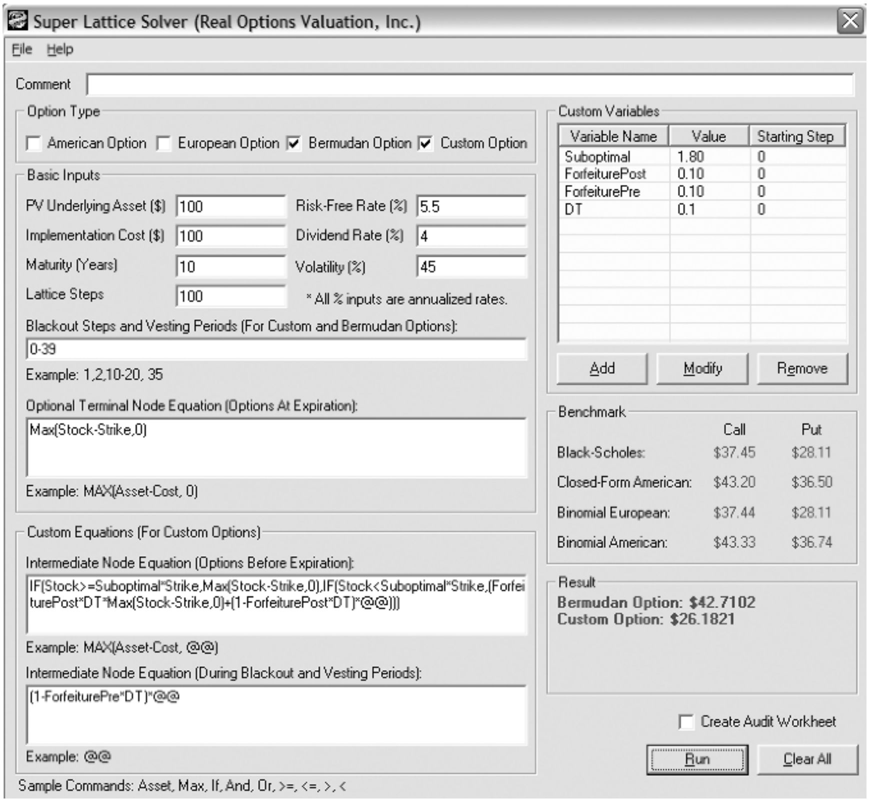


FIGURE 10.55 SLS Results of a Call Option Accounting for Vesting, Forfeiture, Suboptimal Behavior, and Blackout Periods

but the option is kept open otherwise. This means that the intermediate step is a probability weighted average of these occurrences. Finally, when an employee forfeits the option during the vesting period, all options are forfeited, with a prevesting forfeiture rate. In this example, we assume identical pre- and postvesting forfeitures so that we can verify the results using the ESO Toolkit (Figure 10.56). In certain other cases, a different rate may be assumed.

This concludes the SLS software application chapter. The next chapter applies these methodologies to solve actual business cases, where the process of options framing can be seen.

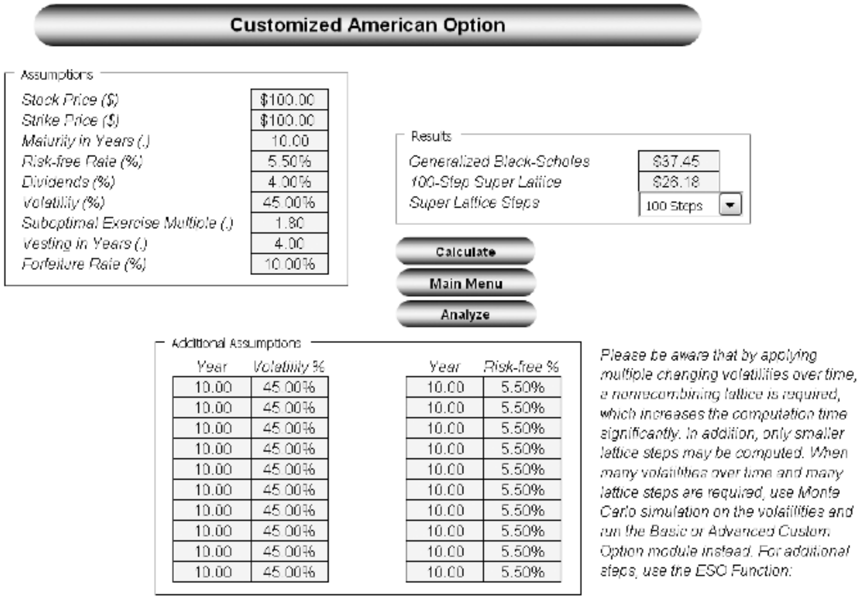


FIGURE 10.56 ESO Toolkit Results after Accounting for Vesting, Forfeiture, Suboptimal Behavior, and Blackout Periods

CHAPTER 11

Real Options Case Studies

This chapter shows several actual cases and real-life applications of real options, financial options, and employee stock options, solved using the Super Lattice Solver (SLS) software and Risk Simulator software. The cases included in this chapter are:

- High-Tech Manufacturing—Build or Buy Decision
- Financial Options—Convertible Warrants with a Vesting Period and Put Protection
- Pharmaceutical Development—Value of Perfect Information and Optimal Trigger Values
- Oil and Gas—Farm Outs, Options to Defer, and Value of Information
- Valuing Employee Stock Options Under 2004 FAS 123
- Integrated Risk Modeling—Applying Simulation, Forecasting, and Optimization on Real Options
- Biopharmaceutical Industry—Valuing Strategic Manufacturing Flexibility
- Real Estate—Alternative Use and Development
- United States Navy—Strategic Flexibility in Mission Control Centers

CASE 1: HIGH-TECH MANUFACTURING—BUILD OR BUY DECISION WITH REAL OPTIONS

Microtech, Inc. is a billion-dollar high-tech manufacturing firm currently interested in developing a new state-of-the-art, high-capacity micro hard drive that fits into the palm of your hand. The problem is, the larger the capacity, the larger the magnetic disk has to be, and the faster it has to spin. Therefore, making small hard drives is hard enough, let alone a high-capacity hard drive that is reliable and state-of-the-art. The risks the firm faces include market risks—will this product sell, will it sell enough, and at what price—and private risks—will the technology work and can we develop it fast enough ahead of the competition—both of which are significant enough to yield serious disasters in the project. In performing its due diligence, the vice president of advanced emerging technologies of this firm found a small start-up firm that is also currently developing such a technology, and it is approximately three-quarters of the way there. This small start-up will initiate a patent process in the next few weeks, and Microtech would like to consider acquiring the start-up prior to the patent process starting. The start-up has shown interest in being acquired and based on preliminary discussions, requested \$50M for the firm.

The question is, should Microtech acquire the firm and mitigate some development risk but still face the market risk and some residual development risk? After all, the start-up has the technology only partially completed. Then again, through the acquisition, Microtech can take a potential rival out of the picture and even mitigate the chances of its competitors acquiring this firm's technologies. How much is this firm really worth, compared to its asking price of \$50M? What options exist to mitigate some of the market and development risks? Are there additional opportunities in the market that Microtech can take advantage of through the acquisition?

The finance staff at Microtech with the assistance of several external consultants began to collect all the relevant data and created a DCF model, and the best-guess present value of the benefits from the firm is \$100M. This means that the NPV of buying the firm is \$50M after accounting for the acquisition cost of \$50M. In the DCF model, the probability of technical success is also modeled (using several binomial distributions and their relevant probabilities of success in several phases multiplied together) as well as the market positioning (triangular distributions were used to simulate the different market conditions in the future). Using the Risk Simulator software, the Monte Carlo simulation's resulting annualized volatility is found to be 25 percent, a somewhat moderate level of risk. The finance staff also created another DCF with which to compare the result. This second DCF models the scenario of building the technology in-house, and the total development cost in present value will be \$40M, a lot less than the acquisition cost of \$50M. At first glance, it might be better off building the technology in-house, providing an NPV of \$60M. However, the volatility is found to be 30 percent as it is riskier to develop the technology from scratch than to buy a firm with the technology almost completed.

The question now becomes what, if any, strategic real options exist for Microtech to consider? Is the NPV analysis sufficient to justify doing it itself? What about all the market and private risks in the project? If acquiring the firm, are there any options to protect Microtech from failure? If building the technology itself, are there any options to mitigate the development risks?

Microtech then proceeded to perform some *real options framing exercises* with its executives, facilitated by an external real options expert consultant. The questions raised included what risks exist and how can they be reduced.

The real options consultant then put a real options framework around the discussions and came up with a preliminary *strategy tree* (Figure 11.1). For the first pass, four main options were conceived: mitigate the development risk of building themselves; mitigate the risk of the market; mitigate the risk of failure if acquiring the firm; and take advantage of the upside risks whenever possible. These options are compiled into path-dependent strategies in Figure 11.1.

Strategy A is to develop the technology in-house but the R&D risk is mitigated through a stage-gate investment process, where the total $40M required investment is spread into four steps of $10M each (in present values). At any phase of the development, the fate of the next stage is determined; that is, to

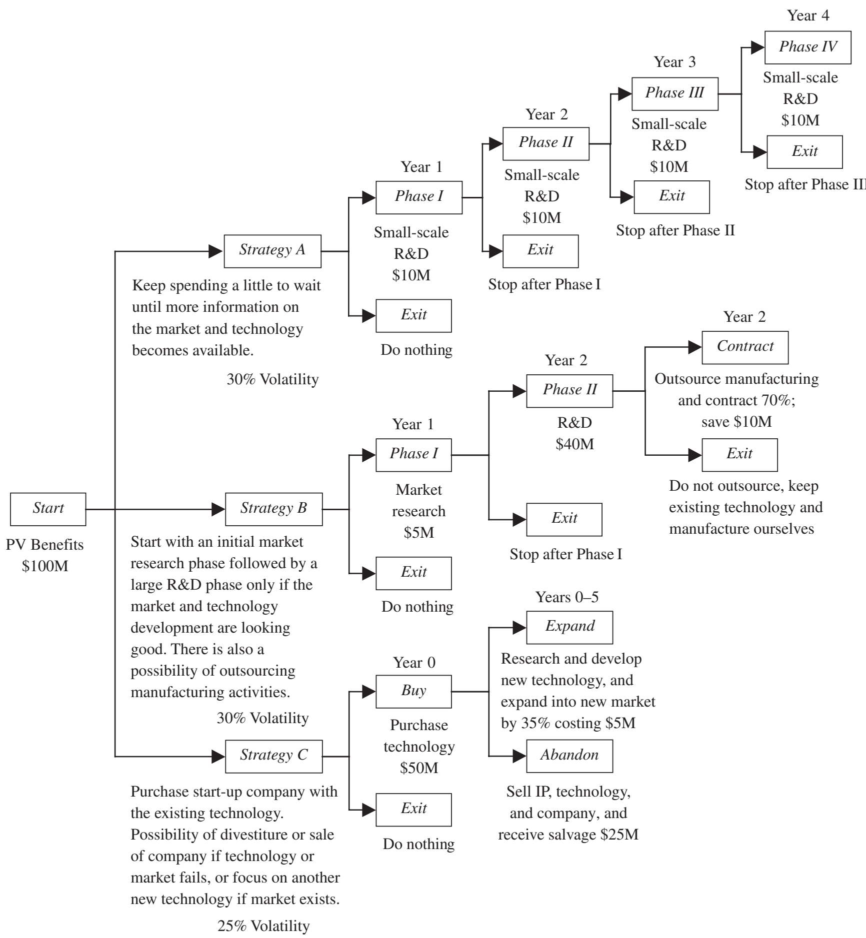


FIGURE 11.1 Strategy Tree for High-Tech Manufacturing

decide if the R&D initiative should continue depending on the outcome of the current phase. The investment can be terminated at any time, and the maximum loss will be the total investment up to that point. That is, if R&D shows bad results after one year, the initiative is abandoned, the firm exits the project, and the maximum loss is \$10M, not the entire \$40M as defined in the DCF model. The questions are: How much is this strategic path worth and is stage-gating the process worth it?

Strategy B is to develop the technology but the market risk is being hedged. That is, a preliminary Phase I market research is performed for \$5M in the first year to obtain competitive intelligence on Microtech's competitors and to see if the market and complementary technologies exist such that the microdrive will indeed be successful. Then, depending on the results of this market research, Phase II's R&D initiative will or will not be executed. Because the market research takes an entire year to complete, further stage-gating the R&D initiative is not an option because it will significantly delay the launch of the product. So, Phase II is a full-scale R&D. In this strategic path, although the market risk is mitigated through market research, the development risk still exists. Hence, a contraction option is conceived. That is, Microtech finds another counterparty to assume the manufacturing risks by signing a two-year contract whereby at any time within the next two years, Microtech can have this counterparty firm take over the development of the microdrive's increased rotational latency and seek times during the R&D process where the counterparty shares in 30 percent of the net profits without undertaking any R&D costs. Microtech will assume the entire \$40M R&D cost but ends up mitigating its highest development risks and also saves \$10M (in present values) by not having to increase its own manufacturing competencies by hiring outside consultants and purchasing new equipment. The questions are: How much is this strategic path worth, is the market research valuable, and how much should Microtech share its net profits with the counterparty?

Strategy C is to purchase the start-up firm for \$50M. However, by acquiring the firm, Microtech obtains additional options. Specifically, if the technology or market does not work out as expected, Microtech can sell the start-up (sell its intellectual property, patents, technology, assets, buildings, and so forth) for an estimated salvage value of \$25M within the first year. As the content of the start-up's intellectual property is expected to increase over time because of added development efforts, the salvage value is expected to increase by \$1M each year. If the technology is successful within the next five years, other products can be created from this microdrive base platform technology. For instance, the microdrive is not only applicable for use as in laptops but with an additional funding of \$5M, the technology can be adapted into handheld global positioning system (GPS) map locators for cars and travel enthusiasts, personal pocket-sized hard drives (where an individual can carry an entire computer on his key chain and all he has to do is plug it into a monitor and the virtual computer comes up), MP3 players, and a multitude of other products, which by Microtech's estimates will increase the NPV of the microdrive by 35 percent. However, this expansion option only exists in Strategy C, as time to market is crucial for these new products and the start-up already has three-quarters of the technology completed, speeding Microtech's time to market tremendously. The questions are: How much is the start-up actually worth to Microtech and is $50M too high a price to pay for the company? Figure 11.1 shows these three strategic paths and the relevant information on each strategy branch.

Strategy A's total strategic value is worth $64.65M using the Multiple Asset SLS software with 100-step lattices as seen in Figure 11.2. The NPV is $60M, indicating that the option value is worth $4.65M. This means that there is definitely value in stage-gating the R&D initiative to hedge downside risks. To follow along, open the MSLS file: *Solution to Chapter 11—Case I Strategy A* from the accompanying CD.

However, when the annualized dividend rate exceeds 2.5 percent, the option value becomes zero and the total strategic value reverts to the NPV of $60M as seen in Figure 11.3. This means that by spending more time and putting off development through a stage-gate process, as long as the maximum losses per year (lost market share and opportunity losses of net revenues from

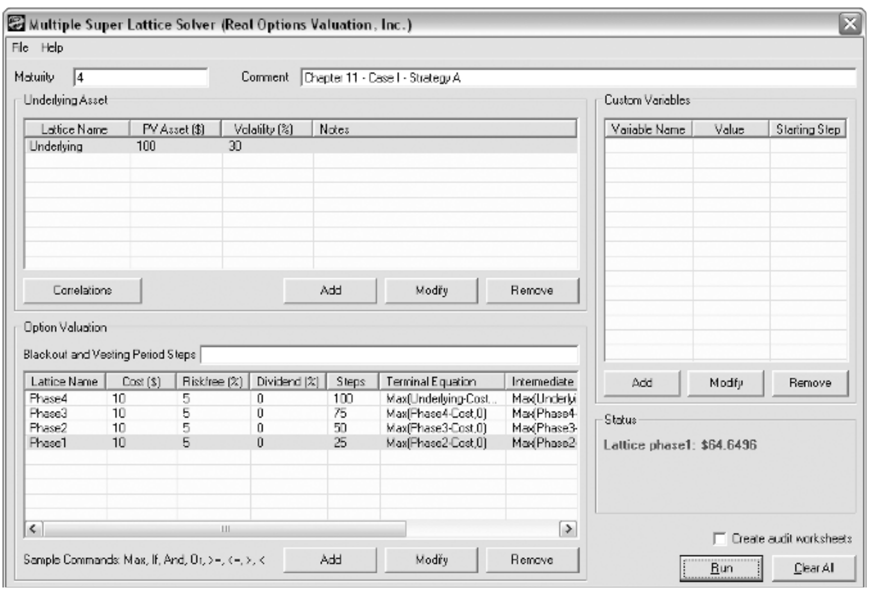


FIGURE 11.2 Value of Strategy A

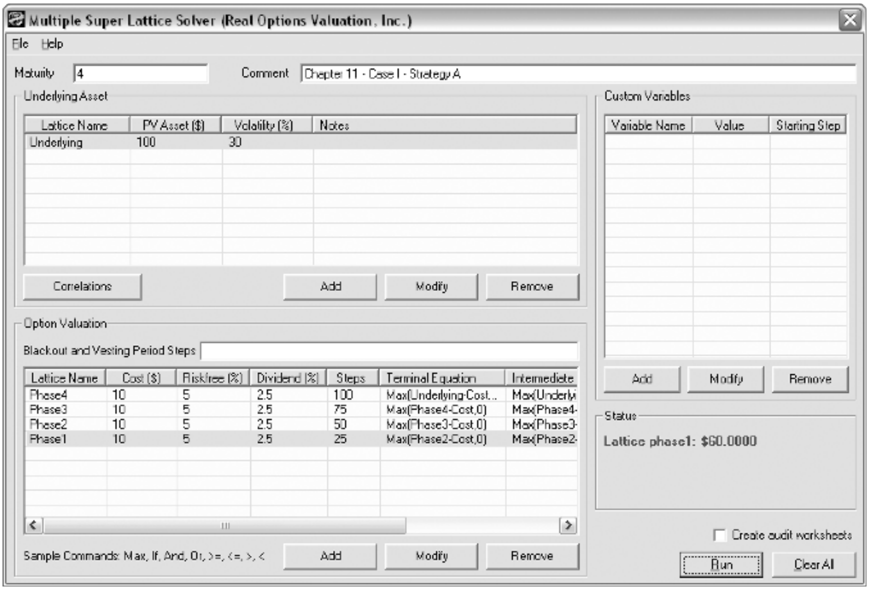


FIGURE 11.3 Strategy A's Break-Even Point

sales) do not exceed $2.5M (2.5 percent of $100M), then stage-gating is valuable. Otherwise, the financial leakage is too severe such that the added risk is worth it and the $40M should be spent immediately on a large-scale development effort. To follow along, open the MSLS file: *Solution to Chapter 11—Case I Strategy A* from the accompanying CD and progressively modify each valuation phase's dividend rate from 0 percent to 2.5 percent.

The total strategic value of Strategy B is valued at $75.90 as seen in Figure 11.4. The NPV is $55M (computed by taking $100M – $5M – $40M), which means that the options are valued at $20.90M. To follow along, open the MSLS file: *Solution to Chapter 11—Case I Strategy B* from the accompanying CD. So, thus far, Strategy B is the better strategic path, with a value of $75.90M. In addition, Figure 11.5 shows the strategic value without the contraction option, worth $59.12M ($4.12M option value to stage-gate with market research and $55M NPV). Thus, the contraction option with the counterparty to hedge the downside technical risk is worth $16.78M. A further analysis can be performed by changing the contraction factor (how much is allocated to the counterparty) and the amount of savings, as seen in Table 11.1.

Finally, the total strategic value for Strategy C is valued at $131.12 (see Figure 11.6) less $50 purchase price of the start-up company or a net strategic value of $81.12M. That is, Microtech should be willing to pay no more than

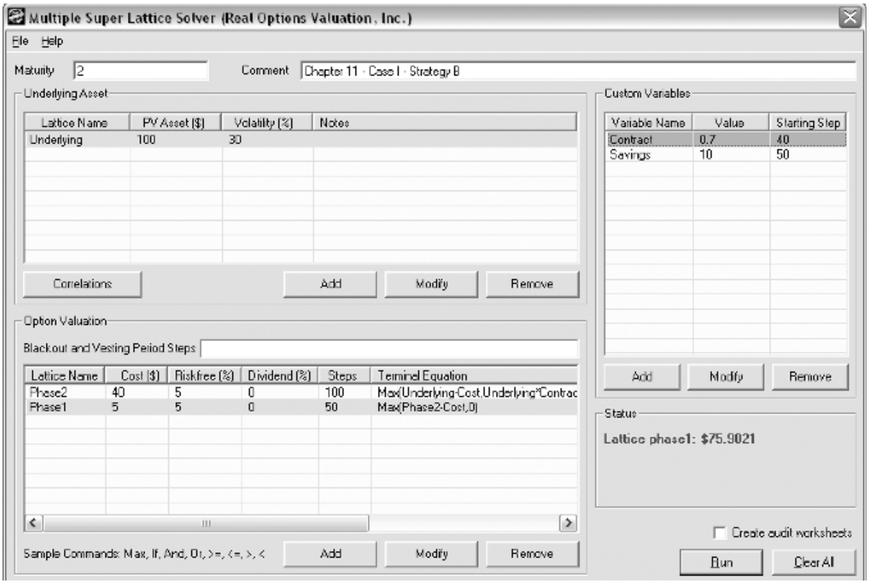


FIGURE 11.4 Value of Strategy B

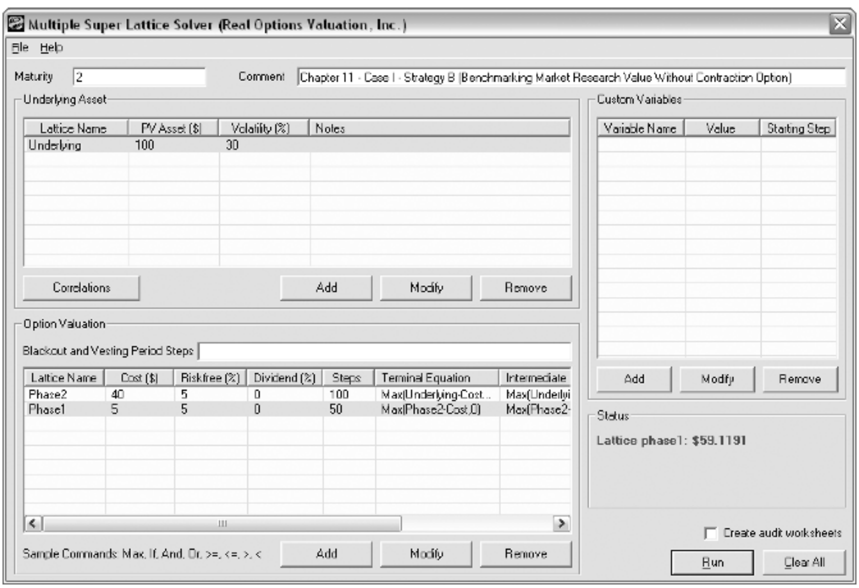


FIGURE 11.5 Strategy B without Contraction

TABLE 11.1 Decision Table on Savings and Contraction Factors

Contraction	Savings										
Factor	$0.00	$5.00	$10.00	$15.00	$20.00	$25.00	$30.00	$35.00	$40.00	$45.00	$50.00
	Strategic Option Value										
0.05	$59.15	$59.27	$59.50	$59.86	$60.39	$61.11	$62.07	$63.25	$64.67	$66.41	$68.40
0.10	59.19	59.35	59.64	60.09	60.74	61.62	62.73	64.13	65.83	67.77	69.94
0.15	59.24	59.47	59.84	60.40	61.19	62.26	63.60	65.24	67.15	69.31	71.83
0.20	59.32	59.62	60.11	60.82	61.80	63.08	64.66	66.53	68.68	71.20	73.97
0.25	59.43	59.82	60.45	61.36	62.56	64.07	65.91	68.06	70.57	73.36	76.39
0.30	59.60	60.12	60.91	62.04	63.50	65.30	67.43	69.94	72.76	75.84	79.15
0.35	59.83	60.53	61.54	62.92	64.68	66.80	69.31	72.15	75.28	78.67	82.30
0.40	60.16	61.09	62.38	64.07	66.18	68.68	71.54	74.73	78.19	81.88	85.80
0.45	60.65	61.86	63.49	65.55	68.05	70.94	74.18	77.72	81.51	85.48	89.65
0.50	61.35	62.91	64.93	67.45	70.38	73.66	77.26	81.14	85.22	89.46	93.83
0.55	62.34	64.32	66.85	69.82	73.18	76.87	80.82	84.98	89.32	93.77	98.31
0.60	63.70	66.25	69.26	72.70	76.49	80.56	84.83	89.24	93.76	98.36	103.01
0.65	65.66	68.71	72.27	76.18	80.35	84.72	89.22	93.81	98.46	103.15	107.87
0.70	68.24	71.88	75.90	80.20	84.68	89.26	93.92	98.61	103.34	108.07	112.82
0.75	71.58	75.72	80.15	84.72	89.37	94.08	98.81	103.55	108.30	113.05	117.81
0.80	75.65	80.18	84.83	89.55	94.28	99.03	103.78	108.54	113.29	118.05	122.81
0.85	80.31	85.02	89.76	94.51	99.27	104.02	108.78	113.54	118.29	123.05	127.81
0.90	85.25	90.00	94.76	99.51	104.27	109.02	113.78	118.54	123.99	128.05	132.81
0.90	85.25	90.00	94.76	99.51	104.27	109.02	113.78	118.54	123.99	128.05	132.81

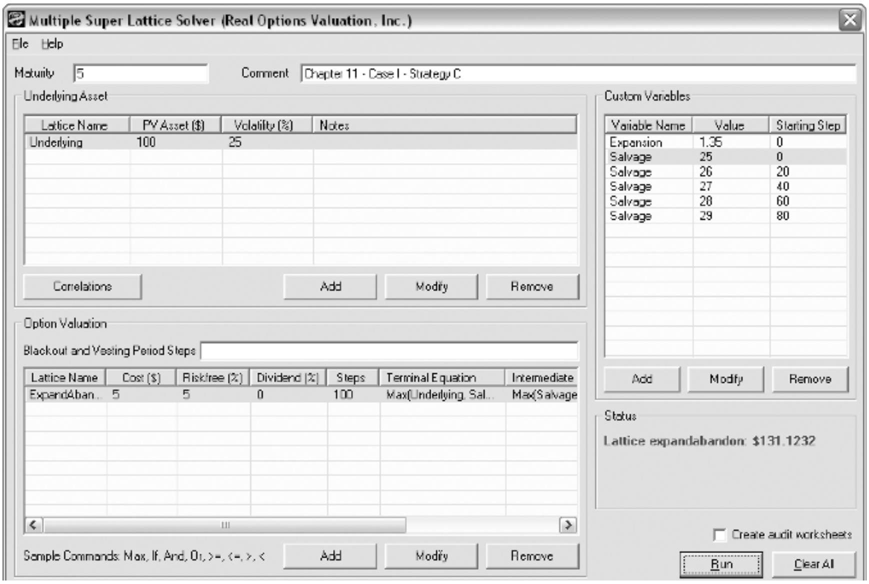


FIGURE 11.6 Value of Strategy C

$55.22M for the start-up (i.e., $50M + $81.12M – $75.90M), otherwise it is better off pursuing Strategy B and building the technology itself.

Thus, the optimal strategy is to purchase the start-up company, go to market quickly with the ability to abandon and sell the start-up should things fail, or to further invest an additional R&D sum later on to develop spin-off technologies. If real options analysis was not performed, Microtech would have chosen to develop the technology itself immediately and spend $40M. This strategy would yield the highest NPV if real options and risk mitigation options are not considered. Microtech would have made a serious decision blunder and taken unnecessary risks. By performing the real options analysis, additional spin-off products and opportunities surface, which prove to be highly valuable.

CASE 2: FINANCIAL OPTIONS—CONVERTIBLE WARRANTS WITH A VESTING PERIOD AND PUT PROTECTION

This case study provides a sample application of the Super Lattice Solver on valuing a warrant (an instrument that can be converted into a stock, similar to a call option) that has a protective put option associated with it. The analysis herein also applies both a customized binomial lattice and a closed-form Black-Scholes model for comparison and benchmarking purposes.

The valuation analysis performed was based on an actual consulting project but all proprietary information provided by the client has been modified except for certain basic publicly available information, and the accuracy of said results is dependent on this factual information at the time of valuation. This case details the input assumptions used as well as some benchmark due diligence to check the results. Certain technical details have been left out to simplify the case.

The client company very recently acquired a small IT firm. The acquisition consisted of both cash as well as some warrants, which are convertible into stocks. But because the client's stocks are fairly volatile, the acquired firm negotiated a protective put to hedge its downside risks. In return for providing this protective put, the client requested that the warrant be exercisable only if the target firm is solvent and its gross margins exceed 33 percent and be no less than $10 million.

Clearly, this problem cannot be solved using a Black-Scholes model because there exist dividends, a vesting period, a threshold price put protection at which the put can be exercised, and the fact that the put cannot be exercised unless the warrant is converted into a stock but only when the stock price is below $33.

To summarize, the following list shows the assumptions and requirements in this exotic warrant:

Stock price on grant date:	$30.12
Warrant strike price:	$15.00
Warrant maturity:	10.00 years (grant date)
Risk-free rate:	4.24% (grant date)
Volatility:	29.50%
Dividend rate:	0.51%
Put threshold price:	$33.00
Vesting for warrant:	3 years
Vesting for put option:	5 years

Further, the following requirements were modeled:

- The protective put option can only be exercised if the warrant is exercised.
- The put option can only be exercised if the stock price is below $33.00 at the time of exercise.
- The warrant can only be exercised if recipient's gross margin equals or exceeds 33 percent and be no less than $10 million. A simulation forecast puts an 85 percent to 90 percent uniform probability of occurrence for this event.

- The warrant can only be exercised if recipient is solvent. Another simulation forecast puts a 90 percent to 95 percent uniform probability of occurrence for this event.
- The protective put payout is the maximum spread between the put threshold price less client's common stock price or the warrant price.

The risk-free rate is based on the 10-year U.S. Treasury note. The volatility estimate is based on the client's historical weekly closing prices for the past one, two, and three years. The volatilities are estimated using the standard deviation of the natural logarithmic relative returns, annualized for a year, and then averaged. The dividend rate is assumed to be 0.51 percent based on available market data on client shares. The total probability of exceeding the gross margin threshold as well as solvency requirements is 80.9375 percent (calculated using the midpoint probability estimates of both independent events 87.50 percent times 92.50 percent, and were results based on simulation forecasts using historical financial data). The only method applicable in valuing such a protective put on a warrant is the use of binomial lattices. However, a Black-Scholes model is used to benchmark the results.

Warrant Valuation

In order to solve the warrant part of the exotic vehicle, high-level analysis rules need to be created:

- If the period ≥ 3 years, then at maturity, the profit maximizing decision is: Max (Exercise the warrant accounting for the probability the requirements are met; or let the warrant expire worthless otherwise).
- If the period ≥ 3 years, then prior to maturity, the profit maximizing decision is: Max (Exercise the warrant accounting for the probability the requirements are met; or keep the option open for future execution).
- If the period < 3 years, then hold on to the warrant as no execution is allowed.

Protective Put Option Valuation

The same is done on the protective put option:

- If the period ≥ 5 years, then at maturity, the profit maximizing decision is: Max (If the stock price is < $33, then exercise the warrant and collect the protective put payout, after accounting for the probability the requirements are met; or let the warrant and put option expire worthless).
- If the period ≥ 5 years, then prior to maturity, the profit maximizing decision is: Max (If the stock price is < $33, then exercise the warrant and

collect the protective put payout, after accounting for the probability the requirements are met; or keep the option open for future execution).

- If the period < 5 years, then hold on to the put option as no execution is allowed.

The binomial model used is a combination of Bermudan vesting nested option, where all the requirements (vesting periods, threshold price, probability of solvency, probability of exceeding gross margin requirements) have to be met before the warrant can be executed, and the put option can only be executed if the warrant is executed. However, the warrant can be executed even if the protective put is not executed.

Analytical Results

A summary of the results of the analysis follows. The results start with a decomposition of the warrant call and the protective put valued independently. These results are then compared to benchmarks to ascertain their accuracy and model reliability. Then, a combination of both instruments is valued in a mutually exclusive nested option model. The results of interest are the combined option model, but we can only obtain such a model by first decomposing the parts. The analysis was performed using the Super Lattice Solver software.

To follow along, you can start the Single Asset Super Lattice Solver software and load the relevant example files: *Case Study - Warrant - Warrant Only; Case Study - Warrant - Put Only*; and *Case Study - Warrant - Combined Value*.

A. Warrant at Grant Date

Naïve Black-Scholes (benchmark)	\$19.71
Adjusted Black-Scholes (benchmark)	\$15.95 (probability adjusted benchmark)
Binomial lattice (100 steps)	\$15.98 (using Super Lattice Solver)

As can be seen, the binomial lattice for the warrant converges to the Black-Scholes results. The reason for this convergence is that the dividend rate is low, making it not optimal to exercise early, but still worth slightly more than a simple European option. See Figure 11.7 for the details.

B. Protective Put Option at Grant Date

Static protection value (total)	\$1.5 million (100,000 warrants granted)
Static protection value (per warrant)	\$15.00 (guaranteed minimum)

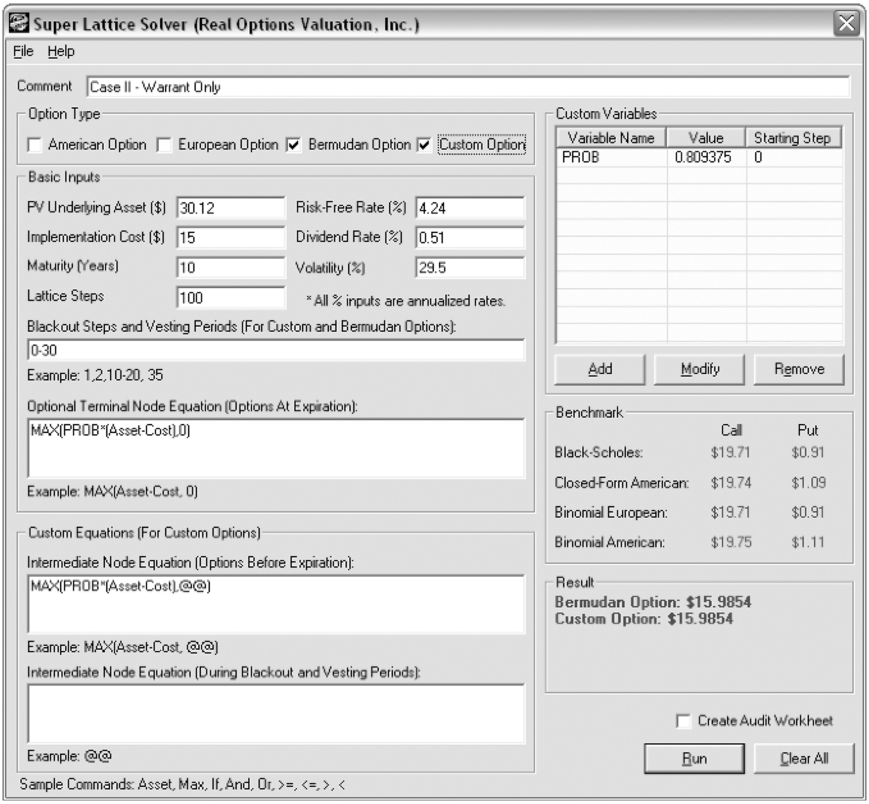


FIGURE 11.7 Warrant Valuation at Grant Date

Adjusted static protection value	\$12.14 (probability adjusted benchmark)
Binomial lattice (100 steps)	\$12.08 (using Super Lattice Solver)

The analysis can be seen in Figure 11.8.

The analysis up to this point decomposes the warrant call and the protective put options and their values are comparable to the static benchmarks, indicating that the models are correctly specified and the results are accurate. However, the warrant issues cannot be separated from the protective put because they are combined into one instrument. Separating them means that at certain points and conditions in time, the holder can both execute the call and also execute the put option protection with another call. This constitutes double-counting. Thus, in such a mutually exclusive condition (either a call is executed or a protective put is executed with the call, not both), a combination

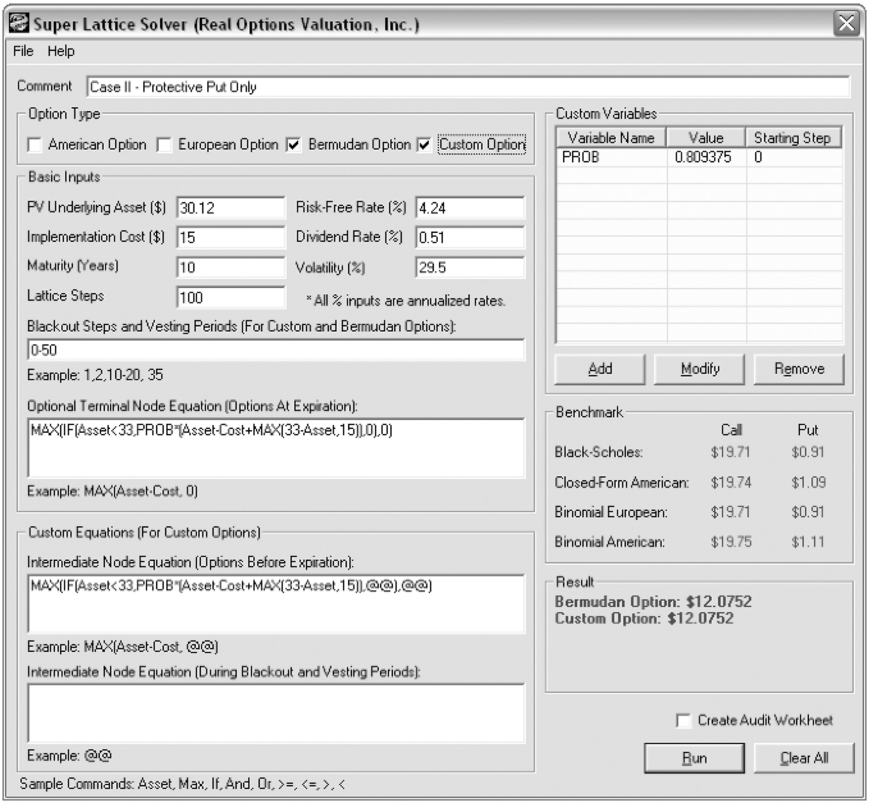


FIGURE 11.8 Protective Put at Grant Date

valuation is performed and the results are shown in the following list. Figure 11.9 illustrates the analysis performed.

C. Combination of Warrant and Protective Put Option at Grant Date

Black-Scholes call option	\$19.71 (benchmark)
Black-Scholes put option	\$ 0.91 (benchmark)
Combination of both Black-Scholes	\$20.62 (sum of both benchmarks)
Binomial lattice (100 steps)	\$22.37

Using Black-Scholes call and put option models as benchmarks, we see that the sought-after result of \$22.37 is valid, after considering that the decompositions of the model are also valid. Clearly the total combination value has to exceed the Black-Scholes as the warrant-put is an American op-

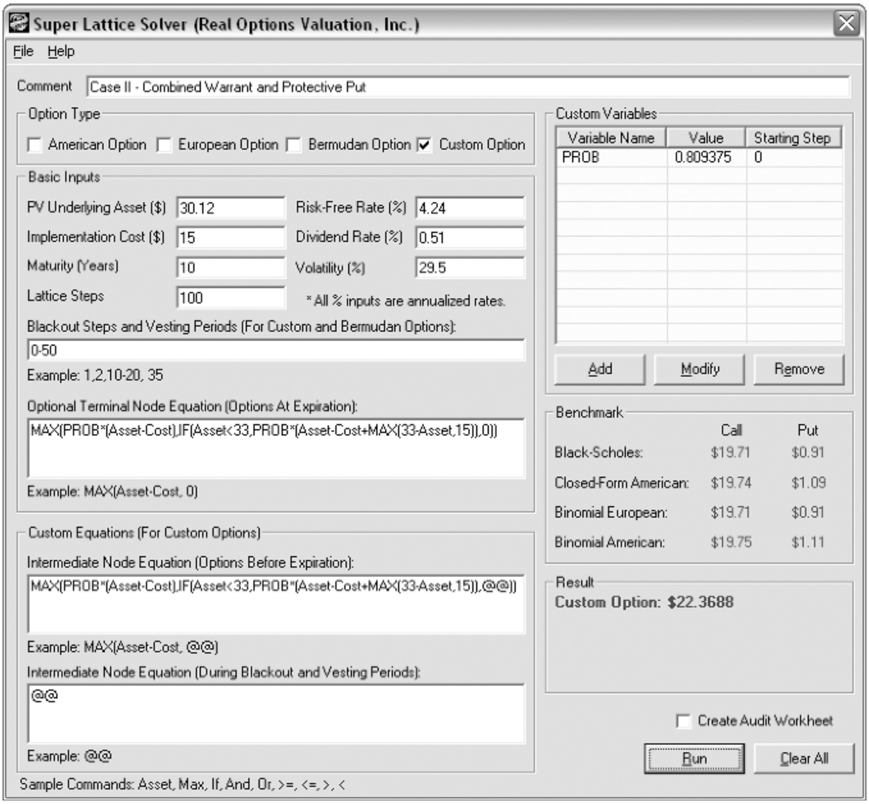


FIGURE 11.9 Combined Warrant with Protective Put at Grant Date

tion (with vesting requirements). To summarize, the analysis cannot be completed without the use of the Single Asset SLS software, and even when solving such complicated instruments, the pricing is relatively straightforward when using the software.

CASE 3: PHARMACEUTICAL DEVELOPMENT—VALUE OF PERFECT INFORMATION AND OPTIMAL TRIGGER VALUES

Suppose BioGen, a large multibillion dollar pharmaceutical firm is thinking of developing a new type of insulin that can be inhaled and the drug will directly be absorbed into the blood stream. This is indeed a novel and honorable idea. Imagine what this means to diabetics who no longer need painful and frequent injections. The problem is that this new type of insulin requires a brand new development effort but if the uncertainties of the market, competition, drug development, and FDA approval are high, perhaps a base insulin drug that can be ingested should first be developed. The ingestible version is a required precursor to the inhaled version. BioGen can decide to either take the risk and fast-track development into the inhaled version or buy an option to defer, to first wait and see if the ingestible version works. If this precursor works, then the firm has the option to expand into the inhaled version. How much should the firm be willing to spend on performing additional tests on the precursor and under what circumstances should the inhaled version be implemented directly?

Suppose that BioGen needs to spend \$100M on developing the inhaled version and if successful, the expected NPV is \$24M (i.e., \$124M PV Asset less the \$100M PV development cost). The probability of technical success, market prices, revenues, and operating expenses are all simulated in the discounted cash flow model and the resulting cash flow stream has a volatility of 22 percent. In contrast, if BioGen first develops the ingestible version, it will cost \$10M and take an entire year to develop, forcing the later phase development of the inhaled version to start one year later. Because this inhaled version uses similar precursors, the development cost is only \$95M and not \$100M. However, by being one year late to market, the PV Asset of doing an ingestible version before attempting the inhaled version will reduce to \$120M. This means that the ingestible–inhaled strategy will yield an NPV of \$15M. Figure 11.10 shows these two competing strategies.

Clearly, under an NPV analysis, the best approach is to pursue the inhaled version directly. However, when a real options analysis is performed by applying a two-phased sequential compound option, the total strategic value is

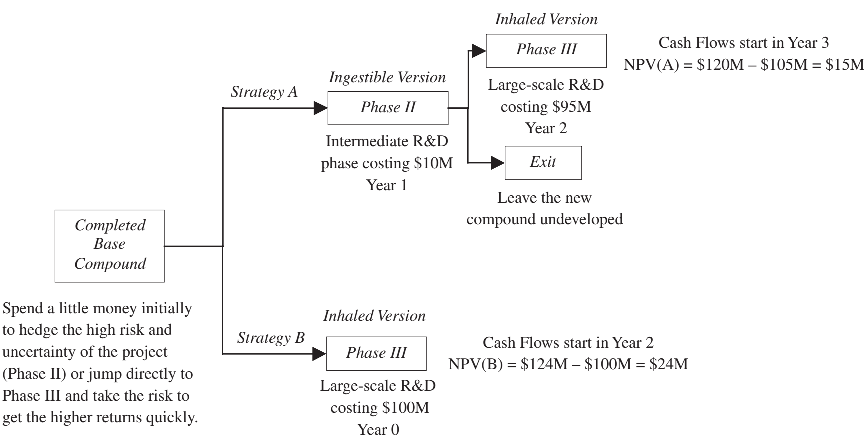


FIGURE 11.10 Strategy Tree for Pharmaceutical Development

found to be $27.24M as seen in Figure 11.11. The strategic option value of being able to defer investments and to wait and see until more information becomes available and uncertainties become resolved is worth $12.24M because the NPV is worth $15M ($120M – $10M – $95M). In other words, the *expected value of perfect information* is worth $12.24M, which indicates that the intermediate phase of developing an ingestible precursor can be used to obtain credible information to decide if further development is possible. The maximum the firm should be willing to spend in the ingestible intermediate phase is on average no more than $22.24M (i.e., $12.24M + $10M). If the cost to obtain the credible information exceeds this value, then it is optimal to take the risk and execute the entire project immediately at $100M or Strategy B. To follow along, open the MSLS file: *Solution to Chapter 11—Case III Strategy A* from the accompanying CD.

In contrast, if the volatility decreases (uncertainty and risk are lower), the strategic option value decreases. In addition, when the cost of waiting (as described by the *Dividend Rate* as a percentage of the *Asset Value*) increases, it is better not to defer and wait that long. Therefore, the higher the dividend rate, the lower the strategic option value. For instance, at a 17.20 percent dividend rate and 22 percent volatility, the resulting value reverts to the NPV of $15M (Figure 11.12), which means that the option value is zero, and that it is better to execute immediately as the cost of waiting far outstrips the value of being able to wait given the level of volatility (uncertainty and risk). Finally,

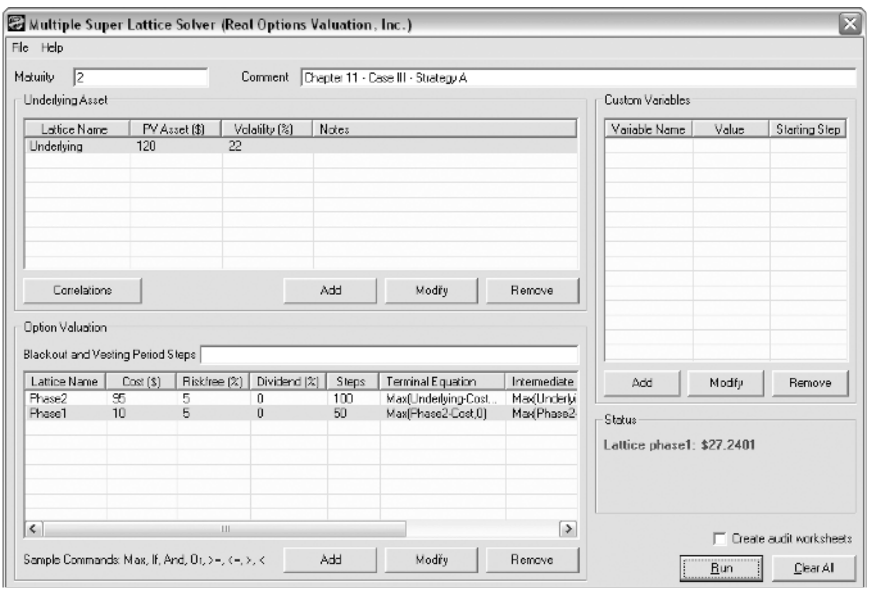


FIGURE 11.11 Value of Strategy A

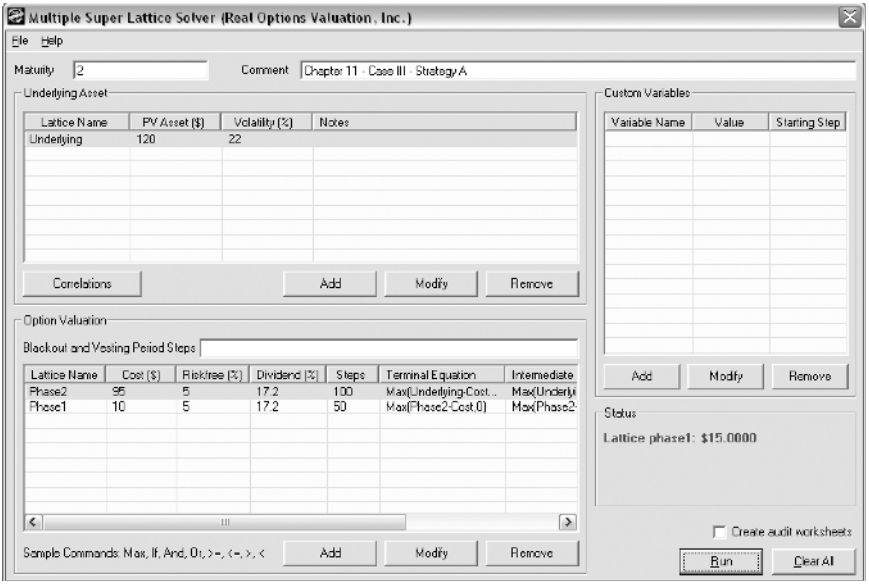


FIGURE 11.12 Sequential Compound Option's Break-Even Point

if risks and uncertainty increase significantly even with a high cost of waiting (e.g., 17.20 percent dividend rate at 30 percent volatility), it is still valuable to wait.

This model provides the decision maker with a view into the optimal balancing between *waiting for more information* (expected value of perfect information) and the *cost of waiting*. You can analyze this balance by creating strategic *options to defer* investments through development stages where at every stage the project is reevaluated as to whether it is beneficial to proceed to the next phase. You can vary the volatility and dividend inputs to determine their interactions—specifically, where the break-even points are for different combinations of volatilities and dividends. Thus, using this information, firms can make better go or no-go decisions (for instance, break-even volatility points can be traced back into the DCF model to estimate the probability of crossing over and this ability to wait becomes valuable).

CASE 4: OIL AND GAS—FARM OUTS, OPTIONS TO DEFER, AND VALUE OF INFORMATION

An oil and gas company, NewOil, is in the process of exploring a new field development. It intends to start drilling and exploring a region in Alaska for oil. Preliminary geologic and aerial surveys indicate that there is an optimal area for drilling. However, NewOil faces two major sources of uncertainties: market uncertainties (oil price volatility and economic conditions) and private uncertainties (geological area, porosity, oil pressure, reservoir size, etc.). Using comparable analysis and oil price forecasts, the development of this new oil field is expected to yield a sum PV of $200M, but drilling may cost up to $100M (in present values). The firm is trying to see if any strategic options exist to mitigate the market and private risks as well as to find the optimal strategic path.

Figure 11.13 represents the outcome of a strategic brainstorming activity at NewOil. Specifically, NewOil can simply take the risk and start drilling, shown as Strategy C. In this case, the NPV is found to be $100M, but there are tremendous amounts of risk in this strategy. To reduce the private risk, either a 3D seismic can be implemented or a series of test wells can be drilled. 3D-seismic studies will cost about $5M and take only 0.5 years to complete. The information obtained is fairly reliable but still contains significant amounts of uncertainty. In contrast, test wells can be drilled but will cost

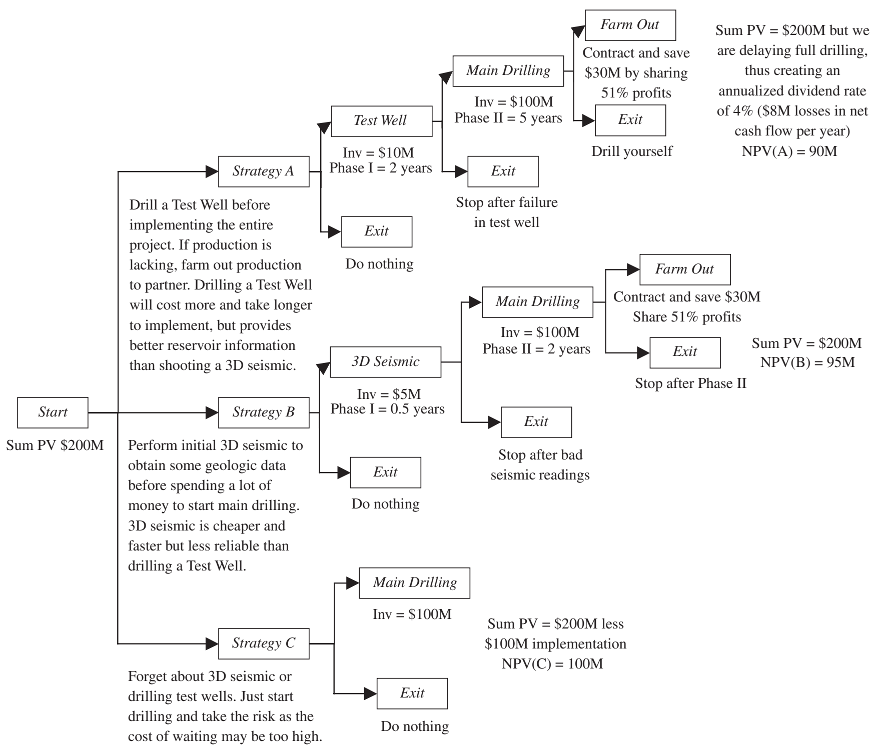


FIGURE 11.13 Strategy Tree for Oil and Gas

NewOil $10M and take two years to complete. However, test wells provide more solid and accurate information than seismic studies as the quality of the oil, the pressure, caps, porosity, and other geologic factors will become known over time as more test wells are drilled. Finally, to reduce the market risk, specifically the oil price reduction and volatility, NewOil has decided to execute a joint venture with LocalOil, a small second-tier oil and gas engineering company specializing in managing and running oil rigs. NewOil will provide LocalOil 49 percent of its gross profits from the oil field provided that LocalOil takes over the entire drilling operation at the request of NewOil. LocalOil benefits from a now fully developed oil field without any investments of its own, while NewOil benefits by pulling out its field personnel and saving $30M in total operating expenses during the remaining economic life of the oil rig, but it still captures a 51 percent share of the field's production. The question now is which strategic path in Figure 11.13 is optimal for NewOil? Should it take some risks and execute the seismic study or completely reduce the private risk through a series of test wells? Perhaps the risk is small enough such that the opportunity losses of putting off immediate development far surpass the risks taken and the oil field should be developed immediately.

Figure 11.14 shows the valuation of Strategy A. The NPV of Strategy A is $90M, which means that the option value is $33.74M ($123.74M – $90M). This means that putting off development by doing some test wells and farming out operations in the future when oil prices are not as high as NewOil expects is worth a lot. This total strategic value of $123.74 is now compared with the total strategic value of Strategy B valued at $129.58M (Figure 11.15). Clearly the cheaper and faster seismic study in Strategy B brings with it a higher volatility (Strategy B has a 35 percent volatility compared to 30 percent for Strategy A), and having the ability to farm out development of the field is worth more under such circumstances. In addition, the added dividend outflow in Strategy A reduces the option value of deferring and getting more valid information through drilling test wells, and, thus, the higher the cost of waiting and holding on to this option, the lower the option value.

CASE 5: VALUING EMPLOYEE STOCK OPTIONS UNDER 2004 FAS 123

In what the *Wall Street Journal* called "among the most far-reaching steps that the Financial Accounting Standards Board (FASB) has made in its 30 year history,"[1] in December 2004, FASB released a final Statement of Financial Accounting Standard 123 (FAS 123) on Share-Based Payment amending the old FAS 123 and 95 issued in October 1995.[2] Basically, the proposal states that starting June 15, 2005, all new and portions of existing employee stock

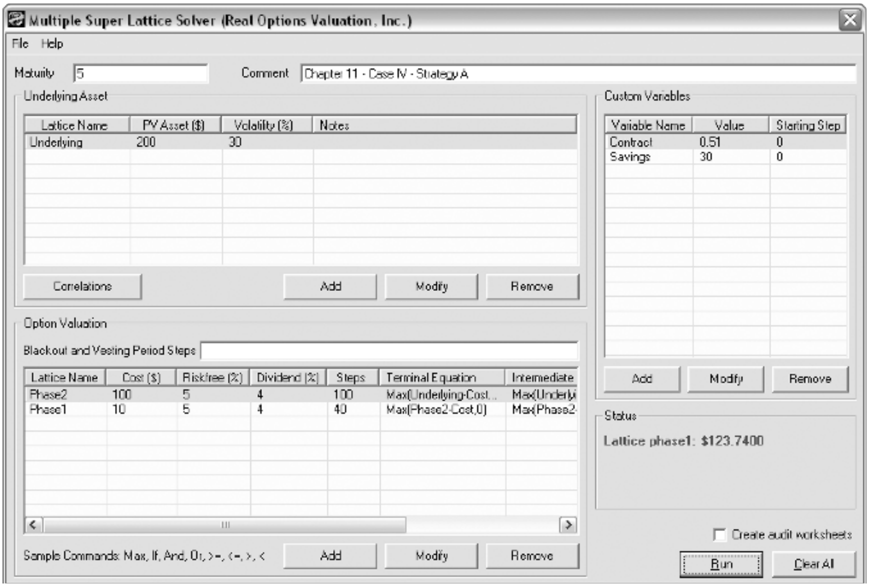


FIGURE 11.14 Value of Strategy A

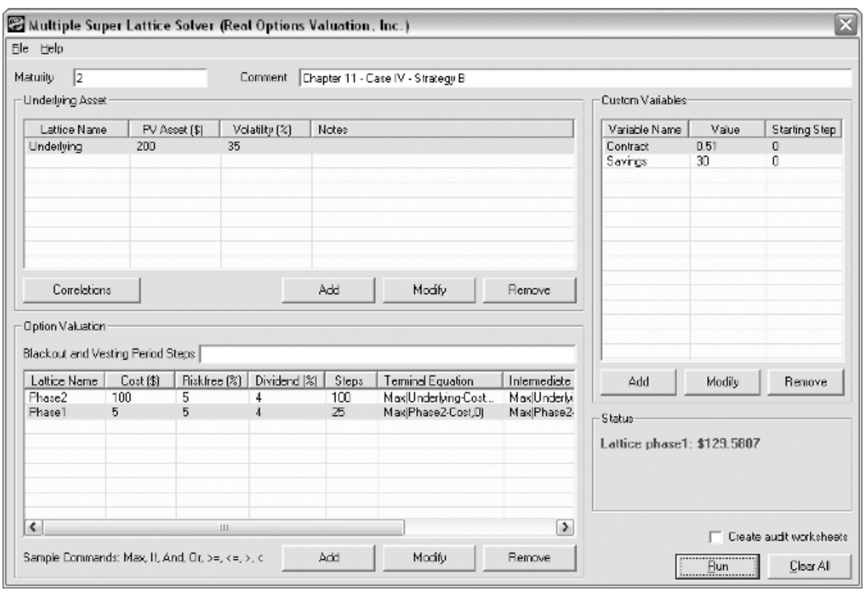


FIGURE 11.15 Value of Strategy B

option (ESO) awards that have not yet vested will have to be expensed. In anticipation of the standard, many companies such as GE and Coca-Cola have already voluntarily expensed their ESOs at the time of writing, while hundreds of other firms are now scrambling to look into valuing their ESOs.

The goal of this case is to provide the reader a better understanding of the valuation applications of FAS 123's preferred methodology—the binomial lattice—through a systematic and objective assessment of the methodology and comparing its results with the Black-Scholes model (BSM). It is shown in this case that with care, FAS 123 valuation can be implemented accurately. The analysis performed uses a customized binomial lattice that takes into account real-life conditions such as vesting, employee suboptimal exercise behavior, forfeiture rates, blackouts, changing dividends, risk-free rates, and volatilities over the life of the ESO. This portion of the chapter introduces the FAS 123 concept, followed by the different ESO valuation methodologies (closed-form BSM, binomial lattices, and Monte Carlo simulation) and their impacts on valuation. It is shown here that by using the right methodology that still conforms to the FAS 123 requirements, firms can potentially reduce their expenses by *millions of dollars* a year by avoiding the unnecessary overvaluation of the naïve BSM by using a modified and customized binomial lattice model that takes into account suboptimal exercise behavior, forfeiture rates, vesting, blackout dates, and changing inputs over time.

FASB used the SLS and ESO Valuation Toolkit created by the author to construct the example in FAS 123's Section A87 examples. It was this software application and the training seminars provided by the author for the Board of Directors at FASB, and one-on-one small group trainings for the project managers and fellows at FASB, that convinced them of the pragmatic applications of ESO valuation. See *Valuing Employee Stock Options* (Wiley, 2004) also by the author, for more technical details of valuing ESOs.

Introduction

One of the areas of concern is the fair-market valuation of ESOs. The binomial lattice is the preferred method in the FAS 123 requirements, but critics argue that companies do not necessarily have the resources in-house or the data availability to perform complex valuations that are both consistent with these new requirements as well as pass an audit. Based on a prior published study by the author that was presented to the FASB Board in 2003, it was concluded that the BSM, albeit theoretically correct and elegant, is insufficient and inappropriately applied when it comes to quantifying the fair-market value of ESOs.[3] This is because the BSM is applicable only to European options without dividends, where the holder of the option can exercise the option only on its maturity date and the underlying stock does not pay any dividends.[4] However, in reality, most ESOs are American-type[5] options with dividends, where the option holder can execute the option at any time up to and including the maturity date while the underlying stock pays dividends. In addition, under real-world conditions, ESOs have a time to *vesting* before the employee can execute the option, which may also be contingent on the firm and/or the individual employee attaining a specific performance level (e.g., profitability, growth rate, or stock price hitting a minimum barrier before the options become live) and are subject to *forfeitures* when the employee leaves the firm or is terminated prematurely before reaching the vested period. In addition, certain options follow a *tranching* or graduated scale, where a certain percentage of the stock option grants become exercisable every year.[6] Also, employees exhibit erratic exercise behavior where the option will be executed only if it exceeds a particular multiple of the strike price. This is termed the *suboptimal exercise behavior multiple*. Next, the option value may be sensitive to the expected economic environment, as characterized by the term structure of interest rates (i.e., the U.S. Treasuries yield curve) where the risk-free rate changes during the life of the option. Finally, the firm may undergo some corporate restructuring (e.g., divestitures, or mergers and acquisitions that may require a stock swap that changes the volatility of the underlying stock). All these real-life scenarios make the BSM insufficient and inappropriate when used to place a fair-market value on the option grant.[7] In summary, firms can implement a variety of provisions that affect the fair value of the options where the foregoing list is only a few examples. The closed-form models such as the BSM or the Generalized Black-Scholes (GBM)—the latter accounts for the inclusion of dividend yields—are inflexible and cannot be modified to accommodate these real-life conditions. Hence, the binomial lattice approach is preferred.

Under very specific conditions (European options without dividends), the binomial lattice and Monte Carlo simulation approaches yield identical values to the BSM, indicating that the two former approaches are robust and exact at the limit. However, when specific real-life business conditions are modeled (i.e., probability of forfeiture, probability the employee leaves or is terminated, time-vesting, suboptimal exercise behavior, etc.), only the binomial lattice with its highly flexible nature will provide the true fair-market value of the ESO. The BSM only takes into account the following inputs: stock price, strike price, time to maturity, a single risk-free rate, and a single volatility. The GBM accounts for the same inputs as well as a single dividend rate. Hence, in accordance to the FAS 123 requirements, the BSM and GBM fail to account for real-life conditions. On the contrary, the binomial lattice can be customized to include the stock price, strike price, time to maturity, a single risk-free rate and/or multiple risk-free rates changing over time, a single volatility and/or multiple volatilities changing over time, a single dividend rate and/or multiple dividend rates changing over time, plus all the other real-life factors including but not limited to vesting periods, suboptimal early exercise behavior, blackout periods, forfeiture rates, stock price and performance barriers, and other exotic contingencies. Note that the binomial lattice results revert to the GBM if these real-life conditions are negligible.

The two most important and most convincing arguments for using binomial lattices are: (1) FASB requires it and states that the binomial lattice is the preferred method for ESO valuation and (2) lattices can substantially reduce the cost of the ESO by more appropriately mirroring real-life conditions. Following is a sample of FAS 123's requirements discussing the use of binomial lattices.

> B64. As discussed in paragraphs A10–A17, closed-form models are one acceptable technique for estimating the fair value of employee share options. However, a lattice model (or other valuation technique, such as a Monte Carlo simulation technique, that is not based on a closed-form equation) can accommodate the term structures of risk-free interest rates and expected volatility, as well as expected changes in dividends over an option's contractual term. *A lattice model also can accommodate estimates of employees' option exercise patterns and postvesting employment termination during the option's contractual term, and thereby can more fully reflect the effect of those factors than can an estimate developed using a closed-form model and a single weighted-average expected life of the options* [author's emphasis].
>
> A15. The Black-Scholes-Merton formula assumes that option exercises occur at the end of an option's contractual term, and that expected volatility, expected dividends, and risk-free interest rates are constant over the option's term. If used to estimate the fair value of instruments in the scope of this Statement, the Black-Scholes-Merton formula must be adjusted to take account of certain characteristics of employee share options and similar instruments that are not consistent with the model's assumptions (for example, the ability to exercise before the end of the option's contractual term). Because of the nature of the formula, those adjustments take the form of weighted average assumptions about those characteristics. *In contrast, a lattice model can be designed to accommodate dynamic assumptions of expected volatility and dividends over the option's contractual term and estimates of expected option exercise patterns during the option's contractual term, including the effect of blackout periods. Therefore, the design of a lattice model more fully reflects the substantive characteristics of a particular employee share option or similar instrument* [author's emphasis]. Nevertheless, both a lattice model and the Black-Scholes-Merton formula, as well as other valuation techniques that meet the requirements in paragraph A8, can provide a fair value estimate that is consistent with the measurement objective and fair-value-based method of this Statement. However, if an entity uses a lattice model that has been modified to take into account

> an option's contractual term and employees' expected exercise and postvesting employment termination behavior, the expected term is estimated based on the resulting output of the lattice. For example, an entity's experience might indicate that option holders tend to exercise their options when the share price reaches two hundred percent of the exercise price. If so, that entity might use a lattice model that assumes exercise of the option at each node along each share price path in a lattice at which the early exercise expectation is met, provided that the option is vested and exercisable at that point. Moreover, such a model would assume exercise at the end of the contractual term on price paths along which the exercise expectation is not met but the options are in-the-money at the end of the contractual term. *That method recognizes that employees' exercise behavior is correlated with the price of the underlying share. Employees' expected postvesting employment termination behavior also would be factored in. Expected term, which is a required disclosure (paragraph A240), then could be estimated based on the output of the resulting lattice* [author's emphasis].

In fact, some parts of the FAS 123 Final Requirements cannot be modeled with a traditional Black-Scholes model. A lattice is required to model items such as suboptimal exercise behavior multiple, forfeiture rates, vesting, blackout periods, and so forth. This case study and the software used to compute the results use both a binomial (and trinomial) lattice and closed-form Black-Scholes models to compare the results. The specific paragraphs describing the use of lattices include:

> A27. *However, if an entity uses a lattice model that has been modified to take into account an option's contractual term and employees' expected exercise and postvesting employment termination behavior, the expected term is estimated based on the resulting output of the lattice* [author's emphasis]. For example, an entity's experience might indicate that option holders tend to exercise their options when the share price reaches two hundred percent of the exercise price. If so, that entity might use a lattice model that assumes exercise of the option at each node along each share price path in a lattice at which the early exercise expectation is met, provided that the option is vested and exercisable at that point.
>
> A28. Other factors that may affect expectations about *employees' exercise and postvesting employment termination behavior* [author's emphasis here and in list] include the following:
>
> a. The vesting period of the award. An option's expected term must at least include the vesting period.
> b. *Employees' historical exercise and postvesting employment termination behavior* for similar grants.

c. Expected volatility of the price of the underlying share.
d. *Blackout periods* and other coexisting arrangements such as agreements that allow for exercise to automatically occur during blackout periods if certain conditions are satisfied.
e. Employees' ages, lengths of service, and home jurisdictions (that is, domestic or foreign).

Therefore, based on the justifications above, and in accordance with the requirements and recommendations set forth by the revised FAS 123, which prefers the binomial lattice, it is hereby concluded that the customized binomial lattice is the best and preferred methodology to calculate the fair-market value of ESOs.

Application of the Preferred Method

In applying the customized binomial lattice methodology, several inputs must be determined:

- Stock price at grant date.
- Strike price of the option grant.
- Time to maturity of the option.
- Risk-free rate over the life of the option.
- Dividend yield of the option's underlying stock over the life of the option.
- Volatility over the life of the option.
- Vesting period of the option grant.
- Suboptimal exercise behavior multiples over the life of the option.
- Forfeiture and employee turnover rates over the life of the option.
- Blackout dates postvesting when the options cannot be exercised.

The analysis assumes that the employee cannot exercise the option when it is still in the vesting period.

Further, if the employee is terminated or decides to leave voluntarily during this vesting period, the option grant will be forfeited and presumed worthless. In contrast, after the options have been vested, employees tend to exhibit erratic exercise behavior where an option will be exercised only if it breaches the suboptimal exercise behavior multiple.[8] However, the options that have vested must be exercised within a short period if the employee leaves voluntarily or is terminated, regardless of the suboptimal behavior threshold—that is, if forfeiture occurs (measured by the historical option forfeiture rates as well as employee turnover rates). Finally, if the option expiration date has been reached, the option will be exercised if it is in-the-money and expire worthless if it is at-the-money or out-of-the-money. The next section details the results obtained from such an analysis.

Technical Justification of Methodology Employed

This section illustrates some of the technical justifications that make up the price differential between the GBM and the customized binomial lattice models. Figure 11.16 shows a tornado chart and how each input variable in a customized binomial lattice drives the value of the option.[9] Based on the chart, it is clear that volatility is not the single key variable that drives option value. In fact, when vesting, forfeiture, and suboptimal behavior elements are added to the model, their effects dominate that of volatility. The chart illustrated is based on a typical case and cannot be generalized across all cases.[10]

In contrast and as can be seen in Figure 11.17, volatility is a significant variable in a simple BSM because there is less interaction among input variables due to the fewer input variables, and for most ESOs that are issued at-the-money, volatility plays an important part when there are no other dominant inputs.

In addition, the interactions among these new input variables are nonlinear. Figure 11.18 shows a spider chart[11] and it can be seen that vesting, forfeiture rates, and suboptimal exercise behavior multiples have nonlinear effects on option value. That is, the lines in the spider chart are not straight but curve at certain areas, indicating that there are nonlinear effects in the model. This means that we cannot generalize these three variables' effects on option value (for instance, we cannot generalize that if a 1 percent increase in forfeiture rate will decrease option value by 2.35 percent, it means that a

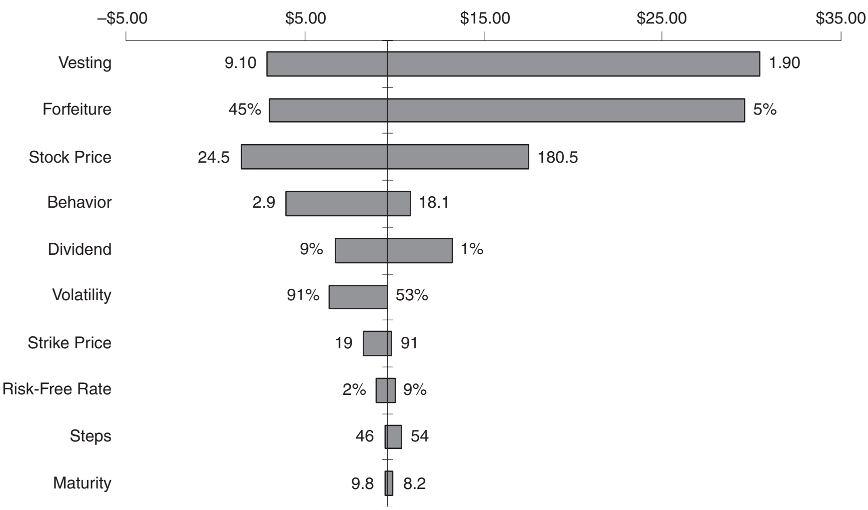


FIGURE 11.16 Tornado Chart Listing the Critical Input Factors of a Customized Binomial Model

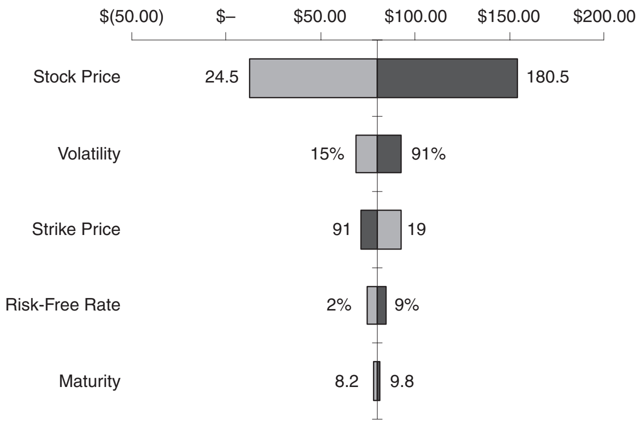


FIGURE 11.17 Tornado Chart Listing the Critical Input Factors of the BSM

2 percent increase in forfeiture rate drives option value down 4.70 percent, and so forth) because the variables interact differently at different input levels. The conclusion is that we really cannot say *a priori* what the direct effects on the magnitude of the final option value are of changing one variable. More detailed analysis will have to be performed in each case.

Although the tornado and spider charts illustrate the impact of each input variable on the final option value, its effects are static. That is, one variable is tweaked at a time to determine its ramifications on the option value.

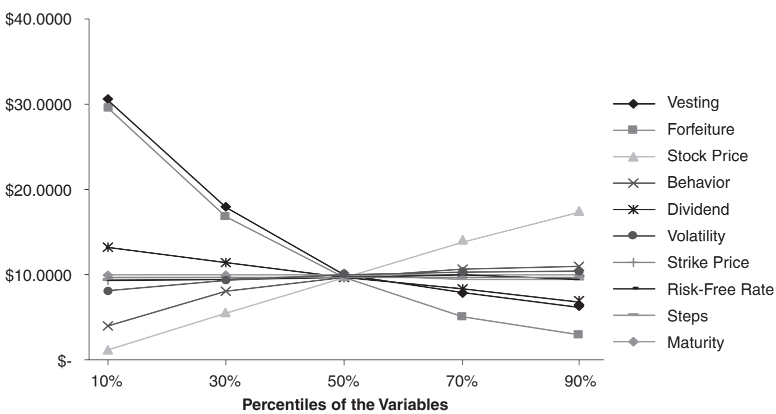


FIGURE 11.18 Spider Chart Showing the Nonlinear Effects of Input Factors in the Binomial Model

However, as shown, the effects are sometimes nonlinear, which means we need to change all variables simultaneously to account for their interactions. Figure 11.19 shows a Monte Carlo simulated dynamic sensitivity chart where forfeiture, vesting, and suboptimal exercise behavior multiple are determined to be important variables, while volatility is again relegated to a less important role. The dynamic sensitivity chart perturbs all input variables simultaneously for thousands of trials, and captures the effects on the option value. This approach is valuable in capturing the net interaction effects among variables at different input levels.

From this preliminary sensitivity analysis, we conclude that incorporating forfeiture rates, vesting, and suboptimal exercise behavior multiple is vital to obtaining a fair-market valuation of ESOs due to their significant contributions to option value. In addition, we cannot generalize each input's effects on the final option value. Detailed analysis has to be performed to obtain the option's value every time.

Options with Vesting and Suboptimal Behavior

Further investigation into the elements of suboptimal behavior and vesting[12] yields the chart shown in Figure 11.20.[13] Here we see that at lower suboptimal exercise behavior multiples (within the range of 1 to 6) the stock option value can be significantly lower than that predicted by the BSM. With a 10-year vesting stock option, the results are identical regardless of the suboptimal exercise behavior multiple—its flat line bears the same value as the BSM result because for a 10-year vesting of a 10-year maturity option, the option reverts to a perfect European option, where it can be exercised only at expiration. The BSM provides the correct result in this case. However, when suboptimal exercise behavior multiple is low, the option value decreases because employees holding the option will tend to exercise the option

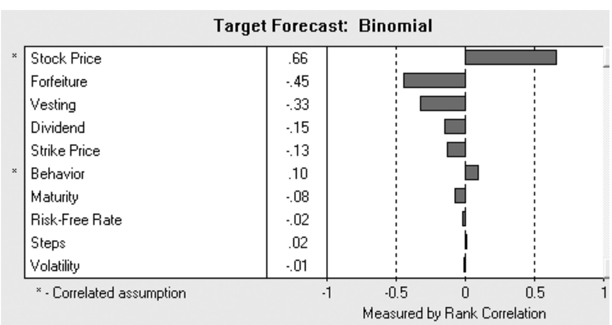


FIGURE 11.19 Dynamic Sensitivity with Simultaneously Changing Input Factors in the Binomial Model

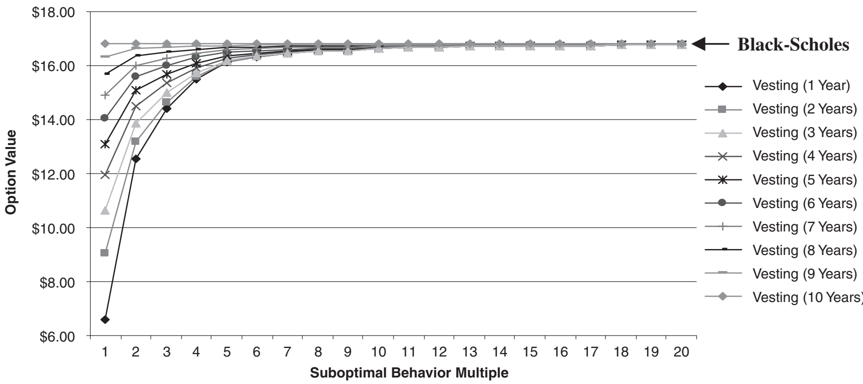


FIGURE 11.20 Impact of Suboptimal Exercise Behavior and Vesting on Option Value in the Binomial Model

suboptimally—that is, the option will be exercised earlier and at a lower stock price than optimal. Hence, the option's upside value is not maximized.

As an example, suppose an option's strike price is $10 while the underlying stock is highly volatile. If an employee exercises the option at $11 (this means a 1.10 suboptimal exercise multiple), he or she may not be capturing the entire upside potential of the option as the stock price can go up significantly higher than $11 depending on the underlying volatility. Compare this to another employee who exercises the option when the stock price is $20 (suboptimal exercise multiple of 2.0) versus one who does so at a much higher stock price. Thus, lower suboptimal exercise behavior means a lower fair-market value of the stock option. This suboptimal exercise behavior has a higher impact when stock prices at grant date are forecast to be high. Figure 11.21 shows that (at the lower end of the suboptimal multiples) a steeper slope occurs the higher the initial stock price at grant date.[14]

Figure 11.22 shows that for higher volatility stocks, the suboptimal region is larger and the impact to option value is greater, but the effect is gradual.[15] For instance, for the 100 percent volatility stock (Figure 11.22), the suboptimal region extends from a suboptimal exercise behavior multiple of 1.0 to approximately 9.0 versus from 1.0 to 2.0 for the 10 percent volatility stock. In addition, the vertical distance of the 100 percent volatility stock extends from $12 to $22 with a $10 range, as compared to $2 to $10 with an $8 range for the 10 percent volatility stock. Therefore, the higher the stock price at grant date and the higher the volatility, the greater the impact of suboptimal behavior will be on the option value. In all cases, the BSM results are the horizontal lines in the charts (Figures 11.21 and 11.22). That is, the BSM will always generate the maximum option value assuming optimal behavior and overexpense the option significantly. A GBM or BSM cannot be modified

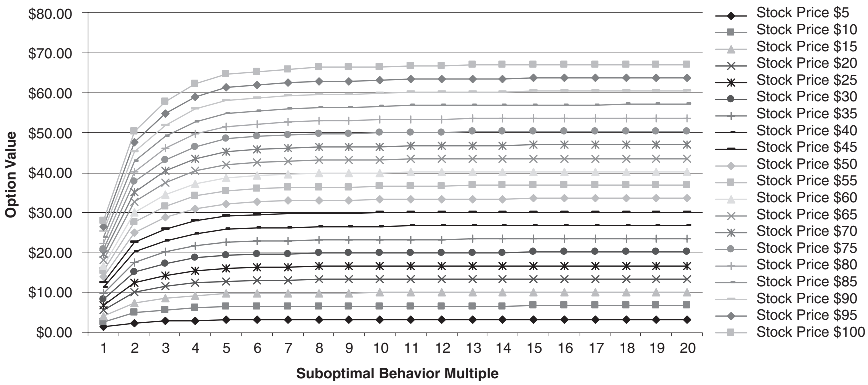


FIGURE 11.21 Impact of Suboptimal Exercise Behavior and Stock Price on Option Value in the Binomial Model

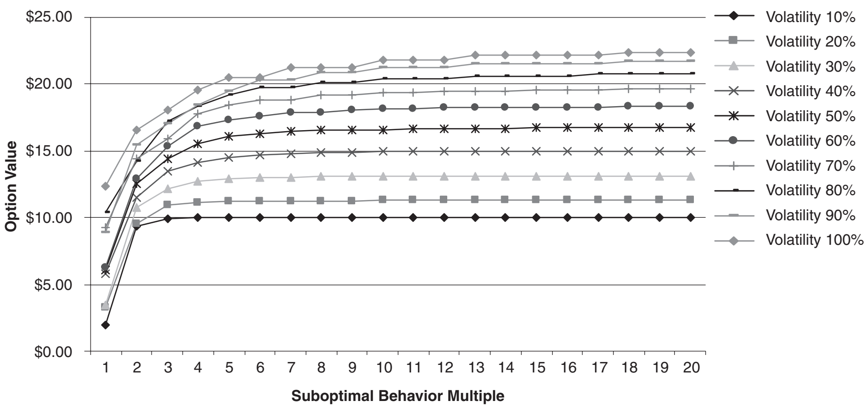


FIGURE 11.22 Impact of Suboptimal Exercise Behavior and Volatility on Option Value in the Binomial Model

to account for this suboptimal exercise behavior. Only the binomial lattice can be used.

Options with Forfeiture Rates

Figure 11.23 illustrates the reduction in option value when the forfeiture rate increases.[16] The rate of reduction changes depending on the vesting period. The longer the vesting period, the more significant the impact of forfeitures will be. This illustrates once again the nonlinear interacting relationship between vesting and forfeitures (that is, the lines in Figure 11.23 are curved and nonlinear). This is intuitive because the longer the vesting period, the lower

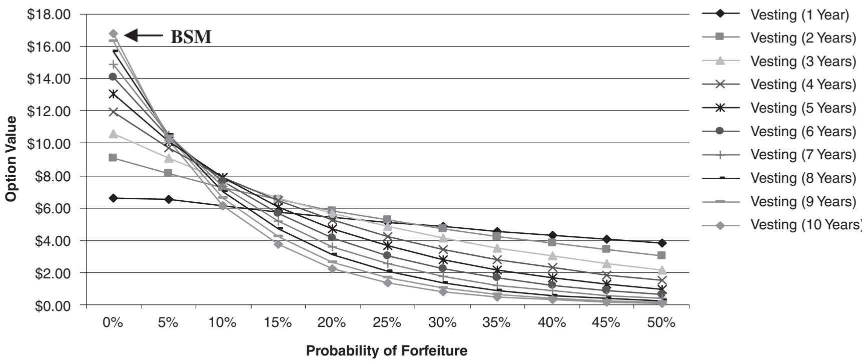


FIGURE 11.23 Impact of Forfeiture Rates and Vesting on Option Value in the Binomial Model

the compounded probability that an employee will still be employed in the firm and the higher the chances of forfeiture, reducing the expected value of the option. Again, we see that the BSM result is the highest possible value assuming a 10-year vesting in a 10-year maturity option with zero forfeiture (Figure 11.23). In addition, forfeiture rates can be negatively correlated to stock price—if the firm is doing well, its stock price usually increases, making the option more valuable and making the employees less likely to leave and the firm less likely to lay off its employees. Because the rate of forfeitures is uncertain (forfeiture rate fluctuations typically occur in the past due to business and economic environments, and will most certainly fluctuate again in the future) and is negatively correlated to the stock price, we can also apply a correlated Monte Carlo simulation on forfeiture rates in conjunction with the customized binomial lattices—this is shown later in this case study. The BSM will always generate the maximum option value assuming all options will fully vest and overexpense the option significantly. The ESO Valuation software can account for forfeiture rates, while the accompanying Super Lattice Solver can account for different prevesting and postvesting forfeiture rates in the lattices.

Options Where Risk-Free Rate Changes Over Time

Another input assumption is the risk-free rate. Table 11.2 illustrates the effects of changing risk-free rates over time on option valuation. When other exotic inputs are added, the changing risk-free lattice model has an overall lower valuation. In addition, due to the time-value-of-money, discounting more heavily in the future will reduce the option's value. In other words, Table 11.2 compares an upward sloping yield curve, a downward sloping yield curve, risk-free rate smile, and risk-free rate frown. When the term struc-

TABLE 11.2 Effects of Changing Risk-Free Rates on Option Value

Basic Input Parameters		Year	Static Base Case (%)	Increasing Risk-Free Rates (%)	Decreasing Risk-Free Rates (%)	Risk-Free Rate Smile (%)	Risk-Free Rate Frown (%)
Stock Price	$100.00	1	5.50	1.00	10.00	8.00	3.50
Strike Price	$100.00	2	5.50	2.00	9.00	7.00	4.00
Maturity	10.00	3	5.50	3.00	8.00	5.00	5.00
Volatility	45.00%	4	5.50	4.00	7.00	4.00	7.00
Dividend Rate	4.00%	5	5.50	5.00	6.00	3.50	8.00
Lattice Steps	1000	6	5.50	6.00	5.00	3.50	8.00
Suboptimal Behavior	1.80	7	5.50	7.00	4.00	4.00	7.00
Vesting Period	4.00	8	5.50	8.00	3.00	5.00	5.00
Forfeiture Rate	10.00%	9	5.50	9.00	2.00	7.00	4.00
		10	5.50	10.00	1.00	8.00	3.50
		Average	5.50	5.50	5.50	5.50	5.50
		BSM using 5.50% Average Rate	$37.45	$37.45	$37.45	$37.45	$37.45
		Forfeiture Modified BSM using 5.5% Average Rate	$33.71	$33.71	$33.71	$33.71	$33.71
		Changing Risk-Free Binomial Lattice	$25.92	$24.31	$27.59	$26.04	$25.76

ture of interest rates increases over time, the option value calculated using a customized changing risk-free rate binomial lattice is lower ($24.31) than that calculated using an average of the changing risk-free rates ($25.92) base case. The reverse is true for a downward-sloping yield curve. In addition, Table 11.2 shows a risk-free yield curve frown (low rates followed by high rates followed by low rates) and a risk-free yield curve smile (high rates followed by low rates followed by high rates). The results[17] indicate that using a single average rate will overestimate an upward-sloping yield curve, underestimate a downward-sloping yield curve, underestimate a yield curve smile, and overestimate a yield curve frown. Therefore, whenever appropriate, use all available information in terms of forward risk-free rates, one rate for each year.

Options Where Volatility Changes Over Time

Table 11.3 illustrates the effects of changing volatilities on an ESO. If volatility changes over time, the BSM ($71.48) using the average volatility over time will *always* overestimate the true option value when there are other exotic inputs. In addition, compared to the $38.93 base case, slowly increasing volatilities over time from a low level has lower option values, while a decreasing volatility from high values and volatility smiles and frowns have higher values than using the average volatility estimate.

Options Where Dividend Yield Changes Over Time

Dividend yield is a simple input that can be obtained from corporate dividend policies or publicly available historical market data. Dividend yield is the total dividend payments computed as a percentage of stock price that is paid out over the course of a year. The typical dividend yield is between 0 percent and 7 percent. In fact, about 45 percent of all publicly traded firms in the United States pay dividends. Of those who pay a dividend, 85 percent have a yield of 7 percent or below, and 95 percent have a yield of 10 percent or below.[18] Dividend yield is an interesting variable with very little interaction with other exotic input variables. It has a close to linear effect on option value, whereas the other exotic input variables do not. For instance, Table 11.4 illustrates the effects of different maturities on the same option.[19] The higher the maturity, the higher the option value, but the option value, increases at a decreasing rate.

In contrast, Table 11.5 illustrates the near-linear effects of dividends even when some of the exotic inputs have been changed. Whatever the change in variable is, the effects of dividends are always very close to linear. While Table 11.5 illustrates many options with unique dividend rates, Table 11.6 illustrates the effects of changing dividends over time on a single option. That is, Table 11.5's results are based on comparing different options with different

TABLE 11.3 Effects of Changing Volatilities on Option Value

Basic Input Parameters		Year	Static Base Case (%)	Increasing Volatilities (%)	Decreasing Volatilities (%)	Volatility Smile (%)	Volatility Frown (%)
Stock Price	$100.00	1	55.00	10.00	100.00	80.00	35.00
Strike Price	$100.00	2	55.00	20.00	90.00	70.00	40.00
Maturity	10.00	3	55.00	30.00	80.00	50.00	50.00
Risk-Free Rate	5.50%	4	55.00	40.00	70.00	40.00	70.00
Dividend Rate	0.00%	5	55.00	50.00	60.00	35.00	80.00
Lattice Steps	10	6	55.00	60.00	50.00	35.00	80.00
Suboptimal Behavior	1.80	7	55.00	70.00	40.00	40.00	70.00
Vesting Period	4.00	8	55.00	80.00	30.00	50.00	50.00
Forfeiture Rate	10.00%	9	55.00	90.00	20.00	70.00	40.00
		10	55.00	100.00	10.00	80.00	35.00
		Average	55.00	55.00	55.00	55.00	55.00
		BSM using 55% Average Rate	$71.48	$71.48	$71.48	$71.48	$71.48
		Forfeiture Modified BSM using 55% Average Rate	$64.34	$64.34	$64.34	$64.34	$64.34
		Changing Volatilities Binomial Lattice	$38.93	$32.35	$45.96	$39.56	$39.71

TABLE 11.4 Nonlinear Effects of Maturity

	1.8 Behavior Multiple, 1-Year Vesting, 10% Forfeiture Rate	
Maturity	**Option Value ($)**	**Change (%)**
1	25.16	—
2	32.41	28.84
3	35.35	9.08
4	36.80	4.08
5	37.87	2.91
6	38.41	1.44
7	38.58	0.43

dividend rates, whereas Table 11.6's results are based on a single option whose underlying stock's dividend yields are changing over the life of the option.[20]

Clearly, a changing-dividend option has some value to add in terms of the overall option valuation results. Therefore, if the firm's stock pays a dividend, then the analysis should also consider the possibility of dividend yields changing over the life of the option.

Options Where Blackout Periods Exist

Another item of interest is blackout periods. These are the dates that ESOs cannot be executed. These dates are usually several weeks before and several weeks after an earnings announcement (usually on a quarterly basis). In addition, only senior executives with fiduciary responsibilities have these blackout dates, and, hence, their proportion is relatively small compared to the rest of the firm. Table 11.7 illustrates the calculations of a typical ESO with different blackout dates.[21] In the case where there are only a few blackout days a month, there is little difference between options with blackout dates and those without blackout dates. In fact, if the suboptimal exercise behavior multiple is small (a 1.8 ratio is assumed in this case), blackout dates at strategic times will actually prevent the option holder from exercising suboptimally and sometimes even increase the value of the option ever so slightly.

Table 11.7's analysis assumes only a small percentage of blackout dates in a year (for example, during several days in a year, the ESO cannot be executed). This may be the case for certain so-called brick-and-mortar companies, and, as such, blackout dates can be ignored. However, in other firms such as those in the biotechnology and high-tech industries, blackout periods play a

TABLE 11.5 Linear Effects of Dividends

	1.8 Behavior Multiple, 4-Year Vesting, 10% Forfeiture Rate		1.8 Behavior Multiple, 1-Year Vesting, 10% Forfeiture Rate		3.0 Behavior Multiple, 1-Year Vesting, 10% Forfeiture Rate		$50 Stock Price, 1.8 Behavior Multiple, 1-Year Vesting, 10% Forfeiture Rate		1.8 Behavior Multiple, 1-Year Vesting, 5% Forfeiture Rate	
Dividend Rate (%)	Option Value ($)	Change (%)	Option Value ($)	Change (%)	Option Value ($)	Change (%)	Option Value ($)	Change (%)	Option Value ($)	Change (%)
0	42.15		42.41		49.07		21.20		45.46	
1	39.94	–5.24	41.47	–2.20	47.67	–2.86	20.74	–2.20	44.46	–2.20
2	37.84	–5.27	40.55	–2.22	46.29	–2.89	20.28	–2.22	43.47	–2.23
3	35.83	–5.30	39.65	–2.24	44.94	–2.92	19.82	–2.24	42.49	–2.25
4	33.92	–5.33	38.75	–2.26	43.61	–2.95	19.37	–2.26	41.43	–2.27
5	32.10	–5.37	37.87	–2.28	42.31	–2.98	18.93	–2.28	40.58	–2.29

TABLE 11.6 Effects of Changing Dividends Over Time

Scenario	Option Value ($)	Change (%)	Notes
Static 3% Dividend	39.65	0.00	Dividends are kept steady at 3%
Increasing Gradually	40.94	3.26	1% to 5% with 1% increments (average of 3%)
Decreasing Gradually	38.39	–3.17	5% to 1% with –1% increments (average of 3%)
Increasing Jumps	41.70	5.19	0%, 0%, 5%, 5%, 5% (average of 3%)
Decreasing Jumps	38.16	–3.74	5%, 5%, 5%, 0%, 0% (average of 3%)

TABLE 11.7 Effects of Blackout Periods on Option Value

Blackout Dates	Option Value ($)
No blackouts	43.16
Every 2 years evenly spaced	43.16
First 5 years annual blackouts only	43.26
Last 5 years annual blackouts only	43.16
Every 3 months for 10 years	43.26

more significant role. For instance, in a biotech firm, blackout periods may extend four to six weeks every quarter, straddling the release of its quarterly earnings. In addition, blackout periods prior to the release of a new product may exist. Therefore, the proportion of blackout dates with respect to the life of the option may reach upward of 35 percent to 65 percent per year. In such cases, blackout periods will significantly affect the value of the option. For instance, Table 11.8 illustrates the differences between a customized binomial lattice with and without blackout periods.[22] By adding in the real-life elements of blackout periods, the ESO value is further reduced by anywhere between 10 percent and 35 percent depending on the rate of forfeiture and volatility. As expected, the reduction in value is nonlinear, as the effects of blackout periods vary depending on the other input variables involved in the analysis.

Table 11.9 shows the effects of blackouts under different dividend yields and vesting periods, whereas Table 11.10 illustrates the results stemming from different dividend yields and suboptimal exercise behavior multiples. Clearly, it is almost impossible to predict the exact impact unless a detailed analysis

TABLE 11.8 Effects of Significant Blackouts (Different Forfeiture Rates and Volatilities)

% Difference between no blackout periods versus significant blackouts	Volatility (25%)	Volatility (30%)	Volatility (35%)	Volatility (40%)	Volatility (45%)	Volatility (50%)
Forfeiture Rate (5%)	–17.33%	–13.18%	–10.26%	–9.21%	–7.11%	–5.95%
Forfeiture Rate (6%)	–19.85%	–15.17%	–11.80%	–10.53%	–8.20%	–6.84%
Forfeiture Rate (7%)	–22.20%	–17.06%	–13.29%	–11.80%	–9.25%	–7.70%
Forfeiture Rate (8%)	–24.40%	–18.84%	–14.71%	–13.03%	–10.27%	–8.55%
Forfeiture Rate (9%)	–26.44%	–20.54%	–16.07%	–14.21%	–11.26%	–9.37%
Forfeiture Rate (10%)	–28.34%	–22.15%	–17.38%	–15.35%	–12.22%	–10.17%
Forfeiture Rate (11%)	–30.12%	–23.67%	–18.64%	–16.45%	–13.15%	–10.94%
Forfeiture Rate (12%)	–31.78%	–25.11%	–19.84%	–17.51%	–14.05%	–11.70%
Forfeiture Rate (13%)	–33.32%	–26.48%	–21.00%	–18.53%	–14.93%	–12.44%
Forfeiture Rate (14%)	–34.77%	–27.78%	–22.11%	–19.51%	–15.78%	–13.15%
Forfeiture Rate (14%)	–34.77%	–27.78%	–22.11%	–19.51%	–15.78%	–13.15%

TABLE 11.9 Effects of Significant Blackouts (Different Dividend Yields and Vesting Periods)

% Difference between no blackout periods versus significant blackouts	Vesting (1)	Vesting (2)	Vesting (3)	Vesting (4)
Dividends (0%)	–8.62%	–6.93%	–5.59%	–4.55%
Dividends (1%)	–9.04%	–7.29%	–5.91%	–4.84%
Dividends (2%)	–9.46%	–7.66%	–6.24%	–5.13%
Dividends (3%)	–9.90%	–8.03%	–6.56%	–5.43%
Dividends (4%)	–10.34%	–8.41%	–6.90%	–5.73%
Dividends (5%)	–10.80%	–8.79%	–7.24%	–6.04%
Dividends (6%)	–11.26%	–9.18%	–7.58%	–6.35%
Dividends (7%)	–11.74%	–9.58%	–7.93%	–6.67%
Dividends (8%)	–12.22%	–9.99%	–8.29%	–6.99%
Dividends (9%)	–12.71%	–10.40%	–8.65%	–7.31%
Dividends (10%)	–13.22%	–10.81%	–9.01%	–7.64%

is performed, but the range can be generalized to be typically between 10 percent and 20 percent. Blackout periods can only be modeled in a binomial lattice and not in the BSM/GBM.

Nonmarketability Issues

The 2004 FAS 123 revision does not explicitly discuss the issue of nonmarketability. That is, ESOs are neither directly transferable to someone else nor freely tradable in the open market. Under such circumstances, it can be argued based on sound financial and economic theory that a nontradable and nonmarketable discount can be appropriately applied to the ESO. However, this is not a simple task, as will be discussed.

A simple and direct application of a discount should not be based on an arbitrarily chosen percentage *haircut* on the resulting binomial lattice result. Instead, a more rigorous analysis can be performed using a put option. A call option is the contractual right, but not the obligation, to purchase the underlying stock at some predetermined contractual strike price within a specified time, while a put option is a contractual right, but not the obligation, to sell the underlying stock at some predetermined contractual price within a specified time. Therefore, if the holder of the ESO cannot sell or transfer the rights of the option to someone else, then the holder of the option has given up his or her rights to a put option (that is, the employee has written or sold the firm a put option). Calculating the put option and discounting this value from the call option provides a theoretically correct and justifiable nonmarketability and nontransferability discount to the existing option.

TABLE 11.10 Effects of Significant Blackouts (Different Dividend Yields and Behaviors)

% Difference between no blackout periods versus significant blackouts	Dividends (0%)	Dividends (1%)	Dividends (2%)	Dividends (3%)	Dividends (4%)	Dividends (5%)	Dividends (6%)	Dividends (7%)	Dividends (8%)	Dividends (9%)	Dividends (10%)
Suboptimal Behavior Multiple (1.8)	–1.01%	–1.29%	–1.58%	–1.87%	–2.16%	–2.45%	–2.75%	–3.06%	–3.36%	–3.67%	–3.98%
Suboptimal Behavior Multiple (1.9)	–1.01%	–1.29%	–1.58%	–1.87%	–2.16%	–2.45%	–2.75%	–3.06%	–3.36%	–3.67%	–3.98%
Suboptimal Behavior Multiple (2.0)	–1.87%	–2.29%	–2.72%	–3.15%	–3.59%	–4.04%	–4.50%	–4.96%	–5.42%	–5.90%	–6.38%
Suboptimal Behavior Multiple (2.1)	–1.87%	–2.29%	–2.72%	–3.15%	–3.59%	–4.04%	–4.50%	–4.96%	–5.42%	–5.90%	–6.38%
Suboptimal Behavior Multiple (2.2)	–4.71%	–5.05%	–5.39%	–5.74%	–6.10%	–6.46%	–6.82%	–7.19%	–7.57%	–7.95%	–8.34%
Suboptimal Behavior Multiple (2.3)	–4.71%	–5.05%	–5.39%	–5.74%	–6.10%	–6.46%	–6.82%	–7.19%	–7.57%	–7.95%	–8.34%
Suboptimal Behavior Multiple (2.4)	–4.71%	–5.05%	–5.39%	–5.74%	–6.10%	–6.46%	–6.82%	–7.19%	–7.57%	–7.95%	–8.34%
Suboptimal Behavior Multiple (2.5)	–6.34%	–6.80%	–7.28%	–7.77%	–8.26%	–8.76%	–9.27%	–9.79%	–10.32%	–10.86%	–11.41%
Suboptimal Behavior Multiple (2.6)	–6.34%	–6.80%	–7.28%	–7.77%	–8.26%	–8.76%	–9.27%	–9.79%	–10.32%	–10.86%	–11.41%
Suboptimal Behavior Multiple (2.7)	–6.34%	–6.80%	–7.28%	–7.77%	–8.26%	–8.76%	–9.27%	–9.79%	–10.32%	–10.86%	–11.41%
Suboptimal Behavior Multiple (2.8)	–6.34%	–6.80%	–7.28%	–7.77%	–8.26%	–8.76%	–9.27%	–9.79%	–10.32%	–10.86%	–11.41%
Suboptimal Behavior Multiple (2.9)	–8.62%	–9.04%	–9.46%	–9.90%	–10.34%	–10.80%	–11.26%	–11.74%	–12.22%	–12.71%	–13.22%
Suboptimal Behavior Multiple (3.0)	–8.62%	–9.04%	–9.46%	–9.90%	–10.34%	–10.80%	–11.26%	–11.74%	–12.22%	–12.71%	–13.22%

However, care should be taken in analyzing this haircut or discounting feature. The same inputs that go into the customized binomial lattice to calculate a call option should also be used to calculate a customized binomial lattice for a put option. That is, the put option must also be under the same risks (volatility that can change over time), economic environment (risk-free rate structure that can change over time), corporate financial policy (a static or changing dividend yield over the life of the option), contractual obligations (vesting, maturity, strike price, and blackout dates), investor irrationality (suboptimal exercise behavior), firm performance (stock price at grant date), and so forth.

Albeit nonmarketability discounts or haircuts are not explicitly discussed in FAS 123, the valuation analysis is performed here anyway, for the sake of completeness. It is up to each firm's management to decide if haircuts should and can be applied. Table 11.11 shows the customized binomial lattice valuation results of a typical ESO.[23] Table 11.12 shows the results from a nonmarketability analysis performed using a down-and-in upper barrier modified put option with the same exotic inputs (vesting, blackouts, forfeitures, suboptimal behavior, and so forth) calculated using the customized binomial lattice model.[24] The discounts range from 22 percent to 53 percent. These calculated discounts look somewhat significant but are actually in line with market expectations.[25] As these discounts are not explicitly sanctioned by FASB, the author cautions against their use in determining the fair market value of the ESOs.

Expected Life Analysis

As seen previously, the 2004 Final FAS 123, Sections A15 and B64 expressly prohibit the use of a modified BSM with a single expected life. This means that instead of using an expected life as the *input* into the BSM to obtain the similar results as in a customized binomial lattice, the analysis should be done the other way around. That is, using vesting requirements, suboptimal exercise behavior multiples, forfeiture or employee turnover rates, and the other standard option inputs, calculate the valuation results using the customized binomial lattice. This result can then be compared with a modified BSM and the expected life can then be *imputed*. Excel's goal-seek function can be used to obtain the imputed expected life of the option by setting the BSM result equal to the customized binomial lattice. The resulting expected life can then be compared with historical data as a secondary verification of the results, that is, if the expected life falls within reasonable bounds based on historical performance. This is the correct approach because measuring the expected life of an option is very difficult and inaccurate.

Table 11.13 illustrates the use of Excel's goal-seek function on the ESO Valuation Toolkit software to impute the expected life into the BSM model by setting the BSM results equal to the customized binomial lattice results.

TABLE 11.11 Customized Binomial Lattice Valuation Results

Customized Binomial Lattice (Option Valuation)	Behavior (1.20)	Behavior (1.40)	Behavior (1.60)	Behavior (1.80)	Behavior (2.00)	Behavior (2.20)	Behavior (2.40)	Behavior (2.60)	Behavior (2.80)	Behavior (3.00)
Forfeiture (0.00%)	$24.57	$30.53	$36.16	$39.90	$43.15	$45.87	$48.09	$49.33	$50.40	$51.31
Forfeiture (5.00%)	$22.69	$27.65	$32.19	$35.15	$37.67	$39.74	$41.42	$42.34	$43.13	$43.80
Forfeiture (10.00%)	$21.04	$25.22	$28.93	$31.29	$33.27	$34.88	$36.16	$36.86	$37.45	$37.94
Forfeiture (15.00%)	$19.58	$23.13	$26.20	$28.11	$29.69	$30.94	$31.93	$32.46	$32.91	$33.29
Forfeiture (20.00%)	$18.28	$21.32	$23.88	$25.44	$26.71	$27.70	$28.48	$28.89	$29.23	$29.52
Forfeiture (25.00%)	$17.10	$19.73	$21.89	$23.17	$24.20	$25.00	$25.61	$25.93	$26.19	$26.41
Forfeiture (30.00%)	$16.02	$18.31	$20.14	$21.21	$22.06	$22.70	$23.19	$23.44	$23.65	$23.82
Forfeiture (35.00%)	$15.04	$17.04	$18.61	$19.51	$20.20	$20.73	$21.12	$21.32	$21.49	$21.62
Forfeiture (40.00%)	$14.13	$15.89	$17.24	$18.00	$18.58	$19.01	$19.33	$19.49	$19.63	$19.73

TABLE 11.12 Nonmarketability and Nontransferability Discount

Haircut (Customized Binomial Lattice Modified Put)	Behavior (1.20)	Behavior (1.40)	Behavior (1.60)	Behavior (1.80)	Behavior (2.00)	Behavior (2.20)	Behavior (2.40)	Behavior (2.60)	Behavior (2.80)	Behavior (3.00)
Forfeiture (0.00%)	$11.33	$11.33	$11.33	$11.33	$11.33	$11.33	$11.33	$11.33	$11.33	$11.33
Forfeiture (5.00%)	$10.76	$10.76	$10.76	$10.76	$10.76	$10.76	$10.76	$10.76	$10.76	$10.76
Forfeiture (10.00%)	$10.23	$10.23	$10.23	$10.23	$10.23	$10.23	$10.23	$10.23	$10.23	$10.23
Forfeiture (15.00%)	$9.72	$9.72	$9.72	$9.72	$9.72	$9.72	$9.72	$9.72	$9.72	$9.72
Forfeiture (20.00%)	$9.23	$9.23	$9.23	$9.23	$9.23	$9.23	$9.23	$9.23	$9.23	$9.23
Forfeiture (25.00%)	$8.77	$8.77	$8.77	$8.77	$8.77	$8.77	$8.77	$8.77	$8.77	$8.77
Forfeiture (30.00%)	$8.34	$8.34	$8.34	$8.34	$8.34	$8.34	$8.34	$8.34	$8.34	$8.34
Forfeiture (35.00%)	$7.92	$7.92	$7.92	$7.92	$7.92	$7.92	$7.92	$7.92	$7.92	$7.92
Forfeiture (40.00%)	$7.52	$7.52	$7.52	$7.52	$7.52	$7.52	$7.52	$7.52	$7.52	$7.52
Nonmarketability and Nontransferability Discount (%)	**Behavior (1.20)**	**Behavior (1.40)**	**Behavior (1.60)**	**Behavior (1.80)**	**Behavior (2.00)**	**Behavior (2.20)**	**Behavior (2.40)**	**Behavior (2.60)**	**Behavior (2.80)**	**Behavior (3.00)**
Forfeiture (0.00%)	46.09%	37.09%	31.32%	28.39%	26.25%	24.69%	23.55%	22.96%	22.47%	22.07%
Forfeiture (5.00%)	47.43%	38.92%	33.43%	30.62%	28.57%	27.08%	25.98%	25.42%	24.95%	24.57%
Forfeiture (10.00%)	48.60%	40.55%	35.35%	32.68%	30.73%	29.32%	28.28%	27.75%	27.31%	26.95%
Forfeiture (15.00%)	49.62%	42.01%	37.08%	34.57%	32.73%	31.40%	30.43%	29.93%	29.53%	29.19%
Forfeiture (20.00%)	50.52%	43.31%	38.66%	36.29%	34.57%	33.33%	32.42%	31.96%	31.59%	31.28%
Forfeiture (25.00%)	51.32%	44.48%	40.09%	37.86%	36.25%	35.10%	34.26%	33.84%	33.49%	33.22%
Forfeiture (30.00%)	52.03%	45.53%	41.38%	39.29%	37.79%	36.72%	35.95%	35.56%	35.25%	35.00%
Forfeiture (35.00%)	52.67%	46.48%	42.56%	40.60%	39.20%	38.21%	37.50%	37.15%	36.86%	36.63%
Forfeiture (40.00%)	53.24%	47.34%	43.64%	41.80%	40.49%	39.57%	38.92%	38.60%	38.34%	38.14%

TABLE 11.13 Imputing the Expected Life for the BSM Using the Binomial Lattice Results Applying Different Suboptimal Behavior Multiples

Stock Price	$20.00	$20.00	$20.00	$20.00	$20.00	$20.00	$20.00
Strike Price	$20.00	$20.00	$20.00	$20.00	$20.00	$20.00	$20.00
Maturity	10.00	10.00	10.00	10.00	10.00	10.00	10.00
Risk-Free Rate	3.50%	3.50%	3.50%	3.50%	3.50%	3.50%	3.50%
Dividend	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Volatility	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%
Vesting	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Suboptimal Behavior	**1.10**	**1.50**	**2.00**	**2.50**	**3.00**	**3.50**	**4.00**
Forfeiture Rate	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Lattice Steps	1000	1000	1000	1000	1000	1000	1000
Binomial	$8.94	$10.28	$11.03	$11.62	$11.89	$12.18	$12.29
BSM	$12.87	$12.87	$12.87	$12.87	$12.87	$12.87	$12.87
Expected Life	**4.42**	**5.94**	**6.95**	**7.83**	**8.26**	**8.74**	**8.93**
Modified BSM	$8.94	$10.28	$11.03	$11.62	$11.89	$12.18	$12.29

Table 11.14 illustrates another case where the expected life can be imputed, but this time the forfeiture rates are not set at zero. In this case, the BSM results will need to be modified. For example, the customized binomial lattice result of $5.41 is obtained with a 15 percent forfeiture rate. This means that the BSM result needs to be BSM(1 – 15%) = $5.41 using the modified expected life method. The expected life that yields the BSM value of $6.36 ($5.41/85% is $6.36, and $6.36[1 – 15%] is $5.41) is 2.22 years.

Dilution

In most cases, the effects of dilution can be safely ignored as the proportion of ESO grants is relatively small compared to the total equity issued by the company. In investment finance theory, the market has already anticipated the exercise of these ESOs and the effects have already been accounted for in the stock price. Once a new grant is announced, the stock price will immediately and fully incorporate this news and account for any dilution that may occur. This means that as long as the valuation is performed after the announcement is made, then the effects of dilution are nonexistent. The 2004 FAS 123 revisions do not explicitly provide guidance in this area. Given that FASB only provides little guidance on dilution (Section A39), and because forecasting stock prices (as part of estimating the effects of dilution) is fairly difficult and inaccurate at best, plus the fact that the dilution effects are minimal (small in proportion compared to all the equity issued by the firm), the effects of dilution are assumed to be minimal and can be safely ignored.

Applying Monte Carlo Simulation for Statistical Confidence and Precision Control

Next, Monte Carlo simulation can be applied to obtain a range of calculated stock option fair values. That is, any of the inputs into the stock options valuation model can be chosen for Monte Carlo simulation if they are uncertain and stochastic. Distributional assumptions are assigned to these variables, and the resulting option values using the BSM, GBM, path simulation, or binomial lattices are selected as forecast cells. These modeled uncertainties include the probability of forfeiture and the employees' suboptimal exercise behavior.

The results of the simulation are essentially a distribution of the stock option values. Keep in mind that the simulation application here is used to vary the inputs to an options valuation model to obtain a range of results, not to model and calculate the options themselves. However, simulation can be applied both to simulate the inputs to obtain the range of options results and also to solve the options model through path-dependent simulation.

Monte Carlo simulation, named after the famous gambling capital of Monaco, is a very potent methodology. Monte Carlo simulation creates artificial futures by generating thousands and even millions of sample paths of

TABLE 11.14 Imputing Expected Life for the BSM Using Lattice Results Under Nonzero Forfeiture Rates and Applying Different Forfeiture Rates

Stock Price	$20.00	$20.00	$20.00	$20.00	$20.00	$20.00	$20.00
Strike Price	$20.00	$20.00	$20.00	$20.00	$20.00	$20.00	$20.00
Maturity	10.00	10.00	10.00	10.00	10.00	10.00	10.00
Risk-Free Rate	3.50%	3.50%	3.50%	3.50%	3.50%	3.50%	3.50%
Dividend	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Volatility	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%
Vesting	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Suboptimal Behavior	1.50	1.50	1.50	1.50	1.50	1.50	1.50
Forfeiture Rate	**0.00%**	**2.50%**	**5.00%**	**7.50%**	**10.00%**	**12.50%**	**15.00%**
Lattice Steps	1000	1000	1000	1000	1000	1000	1000
Binomial	$10.28	$9.23	$8.29	$7.44	$6.69	$6.02	$5.41
BSM	$12.87	$12.87	$12.87	$12.87	$12.87	$12.87	$12.87
Expected Life	**5.94**	**4.71**	**3.77**	**3.03**	**2.45**	**1.99**	**1.61**
Modified BSM[a]	$10.28	$9.23	$8.29	$7.44	$6.69	$6.02	$5.41
Expected Life	**5.94**	**4.97**	**4.19**	**3.55**	**3.02**	**2.59**	**2.22**
Modified BSM[b]	$10.28	$9.23	$8.29	$7.44	$6.69	$6.02	$5.41

[a]Uses the binomial lattice results to impute the expected life for a modified BSM.
[b]Uses the binomial lattice but also accounts for the forfeiture rate to modify the BSM.

outcomes and looks at their prevalent characteristics. Its simplest form is a random number generator that is useful for forecasting, estimation, and risk analysis. A simulation calculates numerous scenarios of a model by repeatedly picking values from a user-predefined *probability distribution* for the uncertain variables and using those values for the model. As all those scenarios produce associated results in a model, each scenario can have a *forecast*. Forecasts are events (usually with formulas or functions) that you define as important outputs of the model.

Simplistically, think of the Monte Carlo simulation approach as picking golf balls out of a large basket repeatedly with replacement, as seen in the example presented next. The size and shape of the basket depend on the distributional *assumptions* (e.g., a normal distribution with a mean of 100 and a standard deviation of 10 versus a uniform distribution or a triangular distribution) where some baskets are deeper or more symmetrical than others, allowing certain balls to be pulled out more frequently than others. The number of balls pulled repeatedly depends on the number of *trials* simulated. For a large model with multiple related assumptions, imagine the large model as a very large basket, where many baby baskets reside. Each baby basket has its own set of golf balls that are bouncing around. Sometimes these baby baskets are linked to each other (if there is a *correlation* between the variables) and the golf balls are bouncing in tandem while others are bouncing independently of one another. The balls that are picked each time from these interactions within the model (the mother of all baskets) are tabulated and recorded, providing a *forecast* result of the simulation. Of course the balls are colored differently for identification and for representing their respective frequencies.

These concepts can be applied to ESO valuation. For instance, the simulated input assumptions are those inputs that are highly uncertain and can vary in the future, such as stock price at grant date, volatility, forfeiture rates, and suboptimal exercise behavior multiples. Clearly, variables that are objectively obtained, such as risk-free rates (U.S. Treasury yields for the next 1 month to 20 years are published), dividend yield (determined from corporate strategy), vesting period, strike price, and blackout periods (determined contractually in the option grant) should not be simulated. In addition, the simulated input assumptions can be correlated. For instance, forfeiture rates can be negatively correlated to stock price—if the firm is doing well, its stock price usually increases, making the option more valuable thus making the employees less likely to leave and the firm less likely to layoff its employees. Finally, the output forecasts are the option valuation results.

The analysis results will be distributions of thousands of option valuation results, where all the uncertain inputs are allowed to vary according to their distributional assumptions and correlations, and the customized binomial lattice model will take care of their interactions. The resulting average (if the distribution is not skewed) or median (if the distribution is highly skewed) option value is used. Hence, instead of using single-point estimates of the inputs to provide a single-point estimate of option valuation, all possible contingencies, scenarios, and possibilities in the input variables will be accounted for in the analysis through Monte Carlo simulation. In fact, Monte Carlo simulation is allowed and recommended in FAS 123 (Sections B64 and B65, and footnotes 48, 52, 74, and 97).

Table 11.15 shows the results obtained using the customized binomial lattices based on single-point inputs of all the variables. The model takes exotic inputs such as vesting, forfeiture rates, suboptimal exercise behavior multiples, blackout periods, and changing inputs (dividends, risk-free rates, and volatilities) over time. The resulting option value is $31.42. This analysis can then be extended to include simulation. Figure 11.24 illustrates the use of simulation coupled with customized binomial lattices.[26]

Rather than randomly deciding on the correct number of trials to run in the simulation, statistical significance and precision control are set up to run the required number of trials automatically. A 99.9 percent statistical confidence on a $0.01 error precision control was selected and 145,510 simulation trials were run.[27] This highly stringent set of parameters means that an

TABLE 11.15 Single-Point Result Using a Customized Binomial Lattice

Risk-Free Rate		Volatility		Dividend Yield		Suboptimal Behavior	
Year	Rate	Year	Rate	Year	Rate	Year	Multiple
1	3.50%	1	35.00%	1	1.00%	1	1.80
2	3.75	2	35.00	2	1.00	2	1.80
3	4.00	3	35.00	3	1.00	3	1.80
4	4.15	4	45.00	4	1.50	4	1.80
5	4.20	5	45.00	5	1.50	5	1.80

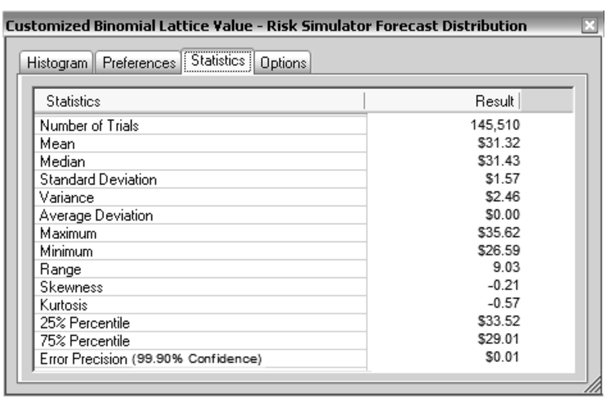
		Forfeiture Rate		Blackout Dates	
		Year	Rate	Month	Step
Stock Price	$100	1	5.00%	12	12
Strike Price	$100	2	5.00	24	24
Time to Maturity	5	3	5.00	36	36
Vesting Period	1	4	5.00	48	48
Lattice Steps	60	5	5.00	60	60
Option Value	**$31.42**				

adequate number of trials will be run to ensure that the results will fall within a \$0.01 error variability 99.9 percent of the time. For instance, the simulated average result was \$31.32 (Figure 11.24). This means that 999 out of 1,000 times, the true option value will be accurate to within \$0.01 of \$31.32. These measures are statistically valid and objective.[28]

A Sample Case Study

The case study here goes through selecting and justifying each input parameter in the customized binomial lattice model and showcases some of the results generated in the analysis. Some of the more analytically intensive but equally important aspects have been omitted for the sake of brevity. This case is based on several real-life consulting projects performed by the author but their values have been sufficiently changed to maintain confidentiality. Nonetheless, the essence of the case remains.

Stock Price and Strike Price The first two inputs into the customized binomial lattice are the stock price and strike price. For the ESOs issued, the strike price is always set at the stock price at grant date such that the ESOs are granted at-the-money. This means obtaining the stock price will also yield the strike price. Table 11.16 was provided by the client firm's investor relations department. A conservative and aggressive closing stock price was provided for a period of 24 months, generated using growth curve estimations. For instance, the closing stock price for December 2004 is estimated to be between \$45.17 and \$50.70. In order to perform due diligence on the stock price forecast at grant date, several other approaches were used. Twelve analyst expectations were obtained and their results were averaged. In addition, econometric



Statistics	Result
Number of Trials	145,510
Mean	\$31.32
Median	\$31.43
Standard Deviation	\$1.57
Variance	\$2.46
Average Deviation	\$0.00
Maximum	\$35.62
Minimum	\$26.59
Range	9.03
Skewness	-0.21
Kurtosis	-0.57
25% Percentile	\$33.52
75% Percentile	\$29.01
Error Precision (99.90% Confidence)	\$0.01

FIGURE 11.24 Options Valuations Result at \$0.01 Precision with 99.9 Percent Confidence

TABLE 11.16 Per Share Stock Price Forecast (in Dollars) from Investor Relations

Grant Date	Conservative	Aggressive	Comment
4-Mar-04	37.51	37.51	Actual
2-Apr-04	33.40	33.40	Actual
May-04	34.87	35.56	Computed
Jun-04	36.34	37.72	Computed
Jul-04	37.81	39.88	Computed
Aug-04	39.28	42.05	Computed
Sep-04	40.75	44.21	Computed
Oct-04	42.22	46.37	Computed
Nov-04	43.69	48.53	Computed
Dec-04	45.17	50.70	Per Investor Relations
Jan-05	45.89	51.52	Computed
Feb-05	46.61	52.34	Computed
Mar-05	47.34	53.16	Computed
Apr-05	48.06	53.98	Computed
May-05	48.78	54.81	Computed
Jun-05	49.51	55.63	Computed
Jul-05	50.23	56.45	Computed
Aug-05	50.95	57.27	Computed
Sep-05	51.68	58.09	Computed
Oct-05	52.40	58.92	Computed
Nov-05	53.13	59.74	Computed
Dec-05	53.85	60.56	Per Investor Relations

modeling with Monte Carlo simulation was used to forecast the stock price. Using a stochastic Brownian Motion simulation model (Figure 11.25), the average stock price was forecast to be $47.22 (Figure 11.26), consistent with the investor relations stock price. The valuation analysis will use all three stock prices and the final result used will be the average of these three stock price forecasts.

Maturity The next input is the option's maturity date. The contractual maturity date is 10 years on each option issue and is consistent throughout the entire ESO plan. Therefore, 10 years is used as the input in the binomial lattice model.

Risk-Free Rates The next input parameter is the risk-free rate. A detailed listing of the U.S. Treasury spot yields were downloaded from www.ustreas.gov.

Using the spot yield curve, the spot rates were bootstrapped to obtain the forward yield curve as seen in Table 11.17. Spot rates are the interest rates from time zero to some time in the future. For instance, a two-year spot rate

Brownian Motion with Drift

Starting Value	**$31.95**
Annualized Drift	**60.00%**
Annualized Volatility	**80.44%**
Forecast Horizon	**0.67**
Granularity	**100.00**
Step Size	**0.0067**

This model illustrates the Brownian Motion stochastic process with a drift rate. An example application includes the simulation of a stock price path. This model requires Risk Simulator to run. Click on the *Single Step* button to perform a stepwise simulation and see why it is so difficult to predict stock prices. Click on *Run Simulation* to estimate the distribution of stock prices at certain time intervals.

Enter some values into the colored input boxes above (default values are $100 for starting value, 10% for annualized drift, 45% for annualized volatility, and 1 for forecast horizon).

Time	Asset Value	Simulate
0.0000	31.95	
0.0067	27.64	-2.11
0.0133	28.82	0.59
0.0200	29.10	0.13
0.0267	30.56	0.65
0.0333	28.40	-1.14
0.0400	30.14	0.87
0.0467	31.72	0.74
0.0533	32.87	0.49
0.0600	34.16	0.54
0.0667	37.27	1.33
0.0733	38.10	0.28
0.0800	36.11	-0.86
0.0867	37.16	0.38
0.0933	37.98	0.28
0.1000	39.03	0.35
0.1067	40.64	0.57
0.1133	39.14	-0.62
0.1200	39.40	0.04
0.1267	40.12	0.22
0.1333	39.67	-0.23
0.1400	42.70	1.10
0.1467	39.88	-1.07
0.1533	39.64	-0.16
0.1600	43.42	1.39
0.1667	43.79	0.07

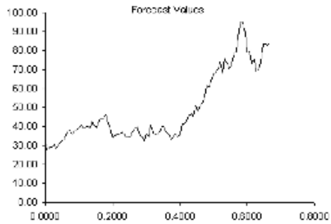


FIGURE 11.25 Stock Price Forecast Using Stochastic Path-Dependent Simulation Techniques

applies from year zero to year two, while a five-year spot rate applies from year zero to year five, and so forth. However, we require the forward rates for the options valuation, which we can obtain from bootstrapping the spot rates. Forward rates are interest rates that apply between two future periods. For instance, a one-year forward rate three years from now applies to the period from year three to year four. Based on the date of valuation, the highlighted risk-free rates below are the rates used in the changing risk-free rate binomial lattice model (i.e., 1.21 percent, 2.19 percent, 3.21 percent, 3.85 percent, and so forth).[29]

Dividends The firm's stocks pay no dividends, and this parameter will always be set to zero. In other cases, if dividend yield exists, these yields are entered

Forecast Stock Price - Risk Simulator Forecast Distribution

Histogram | Preferences | Statistics | Options

Statistics	Result
Number of Trials	10,000
Mean	47.22
Median	38.06
Standard Deviation	34.41
Variance	1183.75
Average Deviation	45.55
Maximum	436.25
Minimum	3.34
Range	432.91
Skewness	2.48
Kurtosis	11.03
25% Percentile	24.56
75% Percentile	59.60
Error Precision at 95% confidence	----

FIGURE 11.26 Results of Stock Price Forecast Using Monte Carlo Simulation

TABLE 11.17 Forward Risk-Free Rates (in Percent) Resulting from Bootstrap Analysis

	Years									
	1	2	3	4	5	6	7	8	9	10
Anual Forward Curve										
2/2/2004	1.29	2.37	3.43	4.01	4.84	4.75	5.27	4.99	5.31	5.63
2/3/2004	1.27	2.29	3.35	3.95	4.78	4.72	5.25	4.94	5.26	5.58
2/4/2004	1.27	2.33	3.37	3.99	4.83	4.72	5.24	4.96	5.28	5.60
2/5/2004	1.29	2.41	3.51	4.03	4.85	4.75	5.26	5.01	5.33	5.65
2/6/2004	1.26	2.28	3.34	3.96	4.80	4.66	5.17	4.94	5.27	5.60
2/9/2004	1.25	2.27	3.27	3.91	4.74	4.65	5.17	4.91	5.24	5.57
2/10/2004	1.27	2.37	3.36	3.94	4.75	4.67	5.18	4.95	5.28	5.61
2/11/2004	1.23	2.23	3.24	3.84	4.65	4.63	5.16	4.87	5.20	5.53
2/12/2004	1.24	2.26	3.29	3.89	4.71	4.61	5.12	4.97	5.32	5.67
2/13/2004	1.21	2.19	3.18	3.84	4.67	4.61	5.14	4.91	5.25	5.59
2/17/2004	1.21	2.19	3.21	3.85	4.68	4.59	5.11	4.91	5.25	5.59
2/18/2004	1.23	2.21	3.23	3.85	4.67	4.60	5.12	4.89	5.23	5.56
2/19/2004	1.23	2.17	3.21	3.85	4.68	4.59	5.11	4.91	5.25	5.59
2/20/2004	1.26	2.24	3.26	3.92	4.76	4.62	5.13	4.96	5.30	5.64
2/23/2004	1.22	2.16	3.26	3.86	4.69	4.60	5.12	4.89	5.23	5.56
2/24/2004	1.23	2.15	3.23	3.83	4.65	4.58	5.10	4.90	5.24	5.58
2/25/2004	1.23	2.11	3.15	3.81	4.64	4.58	5.11	4.88	5.22	5.56
2/26/2004	1.23	2.15	3.17	3.85	4.69	4.61	5.14	4.91	5.25	5.59
2/27/2004	1.21	2.11	3.08	3.90	4.79	4.43	4.90	4.85	5.19	5.53

into the model, including any expected changes to dividend policy over the life of the option.

Volatility Volatility is the next input assumption in the customized binomial lattice model. There are several ways volatility can be measured, and in the interest of full disclosure and due diligence, all methods are used in this study. Figure 11.27[30] shows the first method used to estimate the changing volatility of the firm's stock prices using the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model. The inputs to the model are all available historical stock prices since going public. The results indicate that the standard GARCH (1,1) model is inadequate to forecast the stock's volatility due to the low R-squared[31] and low F-statistics.[32] As such, GARCH analysis is found to be unsuitable for forecasting the volatility for valuing the firm's ESOs and its results are abandoned.

Two additional approaches are used to estimate volatility. The first is to use historical stock prices for the last quarter, last one year, last two years, and last four years (equivalent to the vesting period). These weekly closing prices are then converted to natural logarithmic relative returns and their sample standard deviations are then annualized to obtain the annualized volatilities.

In addition, Long-term Equity Anticipation Securities (LEAPS) can be used to estimate the underlying stock's volatility. LEAPS are long-term stock options, and when time passes such that there are six months or so remaining,

Dependent Variable: LOGRETURNS
Method: ML - ARCH
Date: 04/10/04 Time: 10:48
Sample: 1901 2603
Included observations: 703
Convergence achieved after 30 iterations

	Coefficient	Std. Error	z-Statistic	Prob.
GARCH	-4.360685	1.794356	-2.430222	0.0151
C	0.004958	0.002188	2.266192	0.0234
Variance Equation				
C	3.10E-07	2.12E-06	0.145964	0.8839
ARCH(1)	0.031233	0.005472	5.707787	0.0000
GARCH(1)	0.971900	0.004750	204.5979	0.0000

R-squared	0.010575	Mean dependent var	-0.001054
Adjusted R-squared	0.004905	S.D. dependent var	0.038647
S.E. of regression	0.038552	Akaike info criterion	-3.841624
Sum squared resid	1.037432	Schwarz criterion	-3.809224
Log likelihood	1355.331	F-statistic	1.865084
Durbin-Watson stat	2.125897	Prob(F-statistic)	0.114774

FIGURE 11.27 Generalized Autoregressive Conditional Heteroskedasticity for Forecasting Volatility

LEAPS revert to regular stock options. However, due to lack of trading, the bid-ask spread on LEAPS tends to be larger than for regularly traded equities. After performing due diligence on the estimation of volatilities, it is found that a GARCH econometric model was insufficiently specified to be of statistical validity. Hence, we reverted back to using the implied volatilities of long-term options, or LEAPS, and compared them with historical volatilities. The best single-point estimate of the volatility going forward would be an average of all estimates or 49.91 percent. In addition, volatility data on market comparable firms with similar functions, markets, risks, and geographical location, and within the same sector were calculated and used in the analysis. However, due to this large spread, Monte Carlo simulation was applied by running a *nonparametric* simulation on these volatility rates; thus, every volatility calculated here will be used in the analysis.

Vesting All ESOs granted by the firm vest in two different tranches: one month and six months. The former are options granted over a period of 48 months, where each month, 1/48 of the options vest, until the fourth year when all options are fully vested. The latter is a cliff-vesting grant, where if the employee leaves within the first six months, the entire option grant is forfeited. After the six months, each additional month vests 1/42 additional portions of the options. Consequently, one-month (1/12 years) and six-month (1/2 years) vesting are used as inputs in the analysis. The results of the analysis are simply the valuation of the options. To obtain the actual expenses, each 48-month vesting option is divided into 48 *minigrants* and expensed over the vesting period.

Suboptimal Exercise Behavior Multiple The next input is the suboptimal exercise behavior multiple. In order to obtain this input, data on all options exercised within the past year were collected. We used the past year, as trading from 2000 to 2002 was highly volatile and we believe the high-tech bubble caused extreme events in the stock market to occur that were not representative of our expectations of the future. In addition, only the past year's data are available. Figure 11.28 illustrates the calculations performed. The suboptimal exercise behavior multiple is simply the ratio of the stock price when it was exercised to the contractual strike price of the option. Terminated employees or employees who left voluntarily were excluded from the analysis because employees who leave the firm have a limited time to execute the portion of their options that have vested. In addition, all unvested options will expire worthless. Finally, employees who decide to leave the firm would have potentially known this in advance and hence have a different exercise behavior than a regular employee. Suboptimal behavior does not play a role under these circumstances. The event of an employee leaving is instead captured in the rate of forfeiture. The median behavior multiple is found to be

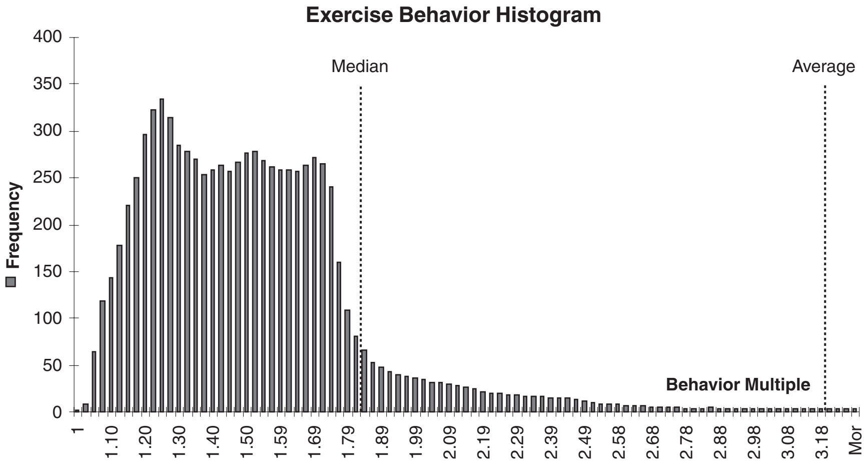


FIGURE 11.28 Estimating Suboptimal Exercise Behavior Multiples

1.85, and is the input used in the analysis. This value is in line with past empirical research, which has shown that the typical suboptimal exercise behavior multiple ranges from 1.5 to 3.0. For instance, Carpenter (1998) provided some empirical evidence that for a 10-year maturity option, the exercise multiple is 2.8 for senior executives.[33] Huddart and Lang (1996) showed that the average multiple was 2.2 for all employees, not just senior executives.[34] In addition, based on the author's own research and consulting activities, the typical multiple was found to lie between 1.5 and 3.0.

The median is used as opposed to the mean value because the distribution is highly skewed (the coefficient of skewness is 39.9), and as means are highly susceptible to outliers, the median is preferred. Figure 11.28 shows that the median is much more representative of the central tendency of the distribution than the average or mean. In order to verify that this is the case, two additional approaches are applied to validate the use of the median: trimmed ranges and statistical hypothesis tests. A trimmed range is created where the range of the suboptimal exercise behavior multiple such that the option holder will exercise at a stock price exceeding $500 is ignored. This is justified because given the current stock price it is highly improbable that it will exceed this $500 threshold.[35] The median calculated using this subjective trimming is 1.84, close to the initial global median of 1.85.

In addition, a more objective analysis, the statistical hypothesis test, was performed using the single-variable one-tailed t-test, and the 99.99th statistical percentile (alpha of 0.0001) from the t-distribution (the t-distribution was used to account for the distribution's skew and kurtosis—its extreme values and fat tails) is found to be 3.92 (Figure 11.29). The median calculated

One-Sample Hypothesis T-Test:
Suboptimal Exercise Behavior
Test of null hypothesis: mean = 3.754
Test of alternate hypothesis: mean < 3.754
Alpha one-tail of 1%

Variable	N	Mean	StDev	SE Mean
Behavior	8530	3.469	11.312	0.122

Variable	99.99% Upper Bound	T	P
Behavior	3.925	-2.33	0.01

Therefore, the 99.99th statistical percentile cutoff is 3.925

The average for the range between 1.000 and 3.925 is

Average	1.7954
Median	1.7689

Therefore, with the three values indicating a suboptimal behavior multiple at around 1.7689, 1.8450 and 1.8531, using the median of all data points provides the best indication as all data are used.
The resulting suboptimal behavior multiple used is
Global Median 1.8531

FIGURE 11.29 Estimating Suboptimal Exercise Behavior Multiples with Statistical Hypothesis Tests

from the suboptimal behavior range between 1.0 and 3.92 yielded 1.76. We therefore conclude that using the global median of 1.85 is the most conservative and best represents the employees' suboptimal exercise behavior.[36]

Forfeiture Rate The rate of forfeiture is calculated by comparing the number of grants that were canceled to the total number of grants. This value is calculated on a monthly basis and the results are shown in Table 11.18. The average forfeiture rate is calculated to be 5.51 percent. In addition, the average employee turnover rate for the past four years was 5.5 percent annually. Therefore, 5.51 percent is used in the analysis.

Number of Steps The higher the number of lattice steps, the higher the precision of the results. Figure 11.30 illustrates the convergence of results obtained using a BSM closed-form model on a European call option without dividends and comparing its results to the basic binomial lattice. Convergence is generally achieved at 1,000 steps. As such, the analysis results will use a minimum of 1,000 steps whenever possible.[37] Due to the high number of steps required to generate the results, software-based mathematical algorithms are used.[38] For instance, a nonrecombining binomial lattice with 1,000 steps has a total of 2×10^{301} nodal calculations to perform, making manual computation

TABLE 11.18 Estimating Forfeiture Rates

							Sum 241,374	Count 133		Years 0.34
Name	ID	Number	Grant Date	Plan	Cancel Date	Cancel Reason	Shares	Price	Total Price	Days to Cancellation
	18292	NI273832	8/4/2003	2003	12/31/2003	Termination-Voluntary	4,300	$30.8800	$132,784.00	149
	18159	00273892	8/4/2003	2003	12/31/2003	Termination-Involuntary	330	30.8800	10,190.40	149
	16794	00273401	7/25/2003	2003	12/31/2003	Termination-Voluntary	2,000	30.7600	61,520.00	159
	16807	NI273719	7/25/2003	2003	12/31/2003	Termination-Voluntary	2,000	30.7600	61,520.00	159
	16666	00273415	7/25/2003	2003	12/31/2003	Termination-Involuntary	2,000	30.7600	61,520.00	159
	16666	00273771	7/25/2003	2003	12/31/2003	Termination-Involuntary	3,134	30.7600	96,401.84	159
	18023	00273288	7/3/2003	2003	12/31/2003	Termination-Voluntary	4,900	29.4200	144,158.00	181
	5257	00273015	5/2/2003	1993	12/31/2003	Termination-Voluntary	333	23.6900	7,888.77	243
	17598	NI272903	5/2/2003	1993	12/31/2003	Termination-Voluntary	2,916	23.6900	69,080.04	243
	16897	00273721	7/25/2003	2003	12/26/2003	Termination-Voluntary	2,750	30.7600	84,590.00	154
	17063	P0002027	6/26/2003	PR98	12/26/2003	Termination-Voluntary	404	28.4600	11,497.84	183
	16897	P0001979	4/1/2003	PR98	12/26/2003	Termination-Voluntary	2,022	24.6300	49,801.86	269
	8092	NI273111	6/4/2003	2003	12/23/2003	Termination-Voluntary	1,281	28.4590	36,455.98	202
	19094	00274451	12/4/2003	2003	12/22/2003	Termination-Voluntary	5,600	37.3200	208,992.00	18
	5428	00272981	5/2/2003	1993	12/19/2003	Termination-Voluntary	462	23.6900	10,944.78	231
	5428	00272795	4/4/2003	1993	12/19/2003	Termination-Voluntary	462	19.0600	8,805.72	259
	18103	00273913	8/4/2003	2003	12/15/2003	Termination-Voluntary	8	30.8800	247.04	133
	8102	NI272622	3/4/2003	1993	12/9/2003	Termination-Voluntary	396	16.0600	6,359.76	280
	5361	00273024	5/2/2003	1993	12/5/2003	Termination-Voluntary	875	23.6900	20,728.75	217
	18911	00274455	12/4/2003	2003	12/4/2003	Termination-Voluntary	750	37.3200	27,990.00	0
	17840	00273283	7/3/2003	2003	12/4/2003	Termination-Voluntary	3,500	29.4200	102,970.00	154
	18721	00274339	11/4/2003	2003	12/3/2003	Termination-Involuntary	900	37.0500	33,345.00	29

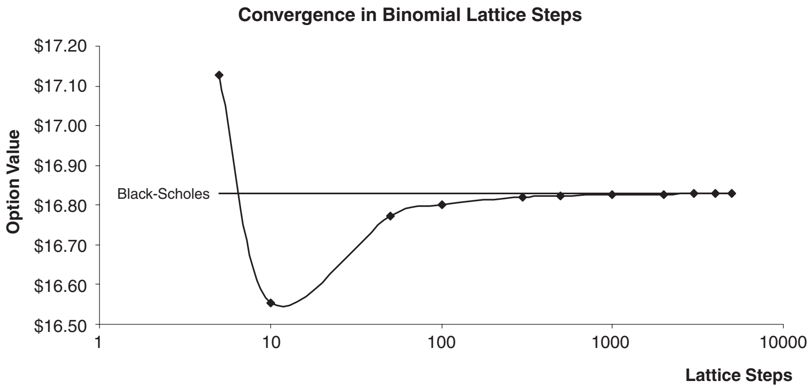


FIGURE 11.30 Convergence of the Binomial Lattice to Closed-Form Solutions

impossible without the use of specialized algorithms.[39] Figure 11.31 illustrates the calculation of convergence by using progressively higher lattice steps. The progression is based on sets of 120 steps (12 months per year multiplied by 10 years). The results are tabulated and the median of the average results are calculated. It shows that 4,200 steps is the best estimate in this customized binomial lattice, and this input is used throughout the analysis.[40]

Treatment of Forfeiture Rates One note of caution in applying the customized binomial lattice is the application of forfeiture rates. The treatment of forfeiture rates will also yield a difference in the option valuation results. Specifically, forfeiture rates can be applied inside a customized binomial lattice model

Stock Price	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17
Strike Price	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17
Maturity	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Risk-free Rate	1.21%	1.21%	1.21%	1.21%	1.21%	1.21%	1.21%	1.21%	1.21%	1.21%	1.21%	1.21%	1.21%	1.21%
Volatility	49.91%	49.91%	49.91%	49.91%	49.91%	49.91%	49.91%	49.91%	49.91%	49.91%	49.91%	49.91%	49.91%	49.91%
Dividend	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Lattice Steps	10	50	100	120	600	1200	1800	2400	3000	3600	4200	4800	5400	6000
Suboptimal Behavior	1.8531	1.8531	1.8531	1.8531	1.8531	1.8531	1.8531	1.8531	1.8531	1.8531	1.8531	1.8531	1.8531	1.8531
Vesting	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
Binomial Option Value	$20.55	$17.82	$17.32	$18.55	$17.55	$13.08	$13.11	$12.93	$12.88	$12.91	$13.00	$13.08	$12.93	$13.06

Segments	Steps	Results	Average
1	120	$18.55	$13.91
5	600	$17.55	$13.45
10	1200	$13.08	$13.00
15	1800	$13.11	$12.99
20	2400	$12.93	$12.97
25	3000	$12.88	$12.97
30	3600	$12.91	$12.99
35	4200	$13.00	$13.00
40	4800	$13.08	$13.02
45	5400	$12.93	$12.99
50	6000	$13.06	$13.06
		Median	$13.00

FIGURE 11.31 Convergence of the Customized Binomial Lattice

(calculations are performed inside the foregoing lattice algorithm) versus outside (adjusting the results after obtaining them from the binomial lattice). The valuation obtained will in most cases and under most conditions be different. Table 11.19 illustrates some of the nontrivial differences in valuation between using forfeitures inside versus outside of the binomial lattice for a typical ESO. Applying forfeiture rates internal to the lattice consistently provides a lower value than when applied outside the lattice.

If the forfeiture rate is applied inside the lattice, which in the author's opinion is the correct method, then when using the customized binomial lattice algorithm, simply input the forfeiture rate as is. In addition, the forfeiture rates can also be allowed to change over time in the customized binomial lattice algorithm. If the forfeiture rate is applied outside the lattice, simply set all forfeiture rates to zero in the binomial lattice and multiply the valuation results or the quantity of options granted by (*1 – Forfeiture*).[41] To understand the theoretical implications of inside versus outside treatments of forfeiture rates, we first need to understand how forfeiture rates are used in the model.

When used inside the lattice, the forfeiture rate is used to condition the customized binomial lattice to zero if the employee is terminated or leaves during the vesting period. Postvesting, the forfeiture rate is used to condition the lattice to execute the option if it is in-the-money or allowed to expire worthless otherwise, regardless of the suboptimal exercise behavior multiple when the employee leaves. This is important because due to the nonlinear interactions among variables, by putting the forfeiture rates inside the lattice, these interactions will be played out in the model—for instance, forfeiture dominates when an employee leaves, but suboptimal exercise and vesting dominate the value when there are no forfeitures, and the employees' actions will depend on the rate of forfeiture and suboptimal behavior. This rate is applied inside the customized binomial lattice. That is, at certain nodes, the lattice value becomes worthless going forward as the option is terminated due to forfeiture. This is more applicable in real life where if an employee who holds a large ESO grant leaves, his or her ESOs become worthless going forward (in the vesting period or postvesting if the ESO is at-the-money or in-the-money). In other words, each option grant has a different expected life (the point where forfeiture occurs is the point where the option value reverts to zero or is executed if in-the-money), and the backward induction calculation used will result in different values compared to applying forfeiture rates outside the lattice.

In contrast, when used outside the lattice, this means that all grants will never be forfeited in the valuation analysis. Forfeiture adjustments will only occur afterwards. In other words, all ESOs will mature and their values will be based on the total length of maturity. Then, these values are adjusted for forfeitures. This is less likely to happen in real life because what this implies is that all employees who are terminated or leave voluntarily will only leave

TABLE 11.19 Comparing the Application of Forfeiture Rates

Stock Price	$50	$50	$50	$50	$50	$50	$50	$50
Strike Price	$50	$50	$50	$50	$50	$50	$50	$50
Maturity	10	10	10	10	10	10	10	10
Risk-Free Rate	3.5%	3.5%	3.5%	3.5%	3.5%	3.5%	3.5%	3.5%
Dividend	0%	0%	0%	0%	0%	0%	0%	0%
Volatility	55%	55%	55%	55%	55%	55%	55%	55%
Lattice Steps	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000
Vesting Period	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Suboptimal Behavior	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80
Forfeiture Rate	**0.00%**	**2.50%**	**5.00%**	**7.50%**	**10.00%**	**12.50%**	**15.00%**	**20.00%**
Naïve BSM	*$34.02*	*$34.02*	*$34.02*	*$34.02*	*$34.02*	*$34.02*	*$34.02*	*$34.02*
Customized Binomial Inside Forfeiture)	*$22.60*	*$21.45*	*$20.40*	*$19.44*	*$18.56*	*$17.75*	*$16.99*	*$15.63*
Customized Binomial (Outside Forfeiture)	*$22.60*	*$22.04*	*$21.47*	*$20.91*	*$20.34*	*$19.78*	*$19.21*	*$18.08*
Difference	*$0.00*	*($0.58)*	*($1.07)*	*($1.46)*	*($1.78)*	*($2.03)*	*($2.22)*	*($2.45)*

at the end of the maturity period. If this were the case, then at maturity, the vesting period would have been over anyway, and, by definition, employees would have been able to exercise their ESOs if they were in-the-money. Thus, adjusting the forfeitures this way makes little sense. In addition, by setting the forfeiture rates outside of the lattice, any and all interactions among forfeiture, vesting, and suboptimal behavior (see the examples on nonlinearity and interactions among variables provided in this case) will be lost. Finally, by setting the forfeiture rates outside the lattice means that the employee's employment status plays no role in determining whether an ESO will be executed. This also makes no sense. If an employee forfeits his or her ESO after being vested, he or she has a limited time to execute the options or lose them. Also, by leaving the forfeiture rate outside, it assumes that employees will execute an ESO when the stock price exceeds the suboptimal exercise threshold regardless of their employment status, which again violates the contractual requirements in the ESO, especially when the employee has already forfeited the option. With that said, no matter how forfeitures are applied, the higher the forfeiture rate, the lower the option value becomes. However, as seen in Table 11.19, valuing the option using forfeiture rates by applying them internally in a lattice reduces the option value more than applying it externally.

Results Using the customized binomial lattice methodology coupled with Monte Carlo simulation, the fair-market value of the options at different grant dates and different forecast stock prices are listed in Table 11.20. For instance, the grant date of January 2005 has a conservative stock price forecast of $45.17 and its resulting binomial lattice result is $17.39 for the one-month vesting option, and $17.42 for the six-month vesting option. In contrast, if we modified the BSM to use the expected life of the option (which was set to the lowest possible value of four years, equivalent to the vesting period of four years),[42] the option's value is still significantly higher at $19.55. This is a $2.16 cost reduction compared to using the BSM, or a 12.42 percent reduction in cost for this simple option alone. When all the options are calculated and multiplied by their respective grants, the total valuation under the traditional BSM is $863,961,092 after accounting for the 5.51 percent forfeiture rate. In contrast, the total valuation for the customized binomial lattice is $813,997,676, a reduction of $49,963,417 over the period of two years. Figure 11.32 shows one sample calculation in detail.

Figure 11.32 illustrates a sample result from a 124,900-trial Monte Carlo simulation run on the customized binomial lattice where the error is within $0.01 with a 99.9 percent statistical confidence that the fair-market value of the option granted at January 2005 is $17.39. Each grant illustrated in Table 11.20 will have its own simulation result like the one in Figure 11.32.

The example illustrated in Table 11.21 shows a naïve BSM result of $26.91 versus a binomial lattice result of $17.39 (the BSM using an adjusted

TABLE 11.20 Analytical Customized Binomial Lattice Results

Per Share Closing Stock Price

Date	Conservative	Aggressive	Average
End Dec 04/Start Jan 05	$45.17	$50.70	$47.93
End Jan 05/Start Feb 05	$45.89	$51.52	$48.70
End Feb/Start Mar	$46.61	$52.34	$49.48
End Mar/Start Apr	$47.34	$53.16	$50.25
End Apr/Start May	$48.06	$53.98	$51.02
End May/Start Jun	$48.78	$54.81	$51.79
End Jun/Start Jul	$49.57	$55.63	$52.57
End Jul/Start Aug	$50.23	$56.45	$53.34
End Aug/Start Sep	$50.95	$57.27	$54.11
End Sep/Start Oct	$51.68	$58.09	$54.89
End Oct/Start Nov	$52.40	$58.92	$55.66
End Nov/Start Dec	$53.13	$59.74	$56.43
End Dec/Start Jan 06	$53.85	$60.56	$57.20
End Jan 06/Start Feb 06	$54.55	$61.36	$57.95
End February/Start March	$55.25	$62.15	$58.70
End March/Start Apr	$55.95	$62.95	$59.45
End Apr/Start May	$56.66	$63.75	$60.20
End May/Start Jun	$57.36	$64.55	$60.95
End Jun/Start Jul	$58.06	$65.34	$61.70
End Jul/Start Aug	$58.76	$66.14	$62.45
End Aug/Start Sep	$59.46	$66.94	$63.20
End Sep/Start Oct	$60.16	$67.74	$63.95
End Oct/Start Nov	$60.87	$68.53	$64.70
End Nov/Start Dec	$61.57	$69.33	$65.45

	48 Months Vest	6 Month Cliff and Then 42 Month Vest		
	New Hire Grant	Acquisition	Focal	Total
Jan-05	550,000			550,000
February	550,000			550,000
March	550,000			550,000
April	550,000			550,000
May	550,000			550,000
June	605,000			605,000
July	605,000			605,000
August	605,000			605,000
September	605,000	2,500,000		3,105,000
October	605,000			605,000
November	605,000		7,492,100	8,097,100
December	605,000			605,000
Jan-06	605,000			605,000
February	605,000			605,000
March	605,000			605,000
April	605,000			605,000
May	605,000			605,000
June	665,500			665,500
July	665,500			665,500
August	665,500			665,500
September	665,500	3,000,000		3,665,500
October	665,500			665,500
November	665,500		8,241,310	8,906,810
December	665,500			665,500

Option Valuation (monthly)

Date	Conservative	Aggressive	Average
Jan-05	$17.39	$19.52	$18.46
February	$17.67	$19.84	$18.76
March	$17.95	$20.16	$19.05
April	$18.23	$20.47	$19.35
May	$18.51	$20.79	$19.65
June	$18.79	$21.11	$19.95
July	$19.06	$21.42	$20.24
August	$19.34	$21.74	$20.54
September	$19.62	$22.05	$20.84
October	$19.90	$22.37	$21.14
November	$20.18	$22.69	$21.43
December	$20.46	$23.00	$21.73
Jan-06	$20.74	$23.32	$22.03
February	$21.01	$23.63	$22.32
March	$21.28	$23.94	$22.61
April	$21.55	$24.24	$22.89
May	$21.82	$24.55	$23.18
June	$22.09	$24.86	$23.47
July	$22.36	$25.16	$23.75
August	$22.63	$25.47	$24.05
September	$22.90	$25.78	$24.34
October	$23.17	$26.09	$24.63
November	$23.44	$26.39	$24.92
December	$23.71	$26.70	$25.20

Option Valuation (cliff vesting)

Date	Conservative	Aggressive	Average
Jan-05	$17.42	$19.55	$18.49
February	$17.70	$19.87	$18.78
March	$17.98	$20.19	$19.08
April	$18.26	$20.50	$19.38
May	$18.54	$20.82	$19.68
June	$18.81	$21.14	$19.98
July	$19.09	$21.45	$20.27
August	$19.37	$21.77	$20.57
September	$19.65	$22.09	$20.87
October	$19.93	$22.41	$21.17
November	$20.21	$22.72	$21.47
December	$20.49	$23.04	$21.76
Jan-06	$20.77	$23.36	$22.06
February	$21.04	$23.66	$22.35
March	$21.31	$23.97	$22.64
April	$21.58	$24.28	$22.93
May	$21.85	$24.59	$23.22
June	$22.12	$24.89	$23.51
July	$22.39	$25.20	$23.80
August	$22.66	$25.51	$24.09
September	$22.93	$25.82	$24.38
October	$23.20	$26.12	$24.66
November	$23.47	$26.43	$24.95
December	$23.75	$26.74	$25.24

Total Options Expense (binomial): $813997675.53

Date	Conservative	Aggressive	Average
Jan-05	$9,566,052.74	$10,737,306.72	$10,151,679.73
February	$9,719,311.21	$10,911,406.31	$10,315,358.76
March	$9,872,590.87	$11,085,484.73	$10,479,037.80
April	$10,025,849.35	$11,259,584.33	$10,642,716.84
May	$10,179,107.83	$11,433,662.74	$10,806,395.88
June	$11,365,602.93	$12,768,538.58	$12,067,082.40
July	$11,534,210.56	$12,960,024.83	$12,247,106.05
August	$11,702,794.88	$13,151,534.39	$12,427,152.99
September	$60,926,665.19	$68,479,589.19	$64,703,067.40
October	$12,039,963.53	$13,534,530.21	$12,787,246.87
November	$163,625,443.32	$183,963,675.91	$173,794,559.61
December	$12,337,155.48	$13,917,526.02	$13,147,340.75
Jan-06	$12,545,739.81	$14,109,035.58	$13,327,387.69
February	$12,709,221.87	$14,294,860.42	$13,502,052.80
March	$12,872,727.23	$14,480,685.27	$13,676,694.60
April	$13,036,232.59	$14,666,510.11	$13,851,359.70
May	$13,199,714.65	$14,852,311.66	$14,026,024.81
June	$14,699,542.02	$16,541,950.16	$15,620,733.27
July	$14,879,372.29	$16,746,357.48	$15,812,864.89
August	$15,059,202.56	$16,950,764.81	$16,004,996.50
September	$83,935,039.46	$94,488,780.60	$89,211,839.45
October	$15,418,914.35	$17,359,579.47	$16,389,234.10
November	$209,062,661.14	$235,401,537.28	$222,232,270.95
December	$15,778,600.52	$17,768,368.50	$16,773,471.70

Option Valuation (Black-Scholes)

Date	Conservative	Aggressive	Average
Jan-05	$19.55	$21.94	$20.75
February	$19.86	$22.30	$21.08
March	$20.18	$22.66	$21.42
April	$20.49	$23.01	$21.75
May	$20.80	$23.37	$22.09
June	$21.12	$23.72	$22.42
July	$21.43	$24.08	$22.75
August	$21.74	$24.43	$23.09
September	$22.06	$24.79	$23.42
October	$22.37	$25.15	$23.76
November	$22.68	$25.50	$24.09
December	$23.00	$25.86	$24.43
Jan-06	$23.31	$26.21	$24.76
February	$23.61	$26.56	$25.09
March	$23.92	$26.90	$25.41
April	$24.22	$27.25	$25.73
May	$24.52	$27.59	$26.06
June	$24.83	$27.94	$26.38
July	$25.13	$28.28	$26.71
August	$25.44	$28.63	$27.03
September	$25.74	$28.98	$27.36
October	$26.04	$29.32	$27.68
November	$26.35	$29.67	$28.01
December	$26.65	$30.01	$28.33

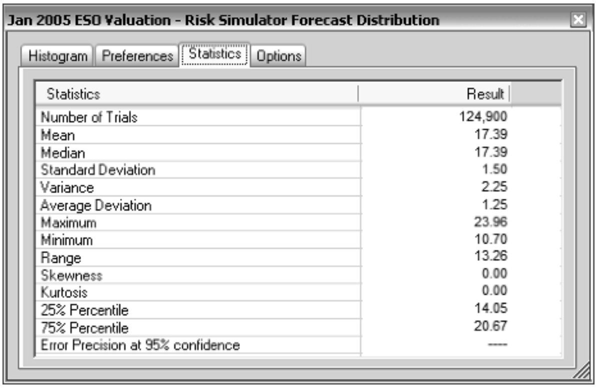
Total Options Expense (Black-Scholes): $914341297.69

Date	Conservative	Aggressive	Average
Jan-05	$10,752,660.26	$12,069,200.79	$11,410,930.53
February	$10,924,929.47	$12,264,896.33	$11,594,912.90
March	$11,097,222.49	$12,460,568.06	$11,778,895.27
April	$11,269,491.70	$12,656,263.59	$11,962,877.65
May	$11,441,760.91	$12,851,935.32	$12,146,860.02
June	$12,775,433.13	$14,352,393.94	$13,563,926.63
July	$12,964,955.45	$14,567,632.85	$13,766,281.05
August	$13,154,451.58	$14,782,897.94	$13,968,661.66
September	$68,484,227.49	$76,974,043.30	$72,729,068.19
October	$13,533,443.84	$15,213,401.93	$14,373,422.88
November	$183,662,838.36	$206,491,668.93	$195,077,253.65
December	$13,912,462.28	$15,643,905.92	$14,778,184.10
Jan-06	$14,101,958.41	$15,859,171.01	$14,980,564.71
February	$14,285,719.38	$16,068,046.23	$15,176,895.90
March	$14,469,506.53	$16,276,921.46	$15,373,200.90
April	$14,653,293.69	$16,485,796.69	$15,569,532.09
May	$14,837,054.65	$16,694,645.72	$15,765,863.28
June	$16,522,925.99	$18,593,873.04	$17,558,385.11
July	$16,725,063.05	$18,823,635.79	$17,774,349.42
August	$16,927,200.11	$19,053,398.54	$17,990,313.73
September	$94,346,643.11	$106,209,508.20	$100,277,996.32
October	$17,331,531.85	$19,512,924.04	$18,422,213.54
November	$234,664,248.79	$264,228,555.25	$249,446,594.79
December	$17,735,834.78	$19,972,420.73	$18,854,113.35

Main Input Assumptions & Results

Year	Risk-Free Rate
1	1.21%
2	2.19%
3	3.21%
4	3.85%
5	4.68%
6	4.59%
7	5.11%
8	4.91%
9	5.25%
10	5.59%

Time to Maturity	10
Dividend	0.00%
Volatility	49.91%
Suboptimal Behavior	1.8531
Forfeiture Rate	5.51%
Vesting	1 month and 6 months
Steps	4,200
Total Black-Scholes	$914,341,298
Total Binomial	$813,997,676
Adjusted Black-Scholes	$863,961,092
Difference	($49,963,417)



Statistics	Result
Number of Trials	124,900
Mean	17.39
Median	17.39
Standard Deviation	1.50
Variance	2.25
Average Deviation	1.25
Maximum	23.96
Minimum	10.70
Range	13.26
Skewness	0.00
Kurtosis	0.00
25% Percentile	14.05
75% Percentile	20.67
Error Precision at 95% confidence	----

FIGURE 11.32 Monte Carlo Simulation of ESO Valuation Result

four-year life is \$19.55). This \$9.52 differential can be explained by contribution in parts. In order to understand this lower option value as compared to the naïve BSM results, Figure 11.33 illustrates the contribution to options valuation reduction.

The difference between the naïve BSM valuation of \$26.91 versus a fully customized binomial lattice valuation of \$17.39 yields \$9.52. Figure 11.33 illustrates where this differential comes from. About 0.02 percent of the difference comes from vesting and changing risk-free rates over the life of the option. From the employees' suboptimal behavior comes 28.60 percent or \$2.64, and the remaining 71.37 percent or \$6.58 comes from the 5.51 percent annualized forfeiture rate. The total variation directly explained comes to \$9.22. The remaining variation of \$0.30 comes from the nonlinear interactions among the various input variables and cannot be accounted for directly. Table 11.22 shows a sample calculation for the January 2005 grant.

It is shown in this valuation analysis that the fair-market value of the employee options can be overvalued by 6.14 percent in Table 11.20 (\$813.99M using the binomial lattice versus \$863.96M using the BSM with adjusted life) if the GBM or BSM is used, because the GBM cannot take into account real-life conditions of ESOs that could affect their value. A proprietary customized binomial lattice model was used instead. This customized binomial lattice can account for all the GBM inputs (stock price, strike price, risk-free rate, dividend, and volatility) as well as the other real-life conditions such as vesting periods, forfeiture rates, suboptimal exercise behavior, blackout dates, changing risk-free rates, changing dividends, and changing volatilities. Table 11.23 illustrates the allocation of expenses using a BSM over the next six years, for the grants starting in January 2005. Notice that the total expense is \$914,341,298, identical to the total valuation results in Table 11.20. The

TABLE 11.21 Option Valuation Results

	Conservative Stock Price	Aggressive Stock Price	Average of Two Stock Prices	Forfeiture 5.51% Year	Risk Free
Stock Price	$45.17	$50.70	$47.93	1	1.21%
Strike Price	$45.17	$50.70	$47.93	2	2.19%
Maturity	10	10	10	3	3.21%
Risk-Free Rate	1.21%	1.21%	1.21%	4	3.85%
Volatility	49.91%	49.91%	49.91%	5	4.68%
Dividend	0%	0%	0%	6	4.59%
Lattice Steps	4200	4200	4200	7	5.11%
Suboptimal Behavior	1.8531	1.8531	1.8531	8	4.91%
Vesting	0.08	0.08	0.08	9	5.25%
				10	5.59%
Customized Binomial Lattice	$17.39 (Customized Binomial with Changing Risk-Free Rates)				
Naïve Black-Scholes	$26.91 (Black-Scholes model with a naïve 10-year assumption)				
Modified Black-Scholes	$19.55 (Black-Scholes model using expected life of 4 years)				
Cost Reduction	$2.16				

Contribution to Option Value Reduction

Vesting	$0.00	0.02%	Customized Binomial Lattice	$17.39
Suboptimal Behavior	$2.64	28.60%	Naïve Black-Scholes	$26.91
Forfeiture	$6.58	71.37%	Savings	***$9.52***
Changing Risk-Free Rate	$0.00	0.02%		
Total Value	***$9.22***			

*The slight difference between $9.22 and $9.52 is due to the interactions between variables.

Variance with Respect to Black-Scholes

Using the Black-Scholes with naïve assumption of full 10-year maturity	$26.91	
Using the Black-Scholes modified with a 4-year expected average life of the option*	$19.55	
Using the Black-Scholes modified with expected life and reduced by forfeiture rate	$18.47	
Customized Binomial Lattice with changing risk-free, forfeitures, suboptimal, and vesting	$17.39	
Cost reduction obtained by using the binomial versus Black-Scholes (expected life)	$2.16	(11.04% reduction)
Average option issues per month in 2005	550,000	
Cost reduction per year	$14,240,709	

*Note: 4 years is used because the option is on a monthly graded-vesting scale for 48 months.

FIGURE 11.33 Contribution to Option Valuation Reduction

expenses were obtained by applying minigrant allocations on each ESO issue as described previously.

Table 11.24, in contrast, shows the allocation of expenses using a customized binomial lattice approach. Again, notice that the total expenses of $813,997,676 agrees with the results in Table 11.20. The difference between a naïve BSM and a customized binomial lattice approach is fairly significant (Table 11.25). The difference between the total valuations is 12.33 percent ($914.34M versus $813.99M) and this percentage is fairly constant throughout the expensed years. However, the dollar expenses are front-loaded and the total difference of $100.34M will not be spread out equally.

Conclusion

It has been more than 30 years since Fisher Black, Myron Scholes, and Robert Merton derived their Nobel-prize winning option pricing model and significant advancements have been made; therefore, do not restrict stock option pricing to one specific model (the BSM/GBM) while a plethora of other models and applications can be explored. The three mainstream approaches to valuing stock options are closed-form models (e.g., BSM, GBM, and American option approximation models), Monte Carlo simulation, and binomial lattices. This case study details the impacts of using a customized binomial lattice and is based on the author's book, *Valuing Employee Stock Options: Under 2004 FAS 123 Requirements* (Wiley Finance, 2004). The BSM and GBM will typically *overstate* the fair value of ESOs where there is suboptimal early exercise behavior coupled with vesting requirements and option forfeitures. *In fact, firms using the BSM and GBM to value and expense ESOs may be significantly overstating their true expense.* The BSM requires many underlying assumptions before it works, and therefore has significant limitations, including

TABLE 11.22 Option Valuation Comparison

	Conservative Stock Price	Aggressive Stock Price	Average Prices	Comments
Option Value	\$17.39	\$19.52	\$18.46	Customized binomial lattice
Naïve Black-Scholes	26.91	30.20	28.55	Black-Scholes model with a naïve 10-year assumption
Modified Black-Scholes	19.55	21.95	20.75	Black-Scholes model using expected life of 4 years
Cost Reduction	2.16	2.42	2.29	
Cost reduction per year	\$14,240,709	\$15,984,148	\$15,110,852	

Note: Assumed average option issues per month in 2005: 550,000.

TABLE 11.23 Expense Allocation (BSM)

Year	Total Expenses
2005	$103,588,842
2006	304,675,765
2007	300,633,091
2008	141,257,655
2009	54,548,249
2010	9,637,696
Sum of Annual Expenses	914,341,298

TABLE 11.24 Expense Allocation (Customized Binomial Lattice)

Year	Total Expenses
2005	$ 92,197,733
2006	271,222,335
2007	267,662,000
2008	125,765,601
2009	48,567,875
2010	8,582,132
Sum of Annual Expenses	813,997,676

TABLE 11.25 Dollar and Percentage Difference in Expenses

	Total Expenses	
Year	Difference ($)	Difference (%)
2005	(11,391,109)	–12.36
2006	(33,453,430)	–12.33
2007	(32,971,091)	–12.32
2008	(15,492,054)	–12.32
2009	(5,980,374)	–12.31
2010	(1,055,564)	–12.30
Sum of Annual Expenses	(100,343,622)	–12.33

being applicable only for European options without dividends. In addition, American option approximation models are very complex and difficult to create in a spreadsheet. The BSM *cannot* account for American options, options based on stocks that pay dividends (the GBM model can, however, account for dividends in a European option), forfeitures, underperformance, stock price barriers, vesting periods, changing business environments and volatilities, suboptimal early exercise behavior, and a sleuth of other conditions. Monte Carlo simulation when used alone is another option valuation approach, but is restricted only to European options. Simulation can be used in two different ways: solving the option's fair-market value through path simulations of stock prices or used in conjunction with other approaches (e.g., binomial lattices and closed-form models) to capture multiple sources of uncertainty in the model.

Binomial lattices are flexible and easy to implement. They are capable of valuing American-type stock options with dividends but require computational power. Software applications should be used to facilitate this computation. Binomial lattices can be used to calculate American options paying dividends and can be easily adapted to solve ESOs with exotic inputs and used in conjunction with Monte Carlo simulation to account for the uncertain input assumptions (e.g., probabilities of forfeiture, suboptimal exercise behavior, vesting, underperformance) and to obtain a high precision at statistically valid confidence intervals. Based on the analyses throughout this case study, it is recommended that the use of a model that assumes an ESO is European style, when, in fact, the option is American style with the other exotic variables, should not be permitted as this substantially overstates compensation expense. Many factors influence the fair-market value of ESOs, and a binomial lattice approach to valuation that considers these factors should be used. With due diligence, real-life ESOs can absolutely be valued using the customized binomial lattice approach as shown in this case study, where the methodology employed is pragmatic, accurate, and theoretically sound.

CASE 6: INTEGRATED RISK ANALYSIS MODEL—HOW TO COMBINE SIMULATION, FORECASTING, OPTIMIZATION, AND REAL OPTIONS ANALYSIS INTO A SEAMLESS RISK MODEL

One of the main questions in risk analysis is how the individual software applications in our Risk Simulator and Real Options Super Lattice Solver suite can be applied in concert with one another. This case study attempts to illustrate how an integrated risk management model can be constructed using the Real Options Super Lattice Solver and Risk Simulator's Monte Carlo *Simulation, Forecasting,* and *Optimization* tools. Figure 11.34 shows an integrated model process of how time-series forecasting, returns on investment, Monte

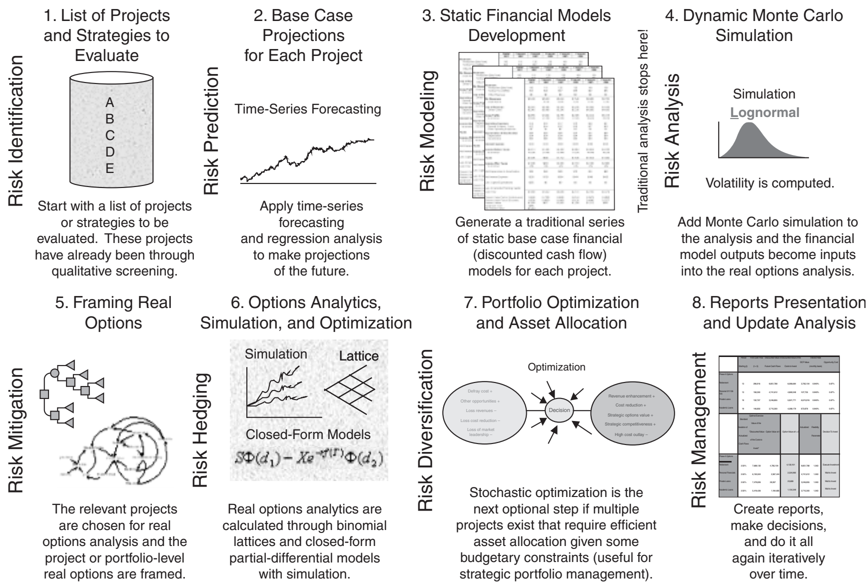


FIGURE 11.34 Integrated Risk Management Process

Carlo simulation, real options, and portfolio optimization are linked within one nicely integrated analytical package. Refer to the software's user guide for more hands-on details as this case study is only meant to illustrate the practical applications of these approaches.

Forecasting

Suppose that some historical annualized revenue data dating from 1985 to 2004 exists. The first task in the modeling is to forecast the next five years of this project or product in question.

In Risk Simulator's *Forecasting* tool, select the area G7 to G26 of the data area (Figure 11.35). Suppose that you know from experience that the seasonality of the revenue stream peaks every four years. Enter the seasonality of four and specify the number of forecast periods to five. The *Forecasting* tool automatically selects the best time-series model to forecast the results. Figure 11.36 illustrates the different forecasting tools available in Risk Simulator. The time-series analysis module is chosen, and running this module on the historical data will generate a report with the five-year forecast revenues (Figure 11.37) that are complete with Risk Simulator assumptions.

Forecasting

Periodicity: Annual

Seasonality: 4 Years

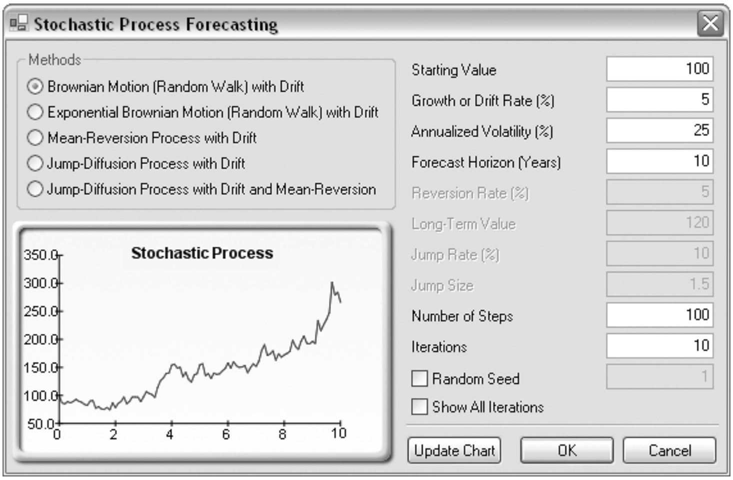
	Year	Revenues
Historical Data	1985	$684.20
	1986	$584.10
	1987	$765.40
	1988	$892.30
	1989	$885.40
	1990	$677.00
	1991	$1,006.60
	1992	$1,122.10
	1993	$1,163.40
	1994	$993.20
	1995	$1,312.50
	1996	$1,545.30
	1997	$1,596.20
	1998	$1,260.40
	1999	$1,735.20
	2000	$2,029.70
	2001	$2,107.80
	2002	$1,650.30
	2003	$2,304.40
	2004	$2,639.40

FIGURE 11.35 Historical Time-Series Data

Monte Carlo Simulation

The second worksheet, Valuation Model, shows a simple DCF that calculates the relevant NPV and IRR values as seen in Figure 11.38. Notice that the output of the forecast worksheet becomes an input into this valuation model worksheet. For instance, cells D19 to H19 (Figure 11.38) are linked from the first forecast worksheet (Figure 11.37). The model uses two discount rates for the two different sources of risk (market risk-adjusted discount rate of 12 percent for the risky cash flows, and a 5 percent cost of capital to account for the private risk of capital investment costs). Further, different discounting conventions are included (midyear, end-year, continuous, and discrete discounting), as well as a terminal value calculation assuming a constant growth model. Finally, the capital implementation costs are separated out of the model (row 46) but the regular costs of doing business (direct costs, cost of goods sold, operating expenses, etc.) are included in the computation of free cash flows. That is, this project has two phases, the first phase in year 2005 costs $5 and the second larger phase costs $2,000. The statistic NPV shows a positive value of $123.14 while the IRR is 15.68 percent, both exceeding the zero-NPV and firm-specific hurdle rate of 15 percent required rate of return. Therefore, at first pass, the project seems to be profitable and justifiable. However, risk has not been considered.

Without applying risk analysis, one cannot determine the chances this NPV and IRR are expected to occur. Figure 11.39 illustrates the same model but now with Risk Simulator input assumptions and output forecasts applied (highlighted cells). This is done by clicking on *Simulation* and selecting *New Simulation Profile*. Then, select the cells you wish to simulate (e.g., D20) and click on *Simulation* and select *Set Assumption*. For illustration purposes, select the *Triangular Distribution* and link to the input parameters



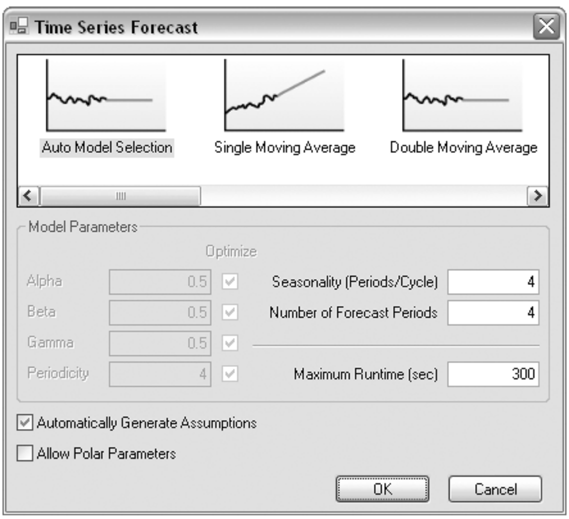
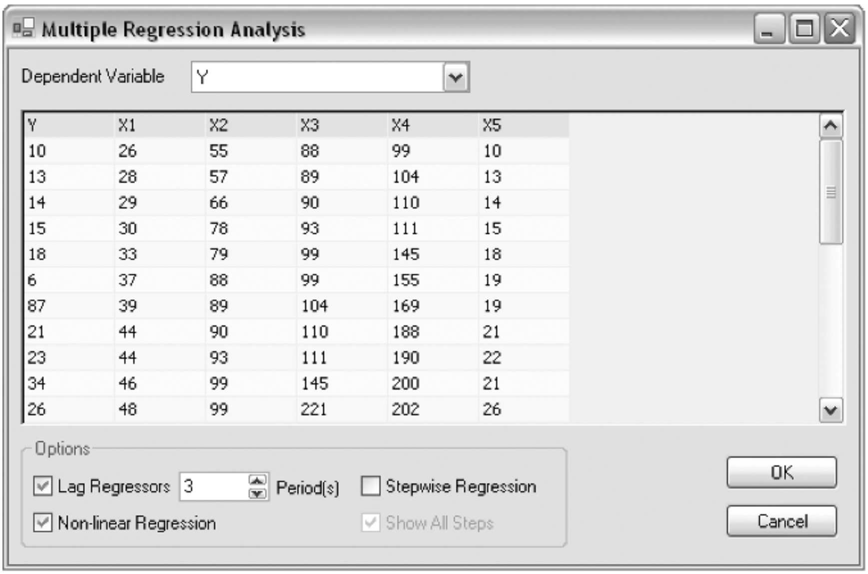


FIGURE 11.36 Forecasting Tool in Risk Simulator

to the appropriate cells. Figure 11.39 illustrates a sample assumption applied to cell D20. Continue the process to define input assumptions on all relevant cells. Then select the output cells such as NPV, IRR, and so forth (cells D14, D15, H14, and H15) and set them as forecasts (i.e., select each cell and click on *Simulation | Set Forecast* and enter the relevant variable names).

The model is then simulated for 5,000 thousand trials and the results are shown in Figures 11.40 to 11.43. As can be seen, there is a 76.40 percent probability of breaking even or better (Figure 11.40). In other words, there is



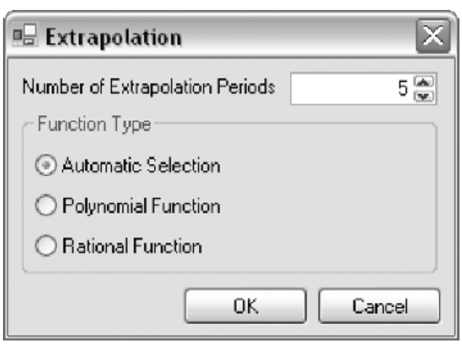


FIGURE 11.36 (*Continued*)

Time-Series Analysis (Holt-Winters Seasonal Multiplicative)

Summary Statistics

Alpha, Beta, Gamma	RMSE	Alpha, Beta, Gamma	RMSE
0.00, 0.00, 0.00	914.824	0.60, 0.60, 0.60	113.974
0.10, 0.10, 0.10	415.322	0.70, 0.70, 0.70	138.884
0.20, 0.20, 0.20	187.202	0.80, 0.80, 0.80	171.881
0.30, 0.30, 0.30	118.795	0.90, 0.90, 0.90	202.578
0.40, 0.40, 0.40	101.794	1.00, 1.00, 1.00	319.759
0.50, 0.50, 0.50	102.143		

Best Fit (Alpha, Beta, Gamma) = 0.2431, 0.9990, 0.7798

Time-Series Analysis Summary

When both seasonality and trend exist, more advanced models are required to decompose the data into their base elements: a base-case level (L) weighted by the alpha parameter; a trend component (b) weighted by the beta parameter; and a seasonality component (S) weighted by the gamma parameter. Several methods exist but the two most common are the Holt-Winters' additive seasonality and Holt-Winters' multiplicative seasonality methods. In the Holt-Winter's multiplicative model, the base case level and trend are added together and multiplied by the seasonality factor to obtain the forecast fit.

The best-fitting test for the moving average forecast uses the root mean squared errors (RMSE). The RMSE calculates the square root of the average squared deviations of the fitted values versus the actual data points.

Mean Squared Error (MSE) is an absolute error measure that squares the errors (the difference between the actual historical data and the forecast-fitted data predicted by the model) to keep the positive and negative errors from canceling each other out. This measure also tends to exaggerate large errors by weighting the large errors more heavily than smaller errors by squaring them, which can help when comparing different time-series models. Root Mean Square Error (RMSE) is the square root of MSE and is the most popular error measure, also known as the quadratic loss function. RMSE can be defined as the average of the absolute values of the forecast errors and is highly appropriate when the cost of the forecast errors is proportional to the absolute size of the forecast error. The RMSE is used as the selection criteria for the best-fitting time-series model.

Mean Absolute Percentage Error (MAPE) is a relative error statistic measured as an average percent error of the historical data points and is most appropriate when the cost of the forecast error is more closely related to the percentage error than the numerical size of the error. Finally, an associated measure is the Theil's U statistic, which measures the naivety of the model's forecast. That is, if the Theil's U statistic is less than 1.0, then the forecast method used provides an estimate that is statistically better than guessing.

Period	*Actual*	*Forecast Fit*
1	684.20	
2	584.10	
3	765.40	
4	892.30	
5	885.40	684.20
6	677.00	667.57
7	1006.60	935.44
8	1122.10	1198.07
9	1163.40	1112.43
10	993.20	887.91
11	1312.50	1348.38
12	1545.30	1546.54
13	1596.20	1572.45
14	1260.40	1299.19
15	1735.20	1704.74
16	2029.70	1976.22
17	2107.80	2026.03
18	1650.30	1637.29
19	2304.40	2245.93
20	2639.40	2643.07
Forecast 21		2713.66
Forecast 22		2114.75
Forecast 23		2900.37
Forecast 24		3293.75
Forecast 25		3346.46

Error Measurements	
RMSE	71.8164
MSE	5157.5910
MAD	53.4084
MAPE	4.50%
Theil's U	0.3055

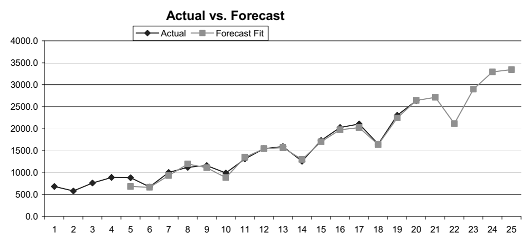
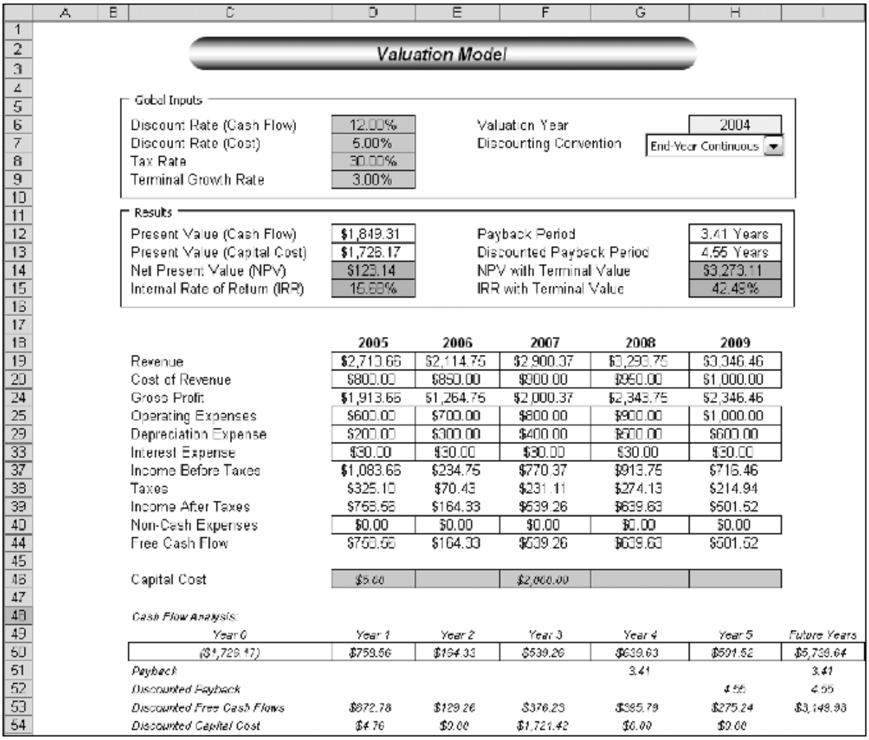


FIGURE 11.37 Forecast Results

no guarantee the project will always make money. In fact, about one out of four times, the project will be making a loss. If the corporate IRR hurdle rate is higher, say at 15 percent, there is only a 42.20 percent probability that the project exceeds this required threshold (Figure 11.41). In fact, when quantifying the single-point NPV estimate of $123.14, there is only a 36.40 percent probability that the project will exceed expectations or a 63.60 percent probability it will be below expectations as captured by the single-point NPV estimate (Figure 11.42).

In fact, Figure 11.43 shows a noble truth in risk analysis: The *expected value is often not the same as the value expected*. That is, the expected value or mean of the distribution of NPV outcomes is $100.83, a far cry from the value expected of $123.14 using a single-point estimate.



Valuation Model

Gobal Inputs

Discount Rate (Cash Flow)	12.00%	Valuation Year	2004
Discount Rate (Cost)	5.00%	Discounting Convention	End-Year Continuous
Tax Rate	30.00%		
Terminal Growth Rate	3.00%		

Results

Present Value (Cash Flow)	$1,849.31	Payback Period	3.41 Years
Present Value (Capital Cost)	$1,726.17	Discounted Payback Period	4.55 Years
Net Present Value (NPV)	$123.14	NPV with Terminal Value	$3,273.11
Internal Rate of Return (IRR)	15.68%	IRR with Terminal Value	42.45%

	2005	2006	2007	2008	2009
Revenue	$2,713.66	$2,114.75	$2,900.37	$3,293.75	$3,346.46
Cost of Revenue	$800.00	$850.00	$900.00	$950.00	$1,000.00
Gross Profit	$1,913.66	$1,264.75	$2,000.37	$2,343.75	$2,346.46
Operating Expenses	$600.00	$700.00	$800.00	$900.00	$1,000.00
Depreciation Expense	$200.00	$300.00	$400.00	$500.00	$600.00
Interest Expense	$30.00	$30.00	$30.00	$30.00	$30.00
Income Before Taxes	$1,083.66	$234.75	$770.37	$913.75	$716.46
Taxes	$325.10	$70.43	$231.11	$274.13	$214.94
Income After Taxes	$758.56	$164.33	$539.26	$639.63	$501.52
Non-Cash Expenses	$0.00	$0.00	$0.00	$0.00	$0.00
Free Cash Flow	$758.56	$164.33	$539.26	$639.63	$501.52
Capital Cost	*$5.00*		*$2,000.00*		

Cash Flow Analysis:

	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	Future Years
	($1,726.17)	*$759.56*	*$164.33*	*$539.26*	*$639.63*	*$501.52*	*$5,739.64*
Payback					*3.41*		*3.41*
Discounted Payback						*4.56*	*4.55*
Discounted Free Cash Flows		*$672.78*	*$129.26*	*$376.23*	*$385.79*	*$275.24*	*$3,149.98*
Discounted Capital Cost		*$4.76*	*$0.00*	*$1,721.42*	*$0.00*	*$0.00*	

FIGURE 11.38 Static Discounted Cash Flow Model

Imagine if you have 20 projects that have similar NPV and IRR values (and similar qualitative strategic values to the firm). It would be impossible to select the best project. However, with the use of Monte Carlo simulation with Risk Simulator, one can delineate the similar projects through their respective risk structures. That is, the first project has a 76.40 percent chance of breaking even, while the second project has only a 35.50 percent chance, and so forth. This way risk analysis is performed and the project with the lowest risk should be chosen. Monte Carlo simulation when applied here will help the decision maker isolate, identify, value, prioritize, and decide on which projects to execute while considering the potential downside of the projects.

Real Options

The next stop is real options analysis. In the previous simulation applications, risk was quantified and compared across multiple projects. The problem is, so what? That is, we have quantified the different levels of risks in different

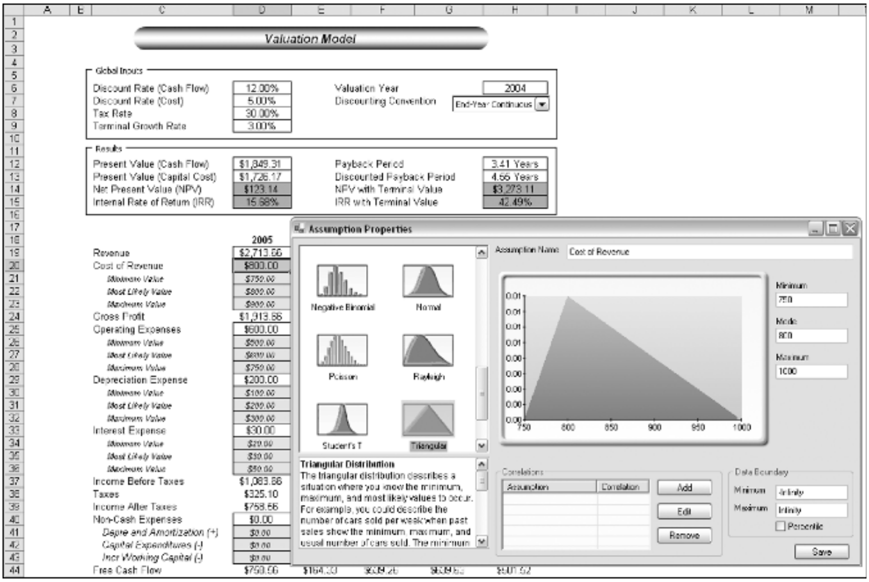


FIGURE 11.39 Simulated Discounted Cash Flow Model

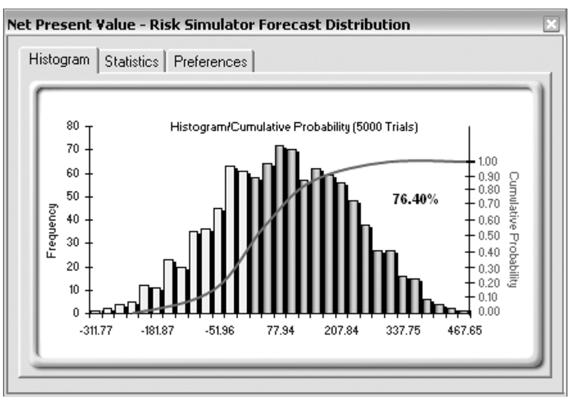


FIGURE 11.40 Probability of at Least Breaking Even

projects. Some are highly risky, some are somewhat risky, and some are not so risky. In addition, the relevant returns are also pretty variable as compared to the risk levels. Real options analysis takes this to the next step. Looking at a specific project, real options identifies if there are ways to mitigate the downside risks while taking advantage of the upside uncertainty. In applying

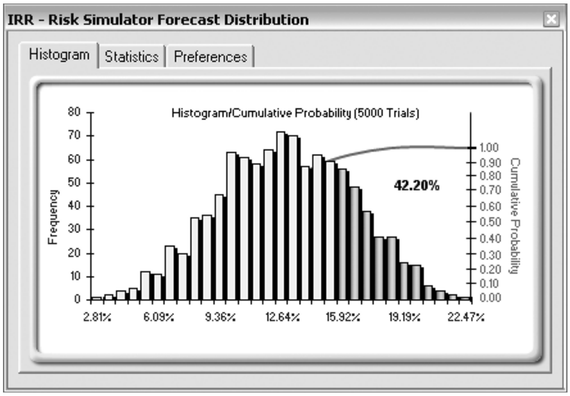


FIGURE 11.41 Probability of Exceeding IRR Hurdle Rate

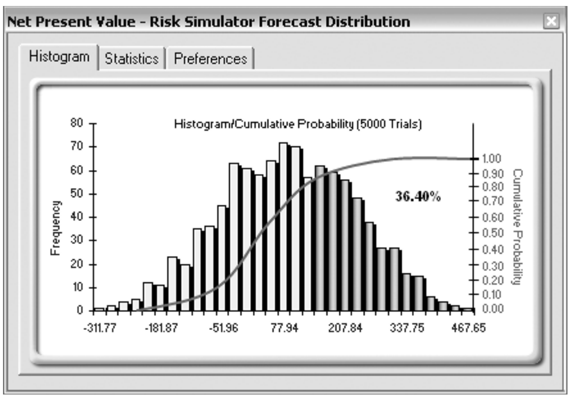


FIGURE 11.42 Probability of Exceeding NPV Expected

Risk Simulator, we have only quantified uncertainties, not risks! Downside uncertainty, if it is real and affects the firm, becomes a risk, while upside uncertainty is a plus that firms should try to capitalize on.

At this point, we can either solve real options problems using the *Multiple Asset Super Lattice Solver* (Figure 11.44) or applying its analytics directly in Excel by using the *SLS Functions* (Figures 11.45 and 11.46). The *MSLS* is used to obtain a quick answer but if a distribution of option values is required, then use the *SLS Functions* and link all the inputs into the function—we use the latter in this case study. Figure 11.45 shows the real options analysis model where the inputs to this model are the outputs from the DCF

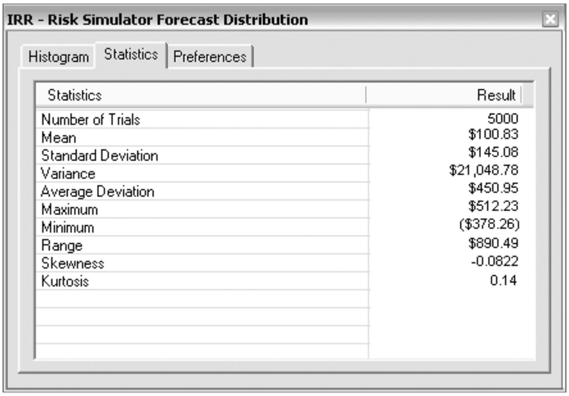


FIGURE 11.43 Calculated NPV Statistics

model in the previous step. For instance, the free cash flows are computed previously in the DCF. The Expanded NPV value in cell C21 is obtained through the SLS Function call (Figure 11.46). Simulation was used to also obtain the volatility required in the real options analysis.

The MSLS results indicate an expanded NPV of $388.10, while the previous NPV was $123.14. This means that there is an additional $264.97 expected value in creating a two-staged development of this project, rather than

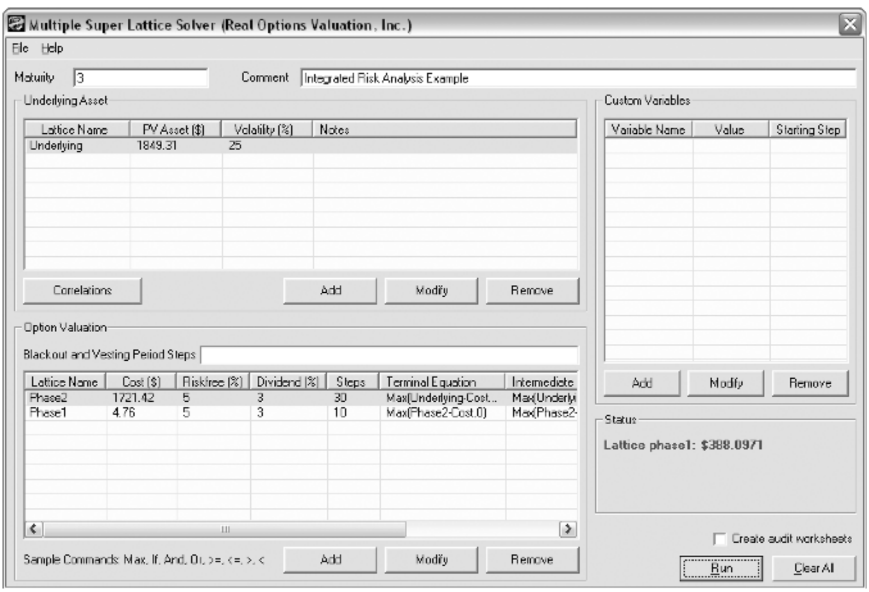


FIGURE 11.44 MSLS Analysis Results

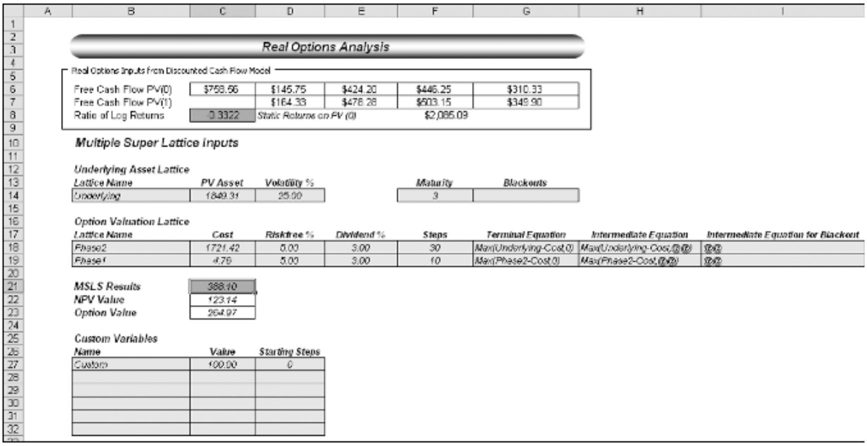


FIGURE 11.45 Worksheet-Based Real Option Analysis Functions

jumping in first and taking all the risk. That is, NPV analysis assumed that one would definitely invest all \$5 in year 1 and \$2,000 in year 3 regardless of the outcome (see Figure 11.47). This view of NPV analysis is myopic. Real options analysis, however, assumes that if all goes well in Phase I (\$5), then continue on to Phase II (\$2,000). Therefore, due to the volatility and uncertainty in the project (as obtained by Monte Carlo simulation), there is a chance that the \$2,000 may not even be spent as Phase II never materializes (due to a bad outcome in Phase I). In addition, there is a chance that a valuation free

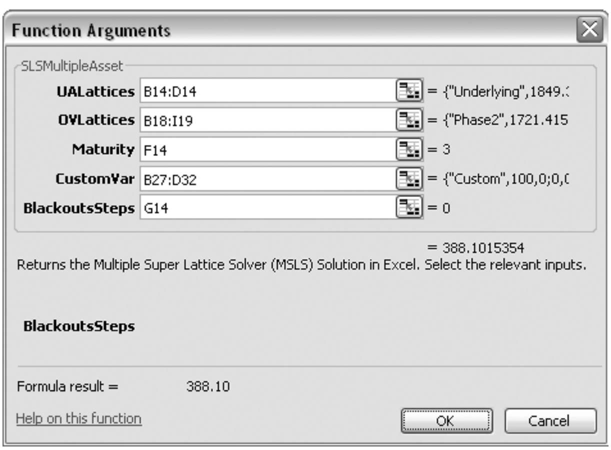


FIGURE 11.46 SLS Function for Cell C21

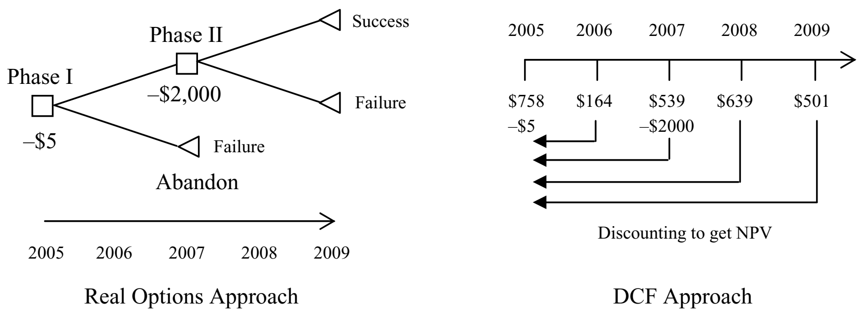


FIGURE 11.47 Graphical Representation of Real Options versus DCF Approaches

cash flow exceeding $1,849 may occur. The expected value of all these happenstances yields an expanded NPV of $388.10 (this is the expected value of the project after accounting for the downside risk mitigation and upside potential after a two-phased stage-gate development process is implemented). The fact that there is an option value means that it is better to perform a stage-gate development process than to take all the risks immediately and invest everything.

Notice that in the real options world, it is an option to execute Phase II, not a requirement, while in the DCF world all investments have been decreed in advance, and, thus, will and must occur. Therefore, management has the legitimate flexibility and ability to abandon the project after Phase I and not continue on Phase II if the outcome of the first phase is bad. Based on the volatility calculated in the real options model, there is a chance Phase II will never be executed, there's a chance that the cash flows could be higher and lower, and there is a chance that Phase II will be executed if the cash flows make it profitable. Therefore, the net expected value of the project after considering all these potential avenues is the option value calculated previously. In this example as well, we assumed a 3 percent annualized dividend yield, indicating that stage-gating the development and taking our time, the firm loses about 3 percent of its PV Asset per year (lost revenues and opportunity costs as well as lower market share due to waiting and deferring action).

Optimization

The next step is portfolio optimization, that is, how to efficiently and effectively allocate a limited budget (budget constraint and human resource constraints) across many possible projects while simultaneously accounting for their uncertainties, risks, and strategic flexibility, and all the while maximizing the portfolio's NPV. To this point we have shown how a single integrated model can be built utilizing simulation, forecasting, and real options. Table 11.26 illustrates a summary of 20 projects (here we assume that you have

TABLE 11.26 Summary of Projects Ready for Optimization

	Project Name	ENPV ($)	NPV ($)	Cost ($)	Risk (%)	Return-to-Risk Ratio	Profitability Index
1	Project A	388.10	123.14	1,732.44	25.00	1552.41	1.07
2	Project B	1,954.00	245.00	859.00	98.00	1993.88	1.29
3	Project C	1,599.00	458.00	1,845.00	97.00	1648.45	1.25
4	Project D	2,251.00	529.00	1,645.00	45.00	5002.22	1.32
5	Project E	849.00	564.00	458.00	109.00	778.90	2.23
6	Project F	758.00	135.00	52.00	74.00	1024.32	3.60
7	Project G	2,845.00	311.00	758.00	198.00	1436.87	1.41
8	Project H	1,235.00	754.00	115.00	75.00	1646.67	7.56
9	Project I	546.00	251.00	364.00	129.00	423.26	1.69
10	Project J	2,250.00	785.00	458.00	85.00	2647.06	2.71
11	Project K	549.00	35.00	45.00	48.00	1143.75	1.78
12	Project L	421.00	75.00	185.00	145.00	290.34	1.41
13	Project M	516.00	451.00	48.00	28.00	1842.86	10.40
14	Project N	499.00	458.00	351.00	94.00	530.85	2.30
15	Project O	859.00	125.00	421.00	65.00	1321.54	1.30
16	Project P	884.00	458.00	124.00	39.00	2266.67	4.69
17	Project Q	956.00	124.00	521.00	154.00	620.78	1.24
18	Project R	854.00	164.00	512.00	210.00	406.67	1.32
19	Project S	195.00	45.00	5.00	12.00	1625.00	10.00
20	Project T	210.00	85.00	21.00	10.00	2100.00	5.05
	Total	$20,618.10	$6,175.14	$10,519.44			

recreated the simulation, forecasting, and real options models for each project), complete with their expanded NPV, NPV, Cost, Risk, Return-to-Risk Ratio, and Profitability Index.

To simplify our analysis and to illustrate the power of optimization under uncertainty, the returns and risk values for projects B to T are simulated, instead of rebuilding this 19 other times.

Then, in Table 11.27, the individual project's human resource requirements as measured by full-time equivalences (FTE) are included. We simplify the problem by listing only the three major human resource requirements: engineers, managers, and salespeople. Their FTE requirements are simulated using Risk Simulator, as are their individual salary costs.

Finally, decision variables (Allocation column in Table 11.27) are constructed, with a minimum of 0 and a maximum of 1, with a discrete step of 1. That is, a project can have an allocation of 1 or 0, representing a go or no-go decision. Table 11.28 shows all 20 projects and their rankings sorted by returns, risk, cost, returns-to-risk ratio, and profitability ratio. Clearly, it is fairly difficult to determine simply by looking at this matrix which projects are the best to select in the portfolio. Figure 11.48 shows this matrix in

TABLE 11.27 Project Allocation and FTE Equivalences

	FTE Equivalence		
Allocation	**Engineers**	**Managers**	**Sales**
1	3.0	4.0	5.0
1	3.0	4.0	5.0
1	3.0	4.0	5.0
1	3.0	4.0	5.0
1	3.0	4.0	5.0
1	4.0	5.0	5.0
1	4.0	5.0	5.0
1	4.0	5.0	5.0
1	4.0	5.0	5.0
1	4.0	5.0	5.0
1	5.0	4.0	4.0
1	5.0	4.0	4.0
1	5.0	4.0	4.0
1	5.0	4.0	4.0
1	5.0	4.0	4.0
1	4.0	3.0	4.0
1	4.0	3.0	4.0
1	4.0	3.0	4.0
1	4.0	3.0	4.0
1	4.0	3.0	4.0
$18,900,000	$6,400,000	$8,000,000	$4,500,000

graphical form, where the size of the balls is the cost of implementation, the x-axis shows the total returns including flexibility, and the y-axis lists the project risk as measured by volatility of the cash flows of each project. Again, it is fairly difficult to choose the right combinations of projects at this point. Optimization is hence required to assist in our decision making.

To set up the optimization process, first select each of the allocation values in Table 11.27 and set the relevant decision variables by clicking on *Simulation* | *Set Decision Variable* (alternatively, you can create one decision variable and copy/paste). Then, click on *Simulation* | *Optimization* and enter the optimization preferences (Figure 11.49).

The linear constraint is such that the total budget value for the portfolio has to be less than $3,500. Next, select Portfolio NPV as the objective to maximize. Set the optimization to run for at least 60 minutes.

The following sections illustrate the results of the optimization under uncertainty and the results interpretation. Further, we can add an additional

TABLE 11.28 Project Selection Metrics Ranking

Project Name	ENPV	NPV	Cost	Risk	Return-to-Risk Ratio	Profitability Index	Optimizer (Go/No-Go)
Project A	18	16	19	3	10	20	1
Project B	4	11	17	14	5	17	1
Project C	5	5	20	13	7	18	1
Project D	2	4	18	6	1	14	1
Project E	11	3	12	15	15	9	1
Project F	12	13	5	9	14	6	1
Project G	1	9	16	19	11	12	1
Project H	6	2	6	10	9	3	1
Project I	14	10	10	16	18	11	1
Project J	3	1	12	11	2	7	1
Project K	13	20	3	7	13	10	1
Project L	17	18	8	17	20	13	1
Project M	15	8	4	4	6	1	1
Project N	16	5	9	12	17	8	1
Project O	9	14	11	8	12	16	1
Project P	8	5	7	5	3	5	1
Project Q	7	15	15	18	16	19	1
Project R	10	12	14	20	19	15	1
Project S	20	19	1	2	9	2	1
Project T	19	17	2	1	4	4	1

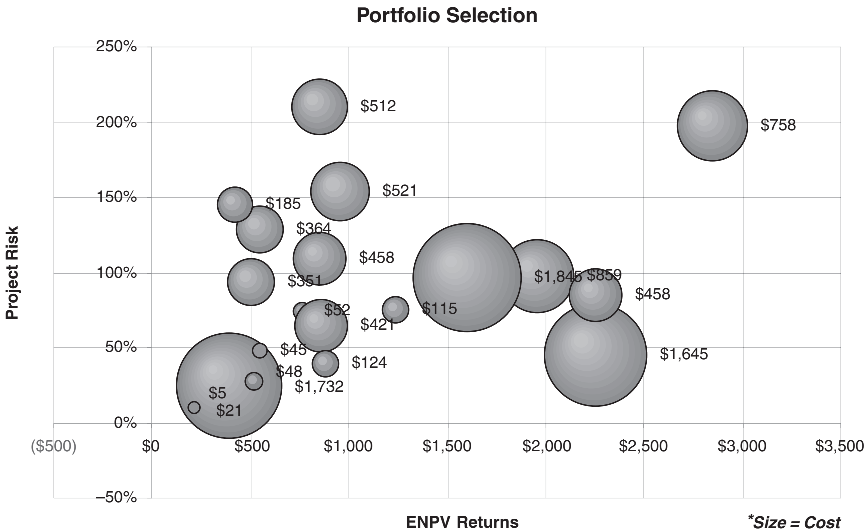


FIGURE 11.48 Graphical Representation of Project Selection Metrics

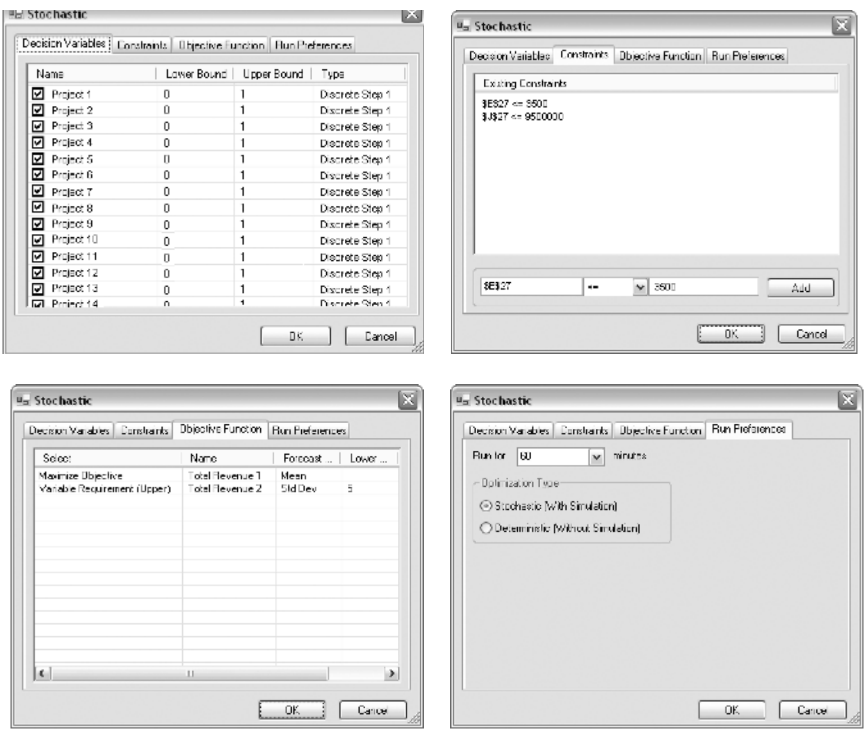


FIGURE 11.49 Optimization Preferences

constraint where the total FTE cannot exceed $9,500,000. We added in a variable requirement because by doing so, we can marginally increase the portfolio risk and see what additional returns the portfolio will obtain. In each run, with a variable requirement inserted, an efficient frontier will be generated. This efficient frontier is generated by connecting a sequence of optimal portfolios under different risk levels. That is, for a particular risk level, points on the frontier are the combinations of projects that maximize the returns of the portfolio. Similarly, these are the points where given a set of returns requirements, these combinations of projects provide the least amount of portfolio risk.

Optimization Results

For the first run, the following parameters were used:

Objective: Maximize Portfolio Returns
Constraint: Total Budget Allocation = $3,500M
Requirement: Total FTE Allocation = $9,500,000
Variable Requirement: 1 to 5 on Portfolio Risk

The results are shown in Figure 11.50 and Table 11.29 to Table 11.31. Figure 11.50 shows a portfolio efficient frontier, where, on the frontier, all the portfolio combinations of projects will yield the maximum returns (portfolio NPV) subject to the minimum portfolio risks. Clearly portfolio P1 is not a desirable outcome due to the low returns. So, the obvious candidates are P2 and

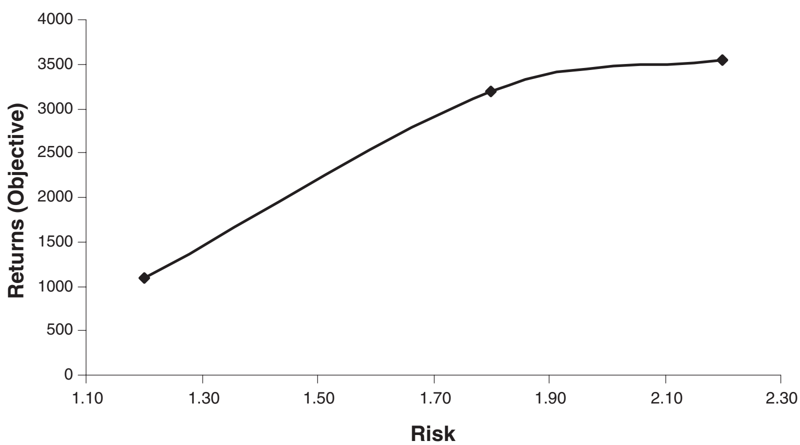


FIGURE 11.50 Efficient Frontier

TABLE 11.29 Project Selection for Portfolio 2

Project Name	ENPV	NPV	Cost	Risk	Return-to-Risk Ratio	Profitability Index	Optimizer (Go/No-Go)
Project A	18	13	19	2	4	20	0
Project B	4	11	17	14	6	17	0
Project C	5	5	20	13	8	18	0
Project D	2	4	18	6	1	14	1
Project E	11	3	12	15	15	9	1
Project F	12	14	5	9	14	6	1
Project G	1	9	16	19	11	12	0
Project H	6	2	6	10	9	3	1
Project I	14	10	10	16	18	11	0
Project J	3	1	12	11	2	7	1
Project K	13	20	3	7	13	10	0
Project L	17	18	8	17	20	13	0
Project M	15	8	4	4	7	1	1
Project N	16	5	9	12	17	8	0
Project O	9	15	11	8	12	16	0
Project P	8	5	7	5	3	5	0
Project Q	7	16	15	18	16	19	0
Project R	10	12	14	20	19	15	0
Project S	20	19	1	3	10	2	0
Project T	19	17	2	1	5	4	0

TABLE 11.30 Project Selection for Portfolio 3

Project Name	ENPV	NPV	Cost	Risk	Return-to-Risk Ratio	Profitability Index	Optimizer (Go/No-Go)
Project A	18	13	19	2	4	20	0
Project B	4	11	17	14	6	17	0
Project C	5	5	20	13	8	18	0
Project D	2	4	18	6	1	14	1
Project E	11	3	12	15	15	9	1
Project F	12	14	5	9	14	6	1
Project G	1	9	16	19	11	12	0
Project H	6	2	6	10	9	3	1
Project I	14	10	10	16	18	11	1
Project J	3	1	12	11	2	7	1
Project K	13	20	3	7	13	10	1
Project L	17	18	8	17	20	13	0
Project M	15	8	4	4	7	1	1
Project N	16	5	9	12	17	8	0
Project O	9	15	11	8	12	16	0
Project P	8	5	7	5	3	5	0
Project Q	7	16	15	18	16	19	0
Project R	10	12	14	20	19	15	0
Project S	20	19	1	3	10	2	0
Project T	19	17	2	1	5	4	1

TABLE 11.31 Summary of Results from Optimization Run I

Portfolio Characteristics	Portfolio 2 (P2)	Portfolio 3 (P3)
Expected NPV (Mean)	$3,208	$3,532
Expected risk (Mean)	182%	218%
Budget used	$2,776	$3,206
90 percentile FTE total cost	$6,368,000	$9,423,000
90% confidence interval for NPV	$2,964 to $3,443	$3,304 to $3,764
90% confidence range for NPV	$479	$460
90% confidence interval for risk	167% to 197%	203 to 234%
90% confidence range for risk	30%	31%
Projects selected	D, E, F, H, J, M	D, E, F, H, I, J, K, M, T

P3. These two portfolios are analyzed in detail in Tables 11.29 to 11.31. It is now up to management to determine what risk–return combination it wants; that is, depending on the risk appetite of the decision makers, these three portfolios are the optimal combinations of projects that maximize returns subject to the least risk, considering the uncertainties (simulation), strategic flexibility (real options), and uncertain future outcomes (forecasting). To summarize the results, Table 11.31 shows the two portfolio combinations side by side. Of course, the results here are only for a specific set of input assumptions, constraints, and so forth. You may choose to change any of the input assumptions to obtain different variants of optimal portfolio allocations.

Many observations can be made from Table 11.31's summary of results. While both optimal portfolios are constrained at under a $3,500M budget and $9,500,000 FTE total cost, P3 uses the budget more effectively as it generates a higher NPV for the entire portfolio but has a higher level of risk (wider range for NPV and risk, higher volatility risk coefficient, requires more projects making it riskier, and higher total cost to implement). For the budget used and the NPV obtained, the lowest risk level required is 218 percent, which is about a 20 percent greater risk compared to P2. In contrast, P2 costs less and has lower risk but comes at the cost of a slightly less NPV and IRR level—these values are obtained from the forecast charts from Risk Simulator (not shown). It is at this point that the decision maker has to decide which risk–return profile to undertake. All other combinations of projects in a portfolio are by definition suboptimal to these two, given the same constraints, and should not be entertained. Hence, from a possible portfolio combination of 20! or 2×10^{18} possible outcomes, we have now isolated the decision down to these two best portfolios. Finally, we can again employ a high-level portfolio real option on the decision. That is, because both portfolios require the implementation of projects D, E, F, H, J, M, do these first! Then, leave the option open to execute the remaining projects (I, K, T) if management decides to pursue P3 later.

CASE 7: BIOPHARMACEUTICAL INDUSTRY—VALUING STRATEGIC MANUFACTURING FLEXIBILITY

This case study was contributed by Uriel Kusiatin, principal and cofounder of 2Value Consulting Group (urielk@twovalue.com), a New York–based management consulting firm that applies advanced financial evaluation and decision analysis techniques to help biopharmaceutical companies significantly improve the way they make and execute strategic decisions. Specifically, 2Value utilizes real options, Monte Carlo simulation, and optimization techniques to evaluate R&D portfolio decisions, licensing opportunities, and major capital investments. 2Value is a strategic partner of the author's firm, Real Options Valuation, Inc. Mr. Kusiatin holds an MBA from the Wharton School and a BSc in industrial engineering from The Engineering Academy of Denmark.

Making decisions on significant investments in manufacturing capacity is a challenging proposition. Biopharmaceutical manufacturing and operations executives are often required to make difficult decisions—decisions that may have significant impact on their company's ability to successfully compete in a complex and highly uncertain business environment.

One of the biggest challenges facing these executives is securing manufacturing capacity for products that are under development and years away from launch. They face a choice between multiple alternatives that include building internal capabilities, outsourcing these to a Contract Manufacturing Organization (CMO), or a combination of the two.

These decisions involve significant capital investments as well as the opportunity cost of allocating funds away from other important initiatives. An internal solution may take four to six years and cost up to $500 million to implement, while there is no guarantee that an outsourced solution will be available when needed, or be sufficiently cost-effective and flexible to meet the needs of the company.

Sizing capacity needs is also challenging. Technology risk associated with biopharmaceutical drug development efforts is high. The probability of a drug candidate in preclinical trials reaching the market is on average less than 20 percent. Market assumptions regarding price, demand, and competition may change dramatically during the lengthy development process and require different capacity than initially anticipated. Build too much and the company will be left with an underutilized asset while trying to identify ways to recoup wasted investment dollars that could have been better utilized elsewhere. Build too little and the company may lose substantial revenue opportunities. Given the risks (and opportunities) inherent in drug development,

- How should manufacturing executives choose the best strategy such that risks are sufficiently mitigated while opportunities are taken full advantage of?
- What can management do to increase its decision-making confidence and improve its capacity-planning capabilities?

The following case study explores how a real options approach can be used to help value and structure strategic investment decisions in a pharmaceutical manufacturing context.

Traditional Valuation Methods and Suboptimal Decisions

Planning for manufacturing capacity typically begins very early in the product development process, often toward the end of preclinical trials or the beginning of Phase I. The key driver used to formulate capacity decisions is a forecast of product demand and anticipated product launch date. Based on these and other assumptions, planners develop a basic timeline that typically incorporates lead times for design and engineering, construction of facility and installation of equipment, and validation and regulatory approval. In addition, they develop initial cost estimates for various alternatives.

Planners must consider a wide range of issues. Typical questions arise:

- What are the available alternatives/strategies that can meet requirements?
- What types of uncertainties from a commercial and product development perspective should be considered?
- How can we secure the appropriate amount of capacity while maintaining the flexibility to expand or contract should business conditions change?
- What is the optimal strategic pathway when strategies have different investment levels, timing, and impact on project value?

For example, suppose a biopharmaceutical company has a product in the preclinical phase of development. The company is initially targeting its most lucrative market—the high-price/low-volume primary market. Marketing has also identified a secondary market—the low-price/high-volume market—that the company may consider expanding into given the right conditions.

Supplying the secondary market with a product will require a significant expansion of capacity.

The company's operations planning group has conducted a feasibility study of various manufacturing capacity strategies and has narrowed the number of alternatives to the following (simplified for illustrative purposes):

Strategy A: Retool an existing facility for launch capacity—commit capital to build new facility to accommodate both primary and secondary markets once the drug has successfully completed the pivotal Phase II clinical trial studies. New facility is expected to be online three years after the product receives FDA approval.

Strategy B: Build a new modular facility—install capacity needed for launch and expand with additional capacity (additional equipment within existing facility) once the drug successfully completes the pivotal Phase II clinical trial studies. Capacity to supply primary and secondary markets is expected to be online with product approval.

Traditional methods used to justify these types of investments are usually based on the discounted cash flow (DCF) model's net present value (NPV) approach.

Given the NPV approach used as seen in Table 11.32, management should focus on the primary low-volume/high-price market only and pursue Strategy A as it provides the highest NPV of both alternatives.

But is this really the optimal strategy?

The problem with the traditional NPV approach is that it is static, that is, it assumes that demand forecasts (and other assumptions driving cash flows) can be projected with 100 percent certainty into the future and that investments are precommitted no matter what happens (i.e., product fails a clinical trial phase, secondary market turns out to be unprofitable). But how certain can planners be of their assumptions when in reality:

- Market factors impacting demand such as competitor moves, pricing and reimbursement, regulatory, and so forth, may change dramatically when the product finally launches.
- Uncertainty increases over time, that is, the further in the future the forecast goes, the more uncertain the forecast. This is especially true when product launch is years away.
- The probability of launching due to product development uncertainty is low—few drugs actually make it to market.

The many uncertainties facing biopharmaceutical decision makers can be described using Monte Carlo simulation Risk Simulator software. These

TABLE 11.32 Manufacturing Capacity Strategies Using Traditional Valuation Approach

	PV of Cash Flow		Investment		Time to Market		NPV	
	Primary Market	Secondary Market	Initial Capacity	Expansion Capacity	Primary Market	Secondary Market	Primary Market	w/Secondary Market
Strategy A—Retool existing facility and expand with new facility	$1,131	$186	$70	$450	Year 9	Year 12	$1,070	$874
Strategy B—Build new modular facility and expand capacity with additional bioreactors	$1,131	$283	$400	$50	Year 9	Year 9	$782	$1,022

uncertainties and their impact on NPV can then be captured and quantified, that is, upside opportunities as well as risks can be identified so that they can be better managed. Real options, as opposed to traditional valuation methods, allows management to value its inherent ability to exploit upside uncertainty while mitigating downside risk.

Valuing Strategic Manufacturing Flexibility Using a Real Options Approach

In reality, management does not have to precommit to making investments before some level of uncertainty clears up, that is, it has the flexibility to make midcourse corrections when better information becomes available.

Manufacturing executives have numerous options at their disposal. For example:

- Build a pilot plant as an option to develop process technology capabilities while outsourcing manufacturing for large-scale production to a partner.
- Buy an option to expand capacity should new opportunities develop.
- Buy an option to sell off excess capacity or use unfinished facilities should business conditions change.
- Use contract manufacturing as a backup/expansion option.
- Abandon/delay facility construction as a response to R&D failure/delay.

In our example, expanding capacity in year 3 is an option contingent on successful completion of Phase II clinical trials and an NPV greater than 0; that is, the decision to commit resources to expansion will happen in year 3 based on better information available at that point in time as seen in Figure 11.51. In addition, the staged nature of manufacturing capacity investments can be linking to key R&D milestones. These investments can be viewed as a series of compound options to abandon the project at any time should NPV become unattractive as seen in Figure 11.52. Obviously, if the R&D effort fails at any point, management can abandon the project and discontinue any further investments.

Volatility of market-based cash flows for both primary and secondary markets was estimated at 40 percent using Monte Carlo simulation with Risk Simulator and applying the Logarithmic Present Value Approach (see Appendix 7A). A market expansion factor was calculated for each strategy to account for the additional cash flows derived from the secondary market. Strategy A's expansion factor was 1.16 times the primary market, while Strategy B's was 1.25. The reason for this difference is the timing of when expansion capacity becomes available. Strategy A's expansion capacity is expected to be online in year 12—it must be built from scratch. Strategy B's expansion capacity is expected to be online in year 9 when product is expected to

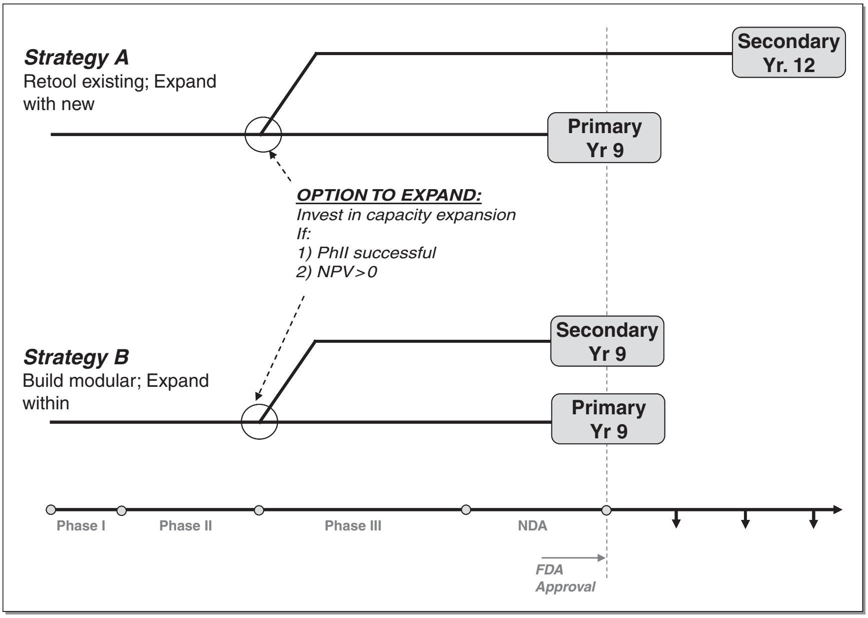


FIGURE 11.51 Strategy Tree for Two Competing Strategies

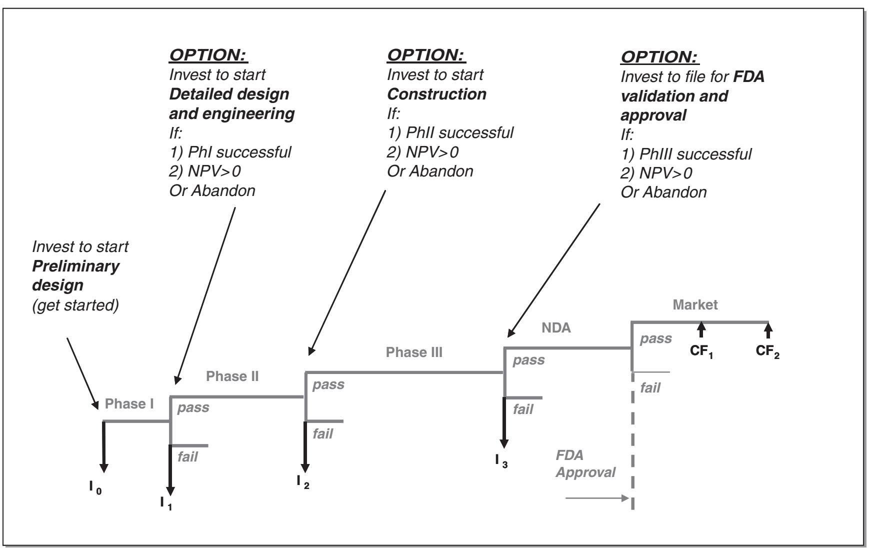


FIGURE 11.52 Linking Manufacturing Capacity Investment Decisions to R&D Clinical Trial Outcomes

launch—additional equipment/suites can be installed and validated in existing buildings to coincide with product launch.

A series of compound options was constructed, each option dependent on the preceding option. The options to start Phase III and FDA filing and validation are chooser options, that is, the option to expand capacity to manufacture to the high-volume/low-price secondary markets, or continue with existing capacity plans to manufacture to the low-volume/high-price primary markets, or abandon the project altogether. For each scenario, NPV is calculated for each option, that is, expand, continue, or abandon. Whichever option provides the highest NPV is the option that will be chosen for that particular scenario. The options to start Phase II and Phase I are basically to continue investing or exit. The decision to continue will depend primarily on the outcome of clinical trials and perceived market value of the opportunity at that point in time.

The resulting NPV of these compound options includes the value of expanding, continuing, or abandoning further investments in capacity only if the NPV in a given scenario is positive or warrants expansion investments (i.e., no precommitment). The option value or value of having this flexibility is the difference between the calculated NPV and the static NPV, that is, the NPV that does *not* account for uncertainty or flexibility (Figure 11.53).

Using a real options approach in our example causes our decision to change—strategy B becomes the optimal strategy. With strategy B, management will commit less total capital for base and expansion capacity while being able to bring expansion capacity (should they need it) to the market with product launch (due to the modular nature of facility layout). Strategy A requires more capital investments in expansion capacity—a new facility must be built from scratch—and due to the build-out lead times, expansion capacity will only be available three years after product launch. That said, Strategy A has more option value, that is, the flexibility to expand, continue, or abandon is more valuable. A greater portion of total investments is deferred until uncertainty is cleared up, post–Phase II. The option to build a new (and costly) facility if, and only if, it makes business sense has value as seen in Table 11.33.

The results seen in Figure 11.53 ($1,279) can be calculated using the MSLS software as seen in Figures 11.54 and 11.55. The former illustrates the same results with a 7-step lattice while the latter illustrates taking the same analysis to 70 steps and beyond, to check for convergence of the results. It would seem that the analysis results are robust and can be simply computed using the MSLS.

This case study illustrates how a real options approach can help managers make better decisions regarding significant investments in manufacturing capacity. It should be mentioned that this approach can be utilized for significantly more complex decisions. In addition, the uncertainties described and

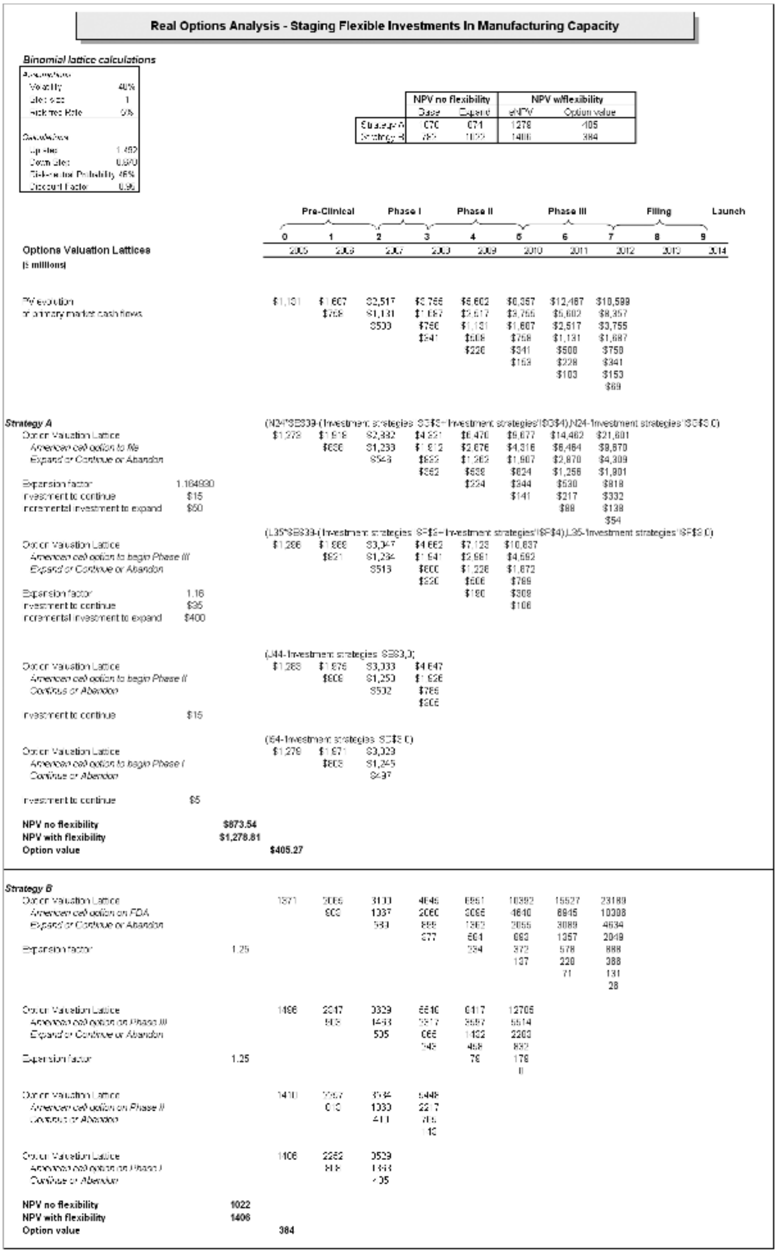


FIGURE 11.53 Real Options Analysis—Compound Chooser Options

TABLE 11.33 Manufacturing Capacity Strategies—Real Options Valuation Approach

	PV of Cash Flow		Investment		Time to Market		NPV		
	Primary	Secondary	Initial	Expansion	Primary	Secondary	wo/flexibiity	w/flexibility	Option Value
Strategy A—Retool existing facility and expand with new facility	$1,131	$186	$70	$450	Year 9	Year 12	$874	$1,279	$405
Strategy B—Build new modular facility and expand capacity with additional bioreactors	$1,131	$283	$400	$50	Year 9	Year 9	$1,022	$1,406	$384

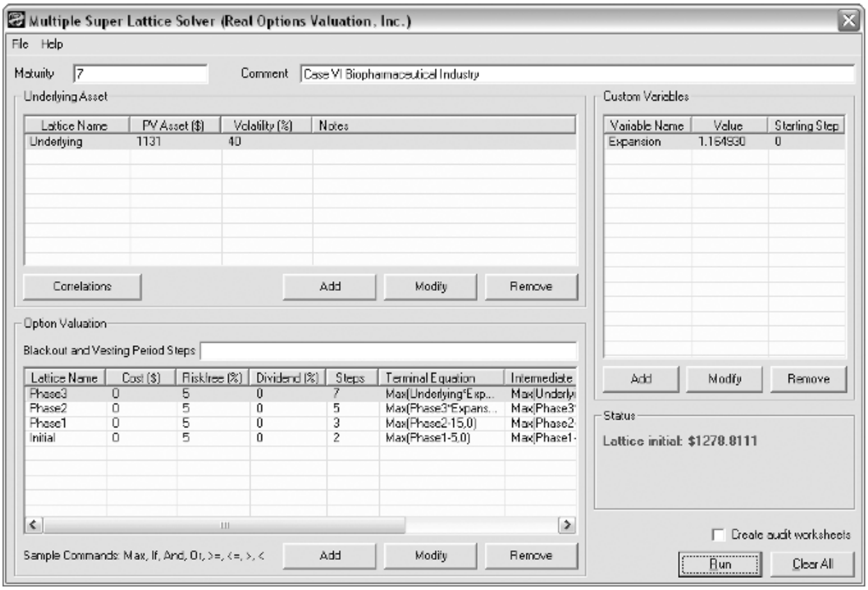


FIGURE 11.54 Multiple SLS Solution (Seven Steps)

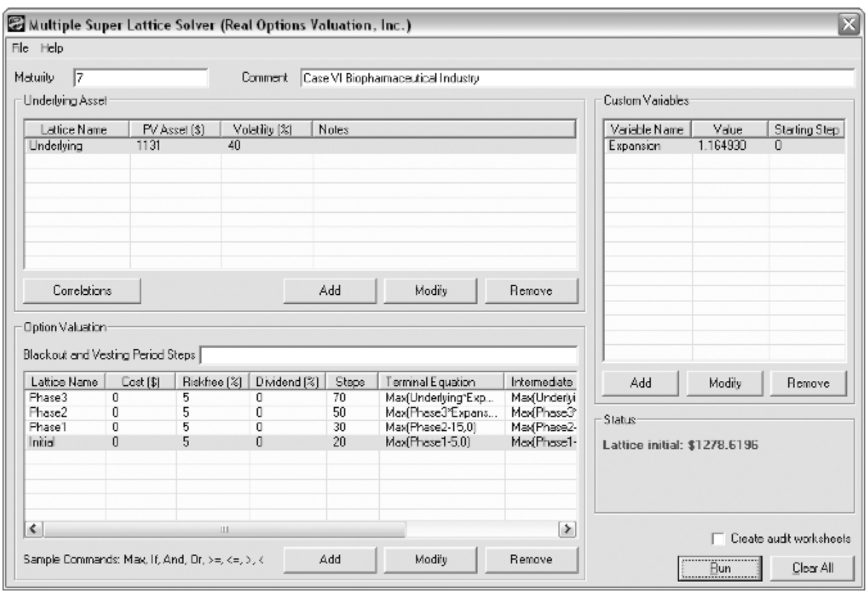


FIGURE 11.55 Multiple SLS Solution (70 Steps)

incorporated in our example are market driven. It is important to also incorporate the probability of technical success, that is, the probability of successfully getting a drug to market. R&D clinical trial success rate probabilities can easily be incorporated into the Monte Carlo simulation.

Implementing a Real Options Approach

The real options approach just described incorporates a learning model, such that management makes better and more informed strategic decisions when some levels of uncertainty are resolved through the passage of time. It also forces management to focus on key decision points or milestones, such as when do we need to make key investment commitments? What are our options at a given point in time that allow us to take advantage of opportunities while reducing risk? What is the cost of waiting or deferring a decision?

This approach sets a premium on obtaining better information before making important decisions. It values flexibility (and identifies the cost of this flexibility) while improving decision makers' risk management capabilities. As such, it can and should be linked to project management execution because value can only be captured if the option is executed optimally.

A real options approach is straightforward to implement and is based on existing inputs and valuation methodologies already used in most companies. Real options analysis adds an additional step to the existing NPV analysis by quantifying the value of the options available to management.

CASE 8: ALTERNATIVE USES FOR A PROPOSED REAL ESTATE DEVELOPMENT—A STRATEGIC VALUE APPRAISAL

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Introduction

It is not uncommon in real estate development for an investor to hold a property where alternative land development uses may be available, subject to planning permission (or rezoning) being obtained. A major element of the overall uncertainty often lies in securing an appropriate permission, and clearly this can convey significant value to the investor. This case example does not focus on mitigating planning uncertainty (as identified in Stage I below) but considers the option value (strategic value) that is conveyed by holding a property where marginal returns on land and profit could be earned as a result of the alternative use opportunities that could be secured.

The case study concerns two mutually exclusive alternative schemes for the same property: a retail warehouse (140,000 square feet) proposal or residential (1,206 units) development. Although the situation is a real-life example, the details have been simplified. The site is located 30 minutes east of central London (United Kingdom) in the suburb of Barking, comprises some 9 acres, and fronts a major road with an interchange giving direct access. A river runs along the eastern and southern boundaries of the property.

The basic simplified details of this case study are as follows:

- The original proposals for redevelopment envisaged the site being developed for retail, theme pub, and restaurant, which would comprise a 100,000 square foot retail warehouse with the necessary market requirement of parking, complemented by 40,000 square feet for restaurant and pub uses.
- Significant remediation works (contamination cleanup) would be necessary to be undertaken across the site together with an internal upgrading of infrastructure and the provision of a riverside walk, which would be an aspect of any redevelopment. This would likely take a year to complete prior to any development.
- Retail warehousing is an attractive investment for funds and developers alike. Building costs are relatively low and speed of construction erection is rapid; an occupier can be trading and paying rent within an 18-month period with pre-leasing construction management being straightforward with little risk for delay. Although the investment market is strong with a shortage of new stock coming into the supply chain, the occupier market is very fickle resulting in development uncertainty.

The owners of the site had concerns over the retail warehouse development as a result of the following considerations:

- The ability to secure appropriate retail occupiers for the scheme.
- The success of nearby residential schemes.
- Encouragement received from the local planning authority to consider a significant residential scheme.
- Demand for residential schemes.

It was anticipated that a residential development of some 1,206 one- and two-bedroom units (904 private and 302 social units) could be accommodated on the site. In addition a small amount of retail, restaurant, and health and fitness uses would be incorporated in an overall scheme.

In summary, a retail warehouse land use of the site would generate a high value, if an operator can be secured, as this would be a sought-after investment. However, the site is well located on the urban fringe, has excellent communication links, and is well placed to take advantage of a rising residential market.

In essence this is an example of a switching or exchange option, that is, the right to buy or take the better of two alternatives. This flexibility provides added value, through risk hedging between the alternatives (subject to a switching cost, that is, the costs of securing an alternative planning permission). The Real Options Super Lattice Solver software is used to ascertain various outcomes, which are then explained in the following text. A manual binomial calculation could also be undertaken but in this case would perhaps be more complex.

A five-step real options analysis (ROA) approach that was adopted comprised the following:

Stage I	Framing the problem
Stage II	Base scoping appraisal (deterministic)
Stage III	Internal and external uncertainty inputs
Stage IV	Real options (quantitative) analysis (stochastic)
Stage V	Explanations and making strategic decisions

Stage I: Framing the Problem

A switching option is another form of delay option, which comes under *flexibility* in the family of real estate development options. In framing this particular situation it is necessary to consider the dynamics of two real estate sectors, residential and retail, and, indeed, their cross-correlation.

A negative correlation provides greater risk diversification and, therefore, should increase the value of the switching option. The opposite is also true. In overall terms, evidence suggested a relatively high correlation between retail warehouse rental values and the level of activity in the housing market, but it was not conclusive, and there was also data to show the correlation varied over time (see Figure 11.58).

It is assumed (as was the case) that the site already has planning permission for a retail warehouse scheme of 140,000 square feet.

A real options analysis strategy matrix in respect to the two alternatives is presented in Table 11.34. From Table 11.34, subject to market factors,

TABLE 11.34 ROA Development Strategy Matrix—Switching Options (Retail Warehouse to Residential)

Strategy/ Approach	Market Factors	Planning Issues (residential)	Timing (development)	Embedded Option Appraisal
		Type of Development: Retail Warehousing or Residential		
Pessimistic	Weak demand from retail sector. Cyclical residential market issues.	Low rise housing with significant (35%+) social housing.	3 years plus (retail) 5 years (residential)	Switch or defer or sell
Cautious	Multiple occupier demand for warehousing but low covenant strength. Improving residential market.	Medium rise housing with 25% social housing.	2 years (retail) 5–7 years (residential)	Switch and develop or switch and defer
Optimistic	Uncertain occupier demand for warehousing. High-growth residential market.	High-rise or densely developed housing. 20% to 25% social.	2 years (retail) 7 years plus (residential)	Switch and develop

timing becomes a major determinant. Although this case study focuses on the switching option, it could also be combined with an option to wait (defer) or abandon or to expand (that is, a larger retail warehouse or residential scheme subject to planning/rezoning).

Stage II: Base Scoping Appraisal (Deterministic)

A standard development appraisal was undertaken in respect of both the retail and residential schemes of which the component elements are summarized in Table 11.35.

Development profit, that is, what a developer would require for the risk of implementing either scheme, is included in the gross development cost (GDC) in order to compare the like-for-like residual land values arising when brought back to the present day. From the foregoing it can be deduced that the static NPV of the switch is the NPV of the difference in the land values less the cost of the switch. In this instance this would equate to £2.53m. Although this would suggest the switch is in-the-money, it is only marginally so, and the two schemes are very different in terms of size and timescale. Is this enough of a reason to change? At what time would change be optimal? What is the current value to the owners of the land of having the ability to switch, now or at a later stage? Sensitivity, scenario, and Monte Carlo analysis may provide a guide to the overall risk structure, which could help quantify to some extent the first of these questions. A real options analysis is a more robust approach to answering these questions given an uncertain market in both sectors.

Stage III: Internal and External Uncertainty Inputs

Internal sensitivities are numerous with the site. It suffers from contamination, which raises some interesting embedded options particularly in respect of timing. This topic is not examined here, but optionality associated with contamination is a considerable area of practical application for ROA.

TABLE 11.35 Base Appraisal Components

Scheme	Retail	Residential
Size	140,000 sq. ft.	1,206 Units (302 social)
Gross Development Value	£34,820,000	£167,950,138
Gross Development Cost (excluding land)	£12,510,000	£135,031,871
Development Time Frame	2 years, 9 months	7 years (approximate)
NPV of Land Value	£18,758,251	£22,134,420
Cost of Switch	£844,121	

As with other real options examples, volatility is a key feature of the switching option. The residential sector, using land registry figures for Barking flats and comparing these to London flats and residential (as a whole) were analyzed. In addition, land transaction prices were analyzed over a similar period. Volatility levels were derived from these and summarized in Figure 11.56.

From the chart, average price volatility for apartment units from 1995 to 2002 for Barking was 11.49 percent. If different periods are taken into account, average volatility rates range between 4.83 percent and 20.21 percent. For the same period it can be seen that land price volatility averages only 3.81 percent for London as a whole. Land prices are notoriously difficult to measure (due to contractual overages and actual timing of payments) and, indeed, if outer London prices are considered on their own, these show an average price volatility of 8.42 percent. It is also a more notable feature of the market that land prices are increasingly correlated to unit price expectations at the time, notwithstanding there is not inconsiderable level of land banking in the U.K. market by developers (that is, developers will delay development until favorable periods for selling but hold the land at historic book value). For the purposes of this case study a volatility of 14 percent is adopted for the residential residual value reflecting the above and future market uncertainties together with a sensitivity range of 5 percent to 25 percent.

Retail warehouse (total return) volatility is indicated in Figure 11.57. For the purposes of this case study, 8 percent is adopted for the volatility of the

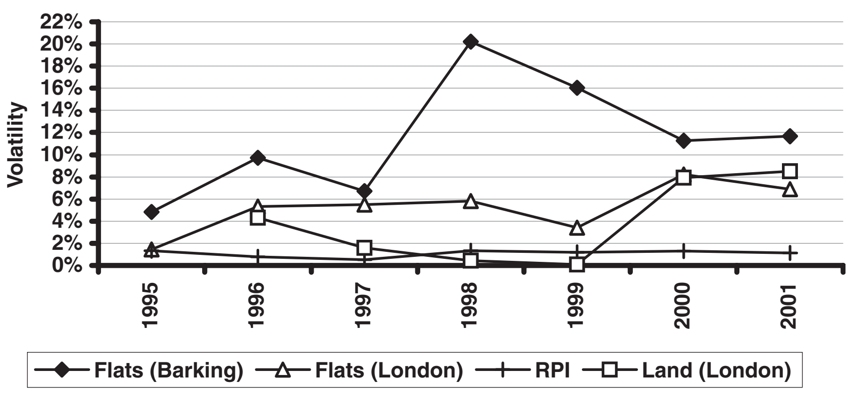


FIGURE 11.56 Residential Unit and Land Sales Volatility 1995 to 2001

Source: Gerald Eve Research.

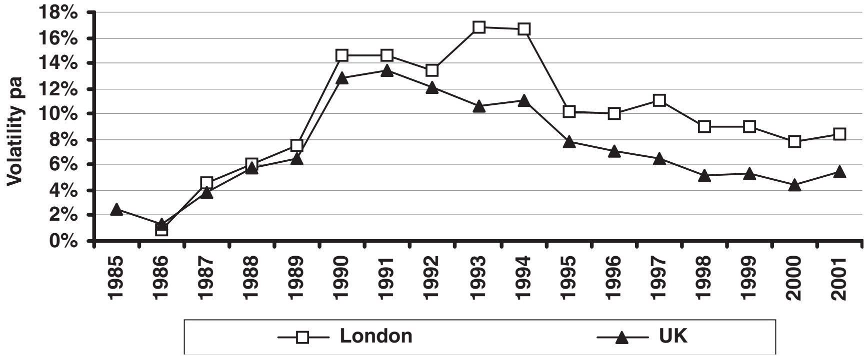


FIGURE 11.57 London and U.K. Retail Warehouse Volatility 1985 to 2001

Source: IPD.

retail residual land value reflecting the foregoing together within a sensitivity range of 5 percent to 20 percent.

The chart in Figure 11.58 provides an indication of the correlation between residential and the retail warehouse market. The synergy between the sectors produces an average positive cross-correlation coefficient of 0.61, which is used for the analysis. However, it is noticeable from the foregoing that there are times where the two sectors are less correlated than the 0.61

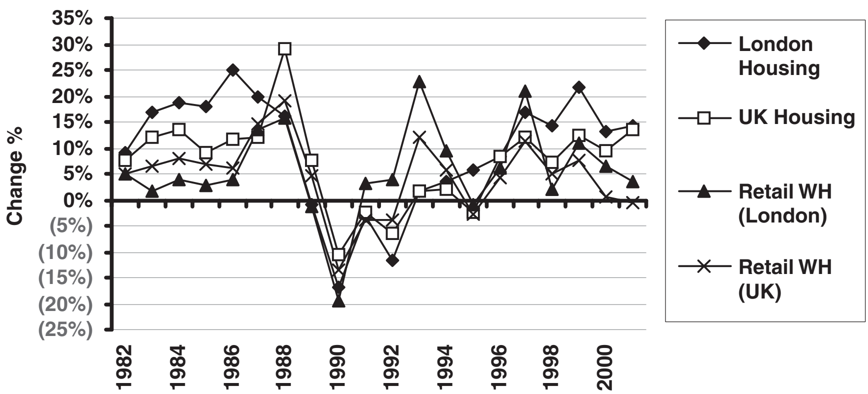


FIGURE 11.58 U.K. and London Retail Warehousing and Residential Correlation

Source: Gerald Eve Research.

would suggest. As previously indicated this would, all other matters being equal, increase the value of the switching option.

Stage IV: Real Options (Quantitative) Analysis

A calculation can be undertaken using the Multiple Asset Super Lattice Solver software with the following inputs: an underlying retail residual land price of £18.758M; a residential residual land price of £22.134M; volatility levels of 8 percent and 14 percent for retail and residential, respectively; a correlation coefficient between retail and residential of 0.61; a risk-free rate of 4.79 percent; three years to maturity (being the time frame in which the land owners would wish to make a decision to develop out either option having secured a residential consent); and cost of switching of £844,121 (being planning, consultants, and other costs).

The switching option value in this instance is calculated at £3.65M. The static NPV (as calculated under Stage II) was £2.53M as seen in Figure 11.59. Therefore the additional switching value created by an ROA is £1.12M.

Stage V: Explanation and Strategic Decision

From the previous section, it can be said that if the switching option is obtained, it is worth £1.12M. That is, the developer should be willing to spend

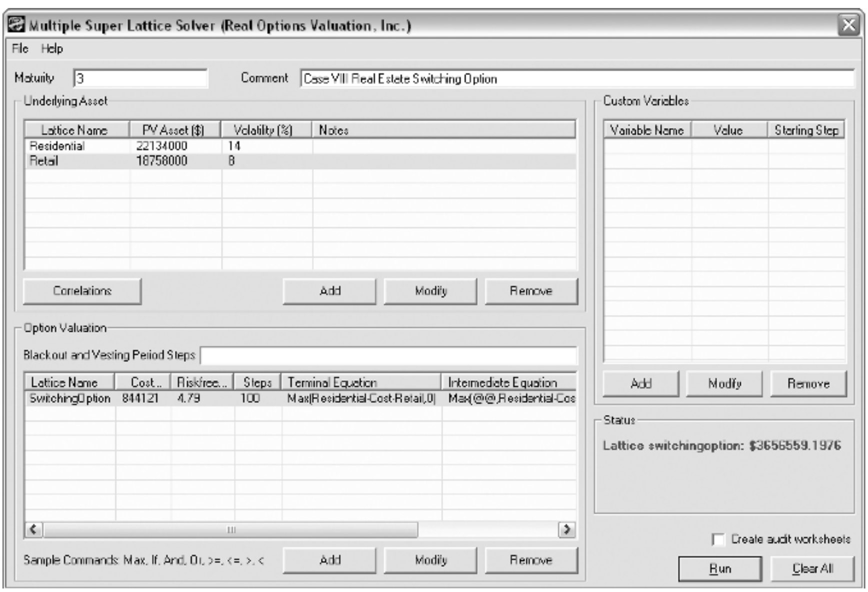


FIGURE 11.59 MSLS Solution to Switching Option

up to an average £1.12M to obtain the planning permission, the contamination cleanup and certification, and all other required legal documentation and requirements such that the option to switch from retail to residential can be executed. If such requirements are not obtained up front, a switching option may not exist. Therefore, the questions that were answered are how much the developer should spend to secure the strategic flexibility such that a new scheme can be implemented quickly and if such a switching scheme carries with it value. Additional real options analysis can be performed to determine the market value of the residential scheme such that executing immediately is valuable (the optimal trigger values and optimal timing analysis).

In real options analysis, if a sensitivity analysis in the form of surface area charts is undertaken, it will provide additional insights into the context of the results obtained. In this instance a Monte Carlo analysis is also provided in order to consider the risk profile and contributing factors to the results obtained.

It is interesting in this case to consider the payoff chart in comparing an option price NPV comparison as illustrated in Figure 11.60. As can be seen from the chart, even where there is a negative static NPV for residential, under a real options analysis there is value attributed to the option of switching.

Volatilities of retail in relation to residential are clearly shown, together with the combined effect, in respect to the switching value in the first of the charts in Figure 11.61. There are very noticeable "frowns" and "smiles" highlighting the importance of volatility analysis and the interaction of

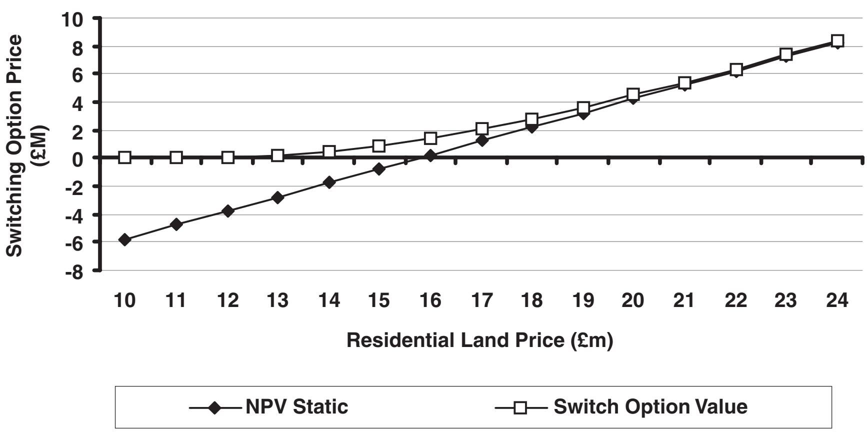


FIGURE 11.60 Switching Option Payoff Charts

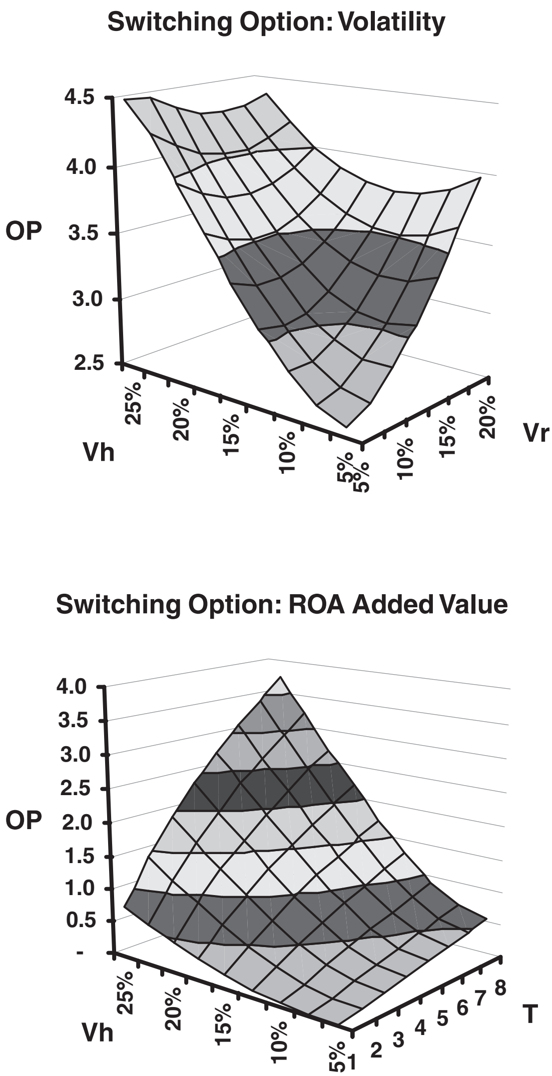


FIGURE 11.61 Sensitivity of Volatility and Timing for Retail and Residential

volatility between residential and retail sectors. Correlation between the sectors is therefore an important issue. In the second of the charts in Figure 11.61, it is also of note that the longer the time period, in this case, the greater the option price, which may suggest that an option to defer even at three years may be better for the land owner.

Finally a Monte Carlo frequency chart using the Risk Simulator software of the switching option price approach is shown in Figure 11.62 with a certainty level of 90 percent. The inputs comprised volatilities of residential and retail; their correlation, cost to switch, and time to maturity. There is a clear right skew and a mean of £1.18m value added above a static NPV as a result of the switch.

The investor, as a result of a real options analysis, can clearly ascertain the value derived from a potential switch notwithstanding that it is in-the-money, albeit only marginally so, on an NPV basis. A real options analysis has regard to future uncertainty with both residential and retail and their respective sensitivities. Again whether this would be enough to release a contingent claim and develop would in practice be subject to a closer assessment, based on optimal timing, having regard to the analysis performed and therefore a strategic decision on switching, now or at a later stage (subject to planning/rezoning).

This case study could provide for numerous other embedded options including the option to expand (or contract) the number of residential units or perhaps seeking an extension of the retail space (an expansion option); the option to phase either the retail or residential; the option associated with contamination mitigation (timing, deferral, or impact of switching); the ability to offset loss of earnings during construction through advertising and temporary use of land (subject to phasing) overcoming to a degree construction time lags; and to combine options (compound or spread options). Although this analysis raises some computational issues, in fully evaluating an appropriate strategy it can be seen that the foregoing analysis can be extended significantly in practice.

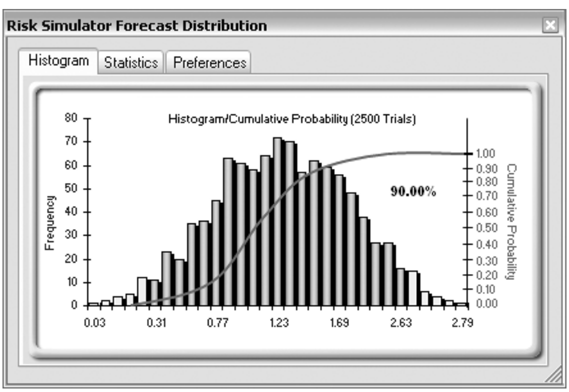


FIGURE 11.62 Real Options Analysis' Monte Carlo Simulation Results

Finally, when the two alternative development options were evaluated and presented against the perceived underlying strategic value of the site with these opportunities, management debate focused on market uncertainty in both sectors and the cost and timing of the switch. A real options analysis provided a quantitative and analytical backdrop for those discussions. The option to keep the ability to switch open until such a time when it was concluded optimal to pursue a residential development and, therefore, the contingent claim will then be exercised.

CASE 9: NAVAL SPECIAL WARFARE GROUP ONE'S MISSION SUPPORT CENTER CASE

This case study was developed by Sarah Nelson, Tom Housel, and Johnathan Mun. Nelson is the CEO of Intellectual Capital Ventures, LLC, a boutique consulting and valuation firm in Chicago, Illinois. Housel is a professor of information sciences at the Naval Postgraduate School in Monterey, California. Both Housel and Mun are strategic partners of the author's firm, Real Options Valuation, Inc., and the results of this project were presented to the Office of Force Transformation, Department of Defense. Proprietary and sensitive information have been removed but the essence of the real options application remains.

We developed the following case study for the Office of Force Transformation, Department of Defense (DoD), to demonstrate the power of applying real options analysis, populated with new raw data gathered using Knowledge Value-Added (KVA), to battlespace strategic planning initiatives. The quantitative analyses provided by pairing KVA and real options analysis enabled the DoD to better understand its return on investment in people and information technology for a technology-heavy mission support center. It also enabled the DoD to gain clarity regarding the many benefits of real options analysis for future planning purposes.

The Naval Special Warfare Group One (NSWG-1) of the United States Navy established and utilized a Mission Support Center (MSC) to assist in conducting mission planning and execution during Operation Enduring Freedom (OEF) and Operation Iraqi Freedom (OIF). The MSC was a reach-back component, located in San Diego, California, that used information technologies to enhance the collaboration between forward and rear units and provided shared situational awareness for war planners and war fighters.

The MSC was designated NSWTG-REAR and was able to generate high-priority requests for information (RFI) that the intelligence community answered. Three new IT tools were also used as an integral part of MSC operations:

- A3, a relational database developed to provide tailored intelligence products.
- WEBBE, a multipoint instant messaging tool with voice-over capabilities.
- Access to Global Broadcast Service (GBS), a satellite downlink that provided for fast transfers of large data files.

Table 11.36 summarizes the people, processes, and technologies that made up the MSC for OIF.

The Mission Planning Cycle supported by the MSC included a number of core processes such as Mission Feasibility Assessment, Warning Order, Fragmentary Order, Concept of Operations, and Execution Order.

We selected the Mission Feasibility Assessment (MFA) cycle for our analysis and made the following assumption: The Mission Feasibility Assessment Cycle was the only segment of the Mission Planning Process in which the MSC-Rear participated and the remainder of the Mission Planning Process occurred forward in the field. This assumption allowed us to equate the total costs and proxy revenues for the MFA cycle with the costs and proxy revenues for the MSC as a whole for use in developing net cash flows for discounted cash flow and real options analyses.

The MFA cycle consisted of ten subprocesses: Receipt of Mission, Mission Feasibility Analysis, Assess SOF Operational Criteria, Develop Courses of Action, Analyze Courses of Action, Compare Courses of Action, Recommend Course of Action, Commander's and Forward Staff's Planning Guidance,

TABLE 11.36 People, Processes, and Technology for OIF MSC

Force Elements	MSC for OIF
Information Sources	Collaborated intelligence/info sources sensors, HUMINT
Value-Added Services	Federated network, Blue Force Tracking, A3, Global Broadcast System, WEBBE, JWICS, SIPRNET for strategic and tactical missions
Command and Control	MSC was permanent and colocated; staffed 24 hours a day, 7 days per week
Effectors	Force composition was 110 staff forward in theater; 75 staff rear at MSC. Supported 600 SOF forces and 7,805 METOC requests
Operating Environment	Desert

HUMINT = human (source) intelligence; JWICS = Joint Worldwide Intelligence Communications System; METOC = a privately owned U.K. company; SIPRNET = Secret Internet Protocol Router Network; SOF = soldier of fortune.

Issue Warning Order, and Issue Feasibility Assessment to JTF/Requested Element. In addition, we included IT infrastructure support in the analysis.

Statement of the Real Options Case Problem

According to a detailed study (June 2004) by Booz Allen Hamilton that assessed the effectiveness of the MSC, the MSC enhanced command and control, increased mission unit effectiveness, altered initial conditions, significantly increased combat power by increasing the number of combat missions that could be simultaneously conducted worldwide, and decisively impacted events in the global war on terror.

For this reason, the U.S. DoD has decided that it needs several more MSCs to assist Joint Forces Special Operations in their warfighting missions. However, the DoD does not know the most effective force mix to use to staff the MSCs and whether the supporting IT should be built, bought, or outsourced. The uncertainties related to acquiring the right MSC analysts and IT, and budgetary constraints were significant.

Instead of simply making a decision on whether implementing the MSCs is prudent and executing it without regard for an ongoing implementation strategy, the DoD has chosen to create a sequential compound option for quantification and review. This stage-gating approach will allow the DoD to halt strategy execution at any given decision node, should that strategy no longer be desirable to pursue.

Three Strategies for Analysis

Three strategies have been selected for analysis. All three have the same initial assumptions:

- The requirements of projected combat potential indicate that Joint Forces will need to enlarge the current MSC to full capacity during Year 1, using current IT.
- Within the next three years, the DoD will also need an additional five MSCs containing 25 analysts each to support five Combatant Command teams of five (i.e., five analysts will be assigned to one Combatant Command team).
- The MSCs will begin to serve all Joint Forces special operations groups, rather than just NSWG-1.

The other critical assumption is that the Mission Feasibility Assessment Cycle is the only segment of the Mission Planning Process in which the MSC will participate. The remainder of the Mission Planning Process will occur strictly in the field. This assumption allows us to equate the total costs and proxy revenues for the MFA cycle with the costs and proxy revenues for the entire MSC. Simplified descriptions of the three strategies are presented next.

Strategy A The increasingly complex technologies and training required to develop, staff, and operate an MSC in support of widely dispersed, ever-changing, asymmetric warfighting scenarios are probably best obtained from the already intensive, mission-specific R&D and long-term training and expertise offered by in-house DoD initiatives. Although Command does not want to utilize warriors from the tip of the spear as MSC analysts, the Reserves have an excellent pool of talent that could be retrained and used in this capacity. In addition, these Reserves would have actual military training and experience and would not need the extensive preparation required for civilian analysts.

The DoD wants to rehab existing military facilities to house the MSCs and will use the current MSC as a prototype for the initial rehab of a physical plant. In addition, the DoD will lease the IT infrastructure and hardware necessary to operate the MSCs and develop customized software over a six-month period, using contract labor under the direction of DoD experts.

Strategy B The increasingly complex technologies and training required to develop, staff, and operate an MSC in support of widely dispersed, ever-changing, asymmetric warfighting scenarios represents a challenge. Command feels that, given the unique nature of Special Forces Operations, the best pool of talent to use in MSC staffing is regular military, preferably with exposure to Special Forces Operations. Neither Reservists nor civilians fit this profile. Command also feels that there is not enough time to develop software in-house or by outsourcing can meet the urgent needs in the field today. So Command has made the decision to purchase off-the-shelf software, utilize intensive training on the software for seasoned military analysts, and adjust as needed at the end of Year 1.

The DoD wants to rehab existing military facilities to house the MSCs and will use the current MSC as a prototype for the initial rehab of a physical plant. In addition, the DoD will lease the IT infrastructure and hardware necessary to operate the MSCs and will enter into a joint venture with the vendor supplying the software in which the vendor will supply the initial software and upgrades at a healthy discount from private sector prices.

Strategy C The increasingly complex technologies and training required to develop, staff, and operate an MSC in support of widely dispersed, ever-changing, asymmetric warfighting scenarios is probably best obtained from the private sector. Here the profit motive, extensive R&D, and technology entrepreneurship will provide a much fuller menu of choices at a lower cost than those offered by DoD R&D initiatives.

The DoD wants to rehab existing military facilities to house the MSCs and will use the current MSC as a prototype for the initial rehab of a physical plant. In addition, the DoD wants to purchase and own the IT infrastructure and hardware necessary to operate the MSCs, but does not want to buy or build the software in-house. A software developer will provide customized software and upgrades for all MSC functions, hire and manage all analysts, and also hire the original five military analysts at equivalent private sector rates to use them as trainers as well as analysts. The vendor will retrain and redeploy the analysts assigned to the MSCs for other DoD functions, should the DoD seek to cancel the contract after Year 1. The strategic tree for this analysis is found in Figure 11.63.

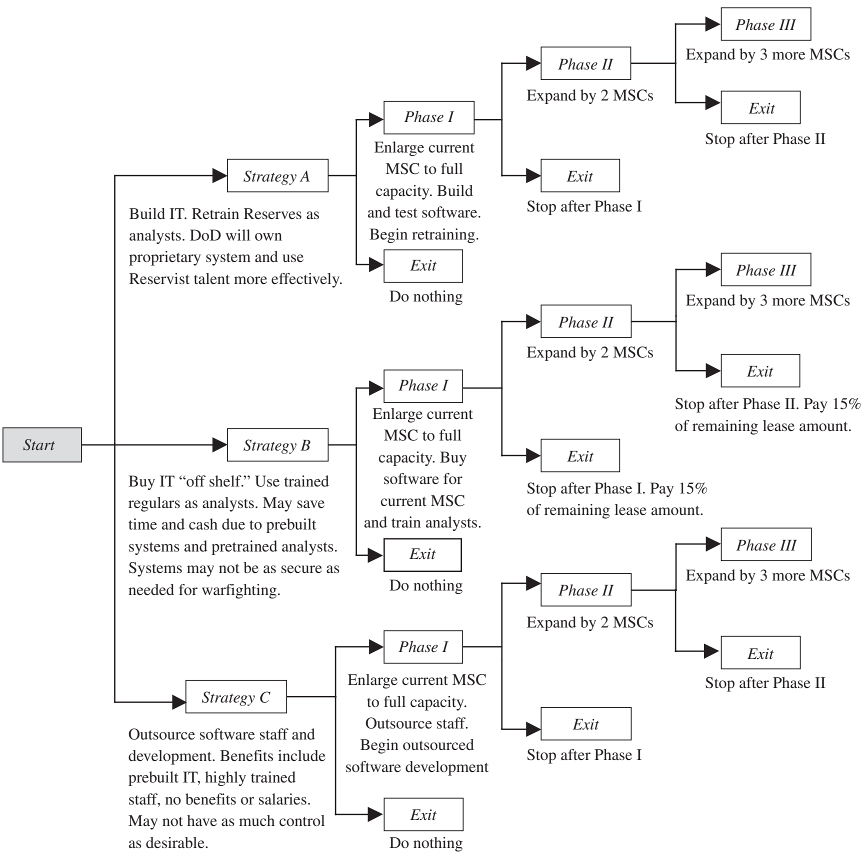


FIGURE 11.63 MSC Strategy Tree

Unique Data Needs

As the future MSC concept will be developed and owned by the DoD, a not-for-profit organization, the analysis requires the use of unique data sets for revenue. Traditionally, for government forecasting, budgeted revenues are equivalent to budgeted cost. This makes it impossible to develop a genuine return on investment (ROI) or to change strategic focus from cost savings to value creation. In addition in this setting, the for-profit capital markets provide no reasonable proxies or comparable data by which to develop the discount rate to be used in Discounted Cash Flow (DCF) analysis or other inputs to the real options analysis model.

To solve these problems, we took two steps: (1) We developed proxy revenues, and (2) we used KVA to assign these proxy revenues to the MFA cycle subprocesses in order to develop a discount rate and real options analysis model inputs.

Developing Proxy Revenues

In the MFA cycle, processes are executed by humans, assisted by information technology. In a very real sense, humans drive the "revenues" (i.e., cash inflows) of the MSC because, without their agency, the MSC would cease to function and it would receive no budget dollars. For this reason, we have chosen to use the private sector "market values" of these human agents as proxy revenues for the MSC. These proxy revenues are a conservative reflection of market expectations for the leverage of human capital in producing revenue (value) for the organization.

Such market values are equivalent to private sector salaries for human agents with similar experience, skill sets, and responsibilities. We developed our proxy revenues by increasing budgeted annual military salaries by a "market premium." This market premium is the percent by which private sector salaries would exceed military salaries for the same levels of experience and responsibility.

KVA and Its Use

KVA is the seminal work of Dr. Tom Housel (Naval Postgraduate School), the coauthor of this case, and Dr. Valery Kanevsky (Agilent Labs). Developed from the complexity theoretic concept of the fundamental unit of change (i.e., a unit of complexity), KVA *provides a means to count the amount of organizational knowledge, in equivalent units, that is required to produce the outputs of the organization*.

The following four assumptions allow KVA to equate units of change (complexity) with units of organizational knowledge and then count them:

1. Humans and technology in organizations take inputs and change them into outputs through core processes.
2. All outputs can be described in terms of the amount of change (i.e., complexity) required to produce them.
3. All outputs can also be described in terms of the time required by an "average" learner to learn how to produce them. Learning time can be considered a surrogate for the amount of organizational knowledge required to produce the outputs. KVA describes these common units of learning time (i.e., units of output) by using the term *knowledge units* (K_μ).
4. A K_μ is proportionate to a unit of complexity, which is proportionate to a unit of change.

By describing all process outputs in common units (i.e., the K_μ required to produce them), it is possible to assign revenue, as well as cost, to those units inside the organizational boundary at any given point in time. This makes it possible to compare all outputs in terms of revenue per unit as well as cost per unit. In addition, once we have assigned both revenue and cost streams to sub-organizational outputs, we can apply standard accounting and financial performance and profitability metrics to them. This methodology applies to not-for-profit as well as for-profit organizations.

KVA and Real Options Analysis for the MSC Sequential Compound Option

The question that remains after building and analyzing the strategy tree is: Which strategy is optimal? The KVA methodology provides the raw inputs (return on knowledge investments as well as assignment of both costs and proxies for revenue). Real options analysis uses these inputs to determine the optimal strategy to execute.

Apply Base Case NPV Analysis and Use Results in Monte Carlo Simulation

First, we modeled the results from the KVA approach into a set of discounted cash flows for the three strategies, resulting in expected net present values (NPVs, i.e., benefits less cost, on a present-dollar value basis), without flexibility, for each. For base case NPV analysis, DCFs were run for each year and then summed to arrive at a total NPV for the three years for each strategy. This base case approach assumes that all future net cash flows are known with certainty and therefore there is zero volatility around input values.

However, the future net cash flows related to the MSC project strategies do involve uncertainty. For example, salary levels may fluctuate over the course of the project. Since we pegged proxy revenues to budgeted salaries, proxy revenue fluctuation will be correlated to the volatility of salaries. In addition, the rate of inflation, modeled in the base case as 4.5 percent, may fluctuate (i.e., exhibit volatility), as may the risk-free rate used to discount future net cash flows. These kinds of input volatilities suggest that we should develop a probabilistic range of NPV values for our analysis, rather than on a single-point estimate of value.

These probabilistic value distributions are generated by using Monte Carlo simulation. All the volatile (i.e., fluctuating) inputs into the model are simultaneously run through 1,000 trials, allowing them to all change at the same time. The results are 1,000 NPV values collated into probabilistic distributions.

For example, Strategy A's NPV is distributed such that its expected NPV is $24.37 million. However, due to the probabilities related to input volatility, the 90 percent statistical confidence range places this NPV at between $23.60 million and $25.13 million.

Table 11.37 shows the expected NPVs and statistical confidence ranges of the three strategies. As we review these results, they indicate that, using the base-case NPV approach, Strategy B is the optimal decision to pursue.

However, NPV analysis only provides a static description of a single decision pathway for each strategy, utilizing a single probability distribution to represent each strategy's input fluctuations. It does not take into account the discrete volatilities and uncertainties related to *staged* MSC implementations or the option to exit and abandon the program if a future stage proves to be unsuccessful. NPV analysis looks at the strategy as a straight path that must be traversed regardless of the learning and changes that occur at a later date. It ignores the inherent flexibility to abandon or expand to the next phase.

So we used real options analysis to look at the complete strategic value of each pathway, accounting for not only the underlying base case input volatilities and uncertainties but also the strategic flexibility embedded in each stage of the pathway.

Develop Volatility Parameter and Calculate Option Results with Simulation

One of the more difficult input parameters to estimate in a real options analysis is volatility. The base-case NPV probability distributions do not tell

TABLE 11.37 Expected NPVs and Statistical Confidence Ranges

	Expected NPV	90% Statistical Confidence Range
Strategy A	$24.37M	$23.60M to $25.13M
Strategy B	26.63M	26.24M to 27.02M
Strategy C	24.75M	24.02M to 25.51M

us what volatility parameters we should apply to inputs into a *staged* MSC implementation. Ordinarily, to get these we could go out to the markets and make estimates based on our informed professional judgment or use historical data to help us build our estimates. However, the DoD has no market comparable data that could reasonably be used for this purpose.

KVA produces an internally generated historical ratio, return on knowledge investment (ROKI), which can be used in a Monte Carlo simulation to generate a volatility parameter. This volatility parameter is a statistical value representing the distilled, integrated effects of all the volatilities and uncertainties inherent in the forecasted values for each MSC strategy stage.

Several methods are available to calculate volatility. We used the Logarithmic Present Value Approach with Monte Carlo simulation, as it was the most robust method and provided a higher degree of results precision. When implied ROKI volatilities were simulated using the Logarithmic Present Value Approach, they produced the volatility parameters shown in Table 11.38. These parameters implied that there were high levels of fluctuation by staged ROKIs around base-case returns, suggesting high degrees of risk inherent in all strategies.

Using these volatility parameters as well as the other inputs associated with each stage of each strategy, we ran real options analysis using binomial lattices and simulation.

Each real options analysis provided us with two new valuations to consider along with the base case NPVs: the total strategic value and the value of the options built into the staged models. Table 11.39 summarizes these values for our three strategies. [Columns (A) + (B) = (C)]

Once the real options analysis was completed for all strategies, we were able to compare total strategic values with base case NPVs under varying levels of volatility, to identify the optimal strategy to execute. Table 11.40 presents the results of this statistical comparison.

An interesting result emerges. When volatilities are low, Strategy B is optimal. However, when the volatility is high, the total strategic value (NPV plus real options value) indicates that Strategy C is optimal. Because the analy-

TABLE 11.38 Volatility Parameters Related to Strategy ROKIs

	Volatility Parameter for ROKI (%)
Strategy A	92
Strategy B	86
Strategy C	92

TABLE 11.39 Base-Case, Option, and Total Strategic Values

	A Base-Case NPVs	B Option Values	C Total Strategic Values
Strategy A	$24.37M	$9.42M	$33.79M
Strategy B	26.63M	8.37M	35.00M
Strategy C	24.75M	14.17M	38.92M

sis of each strategy involved a relatively high volatility for ROKIs (92 percent, 86 percent, and 92 percent), the optimal strategy is C.

Hence, when accounting for the strategic flexibility of the MSC implementations, Strategy C should be undertaken. In fact, Figure 11.64 indicates that 99.90 percent of the time, Strategy C has a higher strategic value than Strategy B.

Problem-Solving Contributions of KVA to Real Options Analysis

At the most aggregated level, real options analysis occurs in four phases over time:

- Phase One—Establish the structure for the problem.
- Phase Two—Plan and frame the options (i.e., lay out the options).
- Phase Three—Implement (exercise) the options over time.
- Phase Four—Track options results and adjust decision paths.

KVA can make significant contributions in Phase One by providing a higher quality of fundamental data inputs to the problem structure. Currently real options analysts use project-level, or even company-level, data for real options analysis. There are currently no specific suborganizational data that can be used. KVA can analyze the effects of core processes on a project and provide fresh raw data based on estimated suborganizational revenues and costs. This suborganizational level data also allows analysts to identify and understand the interdependencies among processes within the project and between the project and the company.

In addition, KVA can make significant contributions in Phase Four. As KVA data is gathered, it can be used to build near real-time option performance assessments. Currently, there has been no direct way for management to measure option performance on an ongoing basis once an option decision path has been selected.

TABLE 11.40 Reversion of Optimal Strategy under Different Volatilities

	Volatilties (%)														
	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
Total Strategic Value	B	B	B	B	C	C	C	C	C	C	C	C	C	C	C
Base-Case NPV	B	B	B	B	B	B	B	B	B	B	B	B	B	B	B

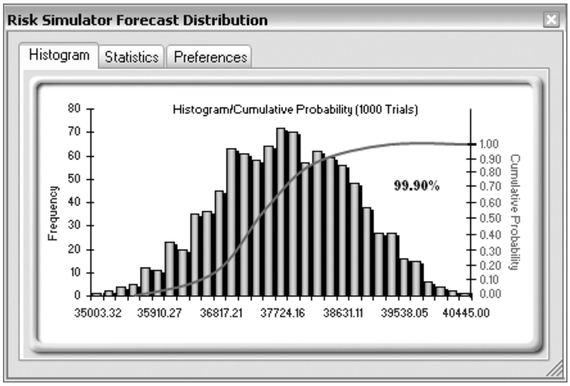


FIGURE 11.64 Strategy C's Statistical Probability of Exceeding Strategy B's Value

Problem-Solving Contributions of Real Options Analysis to KVA

As we stated earlier, KVA uses historical data. KVA has needed the theoretical and practical means by which it could also be used prospectively. Real options analysis provides the rigorous quantitatively based structure in which

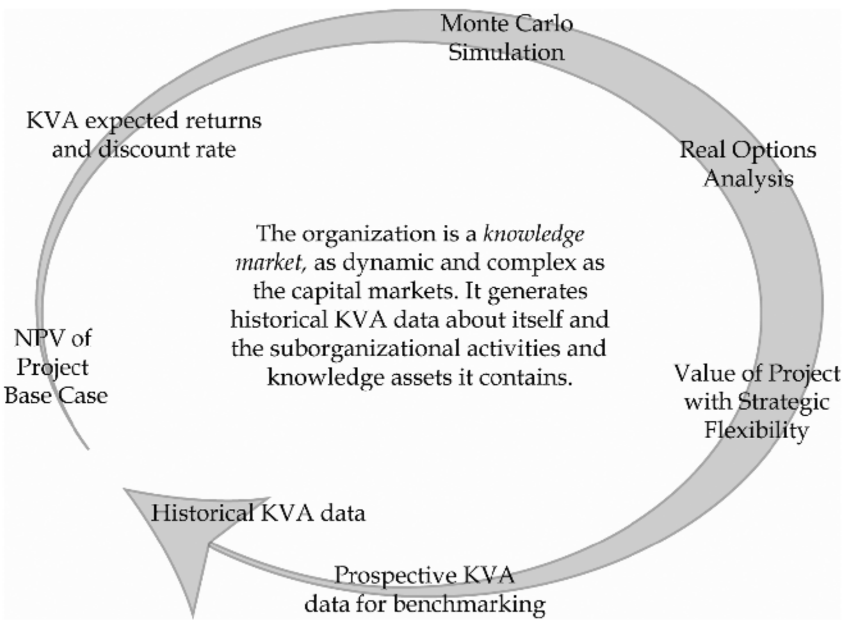


FIGURE 11.65 KVA and Real Options Analysis Cycle

to use KVA data and build effective prospective analyses of suborganizational activities and knowledge assets.

Figure 11.65 offers a diagram of the contributions of KVA and real options analysis to each other. It is intended to demonstrate why they should be applied together as a total solution to strategic planning initiative problems.

KVA provides the required data and real options provide the methodology to analyze the data for making the right decisions.

CHAPTER 12

Results Interpretation and Presentation

INTRODUCTION

Now that you've done some fancy real options analytics, the difficult part comes next: explaining the results to management. It would seem that getting the right results was half the battle. Explaining the results in such a convincing way that management will buy into your recommendations is another story altogether. This chapter introduces some novel approaches to presenting your real options results in a clear and convincing manner, converting black box analytics into nothing more than a series of transparent steps of a logical analytical process. The chapter is arranged in a series of key points that analysts should contemplate when attempting to interpret, present, and defend their results. Each key point is discussed in detail, and examples of how to broach a particular point are also provided. These include high-powered graphics, charts, tables, and explanatory notes. The major key issues and questions that management may ask that are discussed are as follows:

1. How does real options analysis compare with traditional analysis? What are some of the key characteristics? What one-liner or 30-second elevator pitch can you use to convince management that real options are nothing different from traditional analysis but provide increased insights and greater accuracy while taking into consideration the uncertainty of outcomes and risks of projects?
2. What are some of the steps taken in a real options analysis? What is the process flow? Does it make logical sense? Have you discarded the traditional approach and replaced it with real options? Where do the traditional analyses end and the new analytics begin?
3. At the executive summary level, can an analyst differentiate traditional results from the new analytics results? What is the relationship, and how do they compare against one another?

4. How do multiple projects compare against one another? How do you compare larger initiatives with smaller ones? What about project-specific risks?
5. Have you looked at both risk and return profiles of the projects? Which projects have the best bang for the buck?
6. What is the impact to the company's bottom line?
7. What are the major critical success factors driving the decision?
8. What are the risks underlying the results? How confident are you of the analysis results?
9. When will the project pay for itself? What is the break-even point? How long before payback occurs?
10. How did you get the relevant discount rate for the projects? What were some of the assumptions?
11. What were some of the assumptions underlying your real options analysis? Where did the assumptions come from?
12. How was your real options analysis performed? What are some of the key insights obtained through the analysis?
13. How confident are you of the real options analysis results?

This chapter discusses each of these questions in turn, with a series of examples on how to appropriately discuss and broach the issues with management.

COMPARING REAL OPTIONS ANALYSIS WITH TRADITIONAL FINANCIAL ANALYSIS

We begin by discussing the comparison between the real options process and traditional financial analysis. Senior management is always skeptical about using new, fancy analytics when old methods have served them so well in the past. It would seem that the main approach to alleviate management's concerns is to show that the real options methodology is not that far off—in principle, at least—from the conventions of traditional financial analysis. As a matter of fact, traditional discounted cash flow analysis can be seen as a special case of real options analysis when there is negligible uncertainty. That is, when the underlying asset's volatility approaches zero, the real options value approaches zero, and the value of the project is exactly as defined in a discounted cash flow model. It is only when uncertainty exists, and management has the flexibility to defer making midcourse corrections until uncertainty becomes resolved through time, actions, and events, that a project has option value.

Change-management specialists have found that there are several criteria to be met before a paradigm shift in thinking is found to be acceptable in a corporation. For example, in order for senior management to accept a new and novel set of analytical approaches, the models and processes themselves must have applicability to the problem at hand, and not be merely an academic exercise. As we have seen previously, the former is certainly true in that large multinationals have embraced the concept of real options with significant fervor, and that real options is here to stay. It is not simply an academic exercise, nor is it the latest financial analysis fad that is here today and gone tomorrow. In addition, the process and methodology has to be consistent, accurate, and replicable. That is, it passes the scientific process. Given similar assumptions, historical data, and assertions, one can replicate the results with ease and predictability. This is especially so with the use of software programs like the ones included in the CD-ROM.

Next, the new method must provide a compelling value-added proposition. Otherwise, it is nothing but a fruitless and time-consuming exercise. The time, resources, and effort expended must be met and even surpassed by the method's value-add. This is certainly the case in larger capital investment initiatives, where a firm's future or the future of a business unit may be at stake. Other major criteria include the ability to provide the user a comparative advantage over competitors, which is certainly the case when the additional valuable insights generated through real options analysis will help management identify options, value, prioritize, and select strategic alternatives that may otherwise be overlooked.

Finally, in order to accept a change in mindset, the new methodology, process, or model must be easy to explain and understand. In addition, there has to be a link to previously acceptable methods, whether it is an extension of the old or a replacement of the old due to some clear superior attributes. These last two points are the most difficult to tackle for an analyst. The sets of criteria prior to this are direct and easy to define. However, how does one explain to senior management the complexities of real options and that the approach is the next best thing since sliced bread? How do real options extend the old paradigm of discounted cash flow models with which management

In order for a new methodology to be accepted it must be:

- ***Accurate***
- ***Applicable***
- ***Comparable***
- ***Consistent***
- ***Explainable***
- ***Precise***
- ***Replicable***
- ***Understandable***
- ***Value-adding***

has been brought up? An effective method that the author has found useful with clients has been to boil it down to its simplest parts. Figure 12.1 shows a simple example.

In a traditional financial analysis, the analyst usually calculates the net present value (NPV), which simply defined is benefits less cost (first equation), where benefits equal the sum of the present values of future net cash flows after taxes, discounted at some market risk-adjusted cost of capital, and cost equals the present value of investment costs discounted at the risk-free rate or reinvestment rate. Management is usually knowledgeable of NPV and the way it is calculated. Conventional wisdom is such that if benefits outweigh costs, that is, when NPV is positive, one would be inclined to accept a particular project. This is simple and intuitive enough. However, when we turn to options theory, the call option is also nothing but benefits less cost (second equation) with a slight modification. The difference is the introduction of a $\Phi(d)$ multiplier behind benefits and costs. Obviously, the multipliers are nothing but the respective probabilities of occurrence.

Hence, in real options theory, one can very simply define the value of an option as nothing but benefits less costs, taking into account the risk or probabilities of occurrence for each variable. It is easy to understand that option value in this case is far superior to the NPV analysis because it provides an added element of stochastic variability around benefits and costs. It is hubris to say that we know for certain (on a deterministic level) what future benefits and costs will be, when, in reality, business conditions change daily. In addition, we can say that the total strategic value or expanded net present value (eNPV), shown as equation three in Figure 12.1, is the sum of the deterministic base-case NPV and the strategic flexibility option value. The option value takes into account the value of flexibility, that is, the option to execute on a strategic option but not the obligation to do so. The eNPV ac-

$$NPV = Benefits - Cost$$
$$Option = Benefits\ \Phi(d_1) - Cost\ \Phi(d_2)$$
$$eNPV = NPV + Options\ Value$$

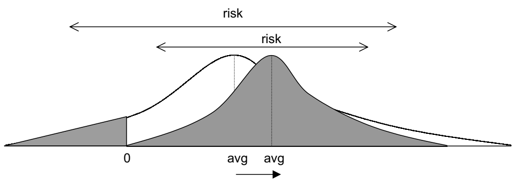


FIGURE 12.1 Enhanced Return with Risk Reduction

counts for both base-case analysis plus the added value of managerial flexibility. If there is negligible uncertainty, volatility approaches zero, which means that the probability multiplier approaches one (outcomes are certain). The options equation reverts back to the NPV equation, indicating that the NPV analysis is a special case of an options analysis when there is no uncertainty.

Finally, the two graphs in Figure 12.1 tell a compelling story of why real options provide an important insight into decision analysis. The graph in the background shows the distribution of the base-case NPV analysis. That is, the first moment or the mean, median, and mode of the graph show the central tendency location of the most likely occurrence of a project's value. Some analysts will call this the expected value of the project (denoted as *avg* for average or expected value). The second moments or the standard deviation, width, variance, and range of the distribution tell of the risk of the project. That is, a wider distribution implies a higher risk because there is a wider range of outcomes the project value may fall between. Clearly, the graph in the foreground shows a much smaller risk structure but a higher average return, which is attributable to real options analysis. We know from many previous illustrations that employing a real options strategy—for example, the passive and active options to wait—will create a higher value because we are hedging project risks by not betting the entire investment outlay now but instead waiting until we get a better idea of the uncertainty that exists over time. Once uncertainty becomes resolved, we can act accordingly. This delaying action helps hedge our losses and thus truncates the distribution in terms of width and moves it to the right because management will never execute a bad strategy assuming they know what will happen if they do. This moves the entire distribution to the right and at the same time reduces the risk, as seen through a reduction in width. Thus, real options, just like its cousins the financial options, help the holder of the option to hedge project risk (lower second moment) while increasing its financial returns leverage (higher first moment).

THE EVALUATION PROCESS

The next issue an analyst should discuss with management pertaining to real options analysis is the steps taken to obtain the results—in other words, what the process flow looks like, how it makes logical sense, and where the traditional analysis ends and the new analytics take over. A thorough understanding of the process flow will make management more comfortable in accepting the results of the analysis. A career-limiting or a high-potential career-ending move is to show management a series of complicated stochastic differential Ito calculus equations, crunch out a number, and then ask the

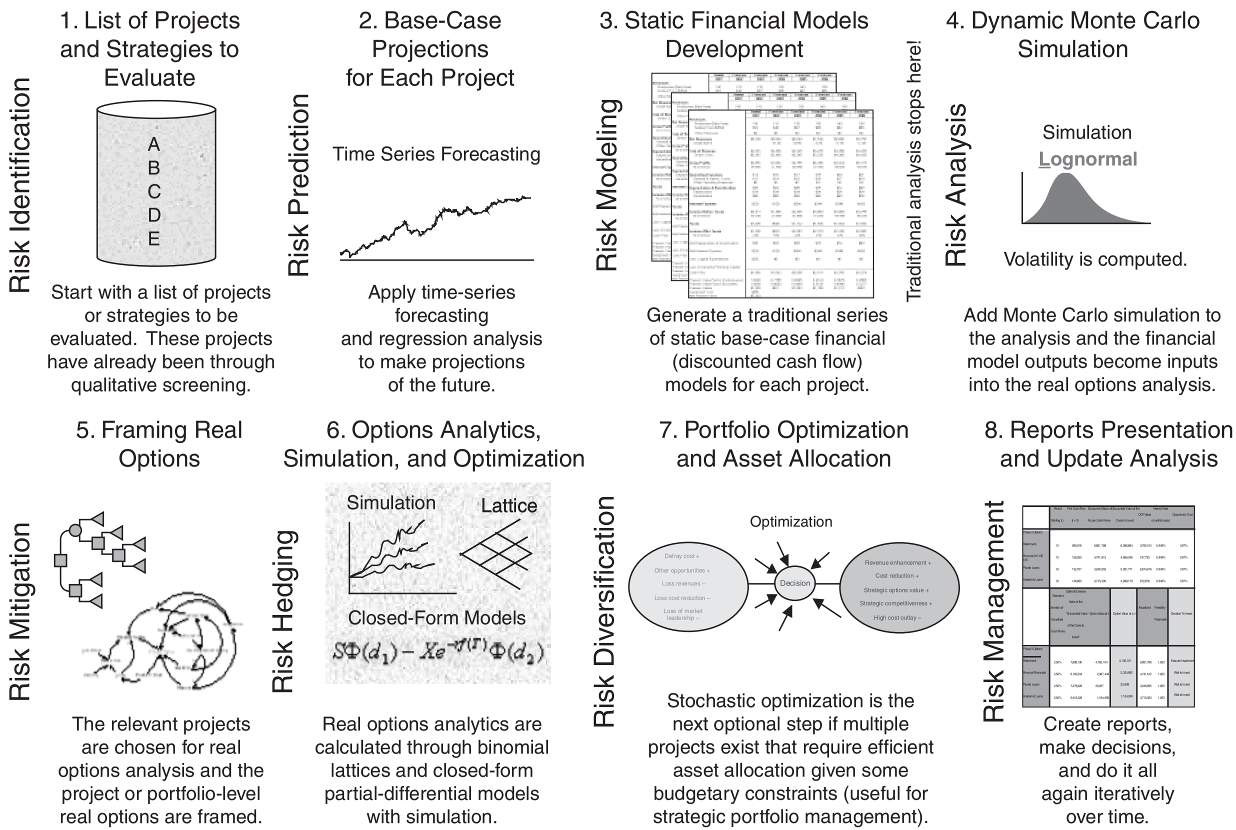


FIGURE 12.2 Real Options Process Summary

CEO to bet the company's future on it. How can management buy in on an analysis if the analyst can't even explain the process flow properly? Figure 12.2 shows a visual representation of the process flow for a robust real options analysis process.

In the first step, the analyst starts off with a list of qualified projects, that is, projects that have been through qualitative screening by management. Having met preset criteria, whether they are strategic visions or goals of the company, these are the projects that need to be analyzed. They may, of course, be different courses of actions, projects, assets, initiatives, or strategies. For each of these strategies or projects, the base-case NPV analysis is performed, as indicated in steps two and three. This could be done in terms of the market, income, or cost approach, using something akin to a discounted cash flow model, as seen in the third step. In certain circumstances, the analyst may elect to perform some intermediate calculation like time-series forecasting and simulation to predict future revenue and cost streams. Depending on the availability of historical data, some fancy econometric, forecasting, regression, time-series, cross-sectional, or stochastic model may be constructed for this purpose. These three steps encapsulate the traditional approach. Using the revenues and cost structures coupled with conventional accounting procedures, the analyst would calculate the net present value of the projects or strategies. Occasionally, other financial metrics may be used, such as an internal rate of return (IRR) or some form of return on investment (ROI) measure. In most cases, a decision will be made based on these deterministic results.

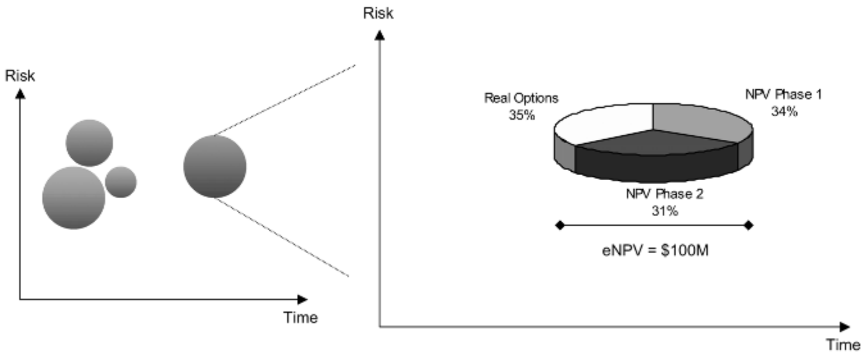
In more advanced financial analysis, specifically, a recommended step for the real options approach is the application of Monte Carlo simulation. Based on some sensitivity analysis, the analyst decides which input variables to the discounted cash flow model previously constructed are most vulnerable to risks and sudden exogenous and systemic shocks. Using historical data, the analyst can take the time-series or cross-sectional data and fit them to a multitude of different distributions. The analyst may also opt to use management assumptions, hunches, experience, or economic behaviors of variables to make the distributional determination. The discounted cash flow model is then simulated. The result is a distribution of the variable of interest, for example, the net present value. Instead of obtaining single-point estimates, the analyst now has a probability distribution of outcomes, indicating with what probabilities certain outcomes will most likely occur. Based on this Monte Carlo simulation, certain intrinsic variables key to the real options analysis are calculated and imported into the real options analysis. These key variables that flow out of the simulation procedures include the volatility of the underlying variable, typically the lognormal returns on the future free cash flows; the implied cross-correlation pairs between the underlying projects; and the expected present value of cash flows.

The fifth and sixth steps involve framing the problem in terms of a real options paradigm. That is, having identified the optionality of the project or strategy, the analyst then chooses the relevant sets of options to analyze. Based on the types of models chosen, the calculation may then proceed automatically in different ways, whether through the use of binomial lattices, closed-form solutions, or path-dependent simulation.

The results are then presented in the seventh and last step of the analysis. The reports may include charts on sensitivity, tables of the financials, including the impact to bottom line, graphs of different risk and returns, and combinations of projects in portfolios. Here, an optional step depending on the type of analysis is the application of portfolio optimization for efficient resource allocation. Portfolio optimization incorporates the interrelationships between projects or strategies as they evolve in a portfolio. Firms usually do not have stand-alone projects; rather, firms usually have multiple projects interacting with each other. Therefore, management is usually more interested in seeing how these projects interact with each other on a rolled-up basis, that is, on a portfolio of options, projects, and strategies. In addition, management usually wants to see what the optimal mix of projects should look like, given budgetary, resource, or timing constraints. The result is usually an optimal portfolio mix plotted on an efficient frontier, where every point along this efficient frontier is an optimal mix of project combinations, depending on management's risk and return appetite. Having shown management the process and steps taken to perform the analysis as well as receiving its buy-in into viability and importance of a real options analysis, we can now proceed with the results presentation.

SUMMARY OF THE RESULTS

Figure 12.3 shows a quick and easy-to-understand executive summary of each project or strategy, allowing management to differentiate between traditional results and the new real options analytics results. It also shows the relationships among projects and how they compare with each other in terms of risk, return, and time horizon. On the right, we see the traditional analysis valuation results (NPV Phases I and II) coupled with the real options results, where together they form the total strategic value or expanded net present value (eNPV) pie chart. On the left, we see a set of summary projects or strategies as delineated by risk on the vertical axis, time on the horizontal axis, and returns (eNPV) as the size of the spheres. Management can very easily view all projects at a glance, with respect to their relevant risk, return, and timing.



Description

This report displays the relative size of each of the project returns. The chart on the left indicates the relative positioning of the projects or strategies with respect to their risk (measured in volatility of free cash flows) and time horizon of their strategic options. The size of the balls indicates the relative dollar amount of their expanded NPV. On the right chart, the diameter of the pie chart indicates the magnitude of the expanded NPV (eNPV), which comprises the projects that make up the strategy as well as their real options strategic value. The pie chart is located on a two-dimensional axis of risk and timing.

FIGURE 12.3 Project Comparison (Risk and Return)

COMPARING ACROSS DIFFERENT-SIZED PROJECTS

Another issue to be discussed is how these multiple projects compare against one another when their relative sizes are dramatically different. That is, how do you compare a $10 million investment that provides a $20 million return to a larger $1 billion investment that returns $100 million? Should the project with the 200 percent return be chosen over the 10 percent return? Should the project returning a higher $100 million face value be chosen? What about the risks inherent in each project?

Can we create a replicating portfolio where we can spend the $1 billion investment on 100 identical $10 million projects and yield $2 billion in return as compared to only $100 million? Obviously, the answer is not a simple one. It strictly depends on whether the firm has the $1 billion budget to begin with. Not to mention that there would need to be 100 projects you can invest in, different projects with similar functions, markets, risks, and so forth. What about diversification effects across different projects? Regardless, we have shown previously in Figure 12.3 that we can depict the absolute revenue, risk, and time horizons across multiple projects. In Figure 12.4, we show the relative comparisons. That is, using some common-size ratios, we can compare across multiple different projects and strategies.

Categories		Project A	Project B	Project C
Growth	Net Revenue	15%	10%	25%
	Net Income	30%	25%	17%
	Total Assets	15%	15%	15%
Profitability	Gross Margin	75%	80%	91%
	EBITDA Margin	70%	69%	82%
	EBIT Margin	65%	60%	80%
	EBT Margin	65%	58%	73%
	Net Income	10%	15%	17%
	Effective Tax Rate	40%	40%	40%
DuPont	Net Income/Pretax Profit	40%	39%	52%
	Pretax Profit/EBIT	43%	50%	55%
	EBIT/Sales	45%	55%	50%
	Sales/Assets	3%	3%	4%
	Assets/Equity	2%	2%	2%
	ROE	12%	15%	
Liquidity	Working Capital	22%	15%	23%

Description

This report details the common size ratios for each of your projects. You can quickly view how your projects stack up against each other on a common basis. Common sized ratios provide a way to analyze and compare different financial statements.

The categories displayed include growth, DuPont, profitability, and liquidity ratios.

FIGURE 12.4 Project Comparison (Common Sizing)

COMPARING RISK AND RETURN OF MULTIPLE PROJECTS

Not only should we common-size the projects with respect to growth rates, profitability ratios, and other accounting ratios, we should also compare each project's return to its risk, that is, the proverbial bang-for-the-buck, as seen in Figure 12.5.

This return-to-risk ratio is important; otherwise, bad projects may be selected depending on management's strategic goals and risk tolerance. For example, using the same analogy presented previously, where Project *X* costs \$10 million but provides \$20 million in return and Project *Y* costs \$1 billion but returns \$100 million. Suppose Project *X* has a standard deviation of \$10 million (we use standard deviation here as a measure of risk) while Project *Y* has a standard deviation of \$100 million. Budget-constrained managers may choose Project *X* because they may have no choice. Returns-driven managers may, however, choose Project *Y* because it is more lucrative, assuming these managers are not resource-constrained. A risk-averse manager may simply choose Project *X* due to the lower risk levels. Add a few more projects with different risk and return characteristics, and you have a conundrum on your hands.

		Project A	Project B	Project C
Categories				
Return	Present Value of Cash Flows	$125M	$137M	$250
	Net Present Value	$75M	$73M	$100
	Expanded NPV	$200M	$210M	$350
	Internal Rate of Return	70%	80%	103%
Risk	Cash Flow Volatility	20%	34%	44%
	Simulated Value at Risk (5%)	$12M	$14M	$34M
	Simulated Value at Risk (1%)	$2M	$2M	$10M
Return to Risk	NPV/Volatility	60%	42%	61%
	ENPV/Volatility	76%	52%	77%
	VaR/ENPV	34%	23%	34%

Description

This report displays a risk and return profile for each project. The risk is measured as cash flow volatility and simulated VaR. Returns are measured as NPV, options value, ENPV (expanded NP V), IRR, and ROI. VaR, or Value at Risk, is defined as worst-case scenario losses 5% of the time. ENPV, or expanded NPV, is defined as a project's NPV and option value.

FIGURE 12.5 Project Comparison (Risk–Return Profiling)

Obviously, the best way is to calculate a standardized ratio such as the return-to-risk ratio.[1] That is, the return to risk ratio of Project *X* is 2.0, while Project *Y* has a ratio of 1.0. Clearly, Project *X* has the higher bang for the buck. That is, for each unit of risk, Project *X* provides two units of return, but Project *Y* only provides one unit of return. Conversely, for each unit of return, Project *X* only requires half a unit of risk, while Project *Y* requires a full unit of risk. The smart manager may simply create a replicating portfolio to maximize profit and minimize risk, by spending $50 million on five identical Project *X*s returning $100 million while only being exposed to $50 million in risk. Compare that to spending $1 billion on a single Project *Y* and receiving $100 million in return while being exposed to $100 million in risk! This shows similar returns can cost less and have less risk when we take risk and return into consideration. Imagine the disastrous decision made by the first returns-driven manager who would only consider Project *Y* due to its whopping absolute return levels.

One powerful visualization method to observe the clumping of project valuation is the use of a 3D option space as shown in Figure 12.6. The horizontal axis is a measure of a project's profitability index (Q-Ratio), that is, the ratio of the sum of present values of future net cash flows to the sum of the present values of investments or implementation costs. The former is discounted at a market-risk adjusted discount rate of return and the latter using the risk-free rate or some appropriate cost of money, to account for differences between market risk and private risks. A Q-Ratio greater than one means that the NPV is positive and the project is financially feasible and profitable, while a Q-Ratio less than one means the NPV is negative. Break-even

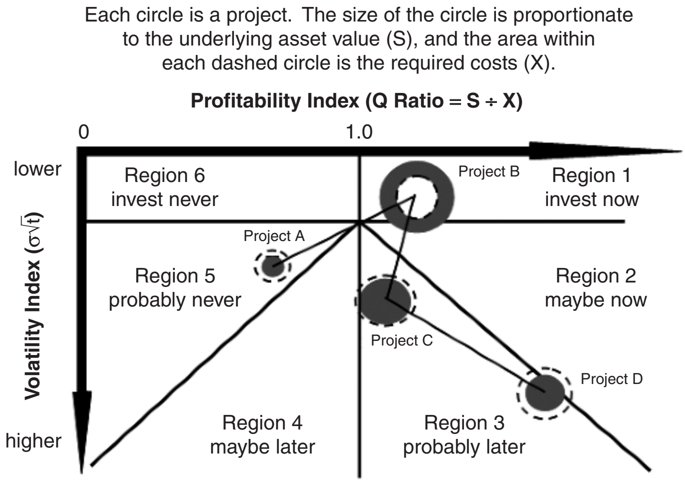


FIGURE 12.6 Project Valuations in 3D Option Space

projects have a Q-Ratio of one. The vertical axis measures the volatility index, essentially the volatility value after performing Monte Carlo simulation, multiplied by the square root of the project's maturity in years. The project's maturity in years can be calculated either as the amount of time left to implement the project, or the project's economic life. Either way, as long as the calculation is applied consistently across all projects, the volatility index indicates a project's uncertainty, adjusted for time. A higher volatility index implies a higher option value but a potentially risky project.

Projects can then be classified into different regions (invest now, invest later, and so forth) depending on where they fall in the 3D space as well as management's risk and investment preferences. Notice that the chart in Figure 12.6 has a width (profitability index), height (volatility index) and depth (size of the circles, indicative of the projects' relative benefits and costs), hence the name 3D option space matrix. Plotting projects' profitability and volatility indices is the first step in the real options framing exercise. The reason is that projects that are deep in-the-money should be executed immediately anyway (subject to budget constraints), and additional analyses are unnecessary. For instance, spending $1 million to make $100 million guaranteed is a no-brainer and should be implemented immediately. All the real options analysis in the world will not yield any more significant value. Conversely, deep out-of-the-money projects should not compel the analyst to perform any more in-depth analyses using real options as sometimes when the project is so unprofitable, all the uncertainties and strategic options in the world are useless in justifying its existence. However, projects that are close to at-the-money or slightly in-the-money or out-of-the-money should be considered for real options analytics. This is so especially in the case where projects tend to clump or cluster together, making it difficult to distinguish which projects ought to be selected. This is where the 3D option space matrix provides valuable insights into which projects ought to undergo more real options scrutiny.

Whereas Figure 12.6 shows individual projects, Figure 12.7 illustrates the portfolio of multiple combined projects. Specifically, Figure 12.7 shows a portfolio efficient frontier where on the frontier, all the portfolio combinations of projects will yield the maximum returns (portfolio eNPV or total strategic value including option value and NPV) subject to the minimum portfolio risks. Clearly portfolio P1 is not a desirable outcome due to the low returns. So, the obvious candidates are P2 and P3. These two portfolios are analyzed in detail in Table 12.1. It is now up to management to determine what risk–return combination it wants; that is, depending on the risk appetite of the decision makers, these three portfolios are the optimal combinations of projects that maximize returns subject to the least risk, while considering the uncertainties (simulation), strategic flexibility (real options), and uncertain future outcomes (forecasting).

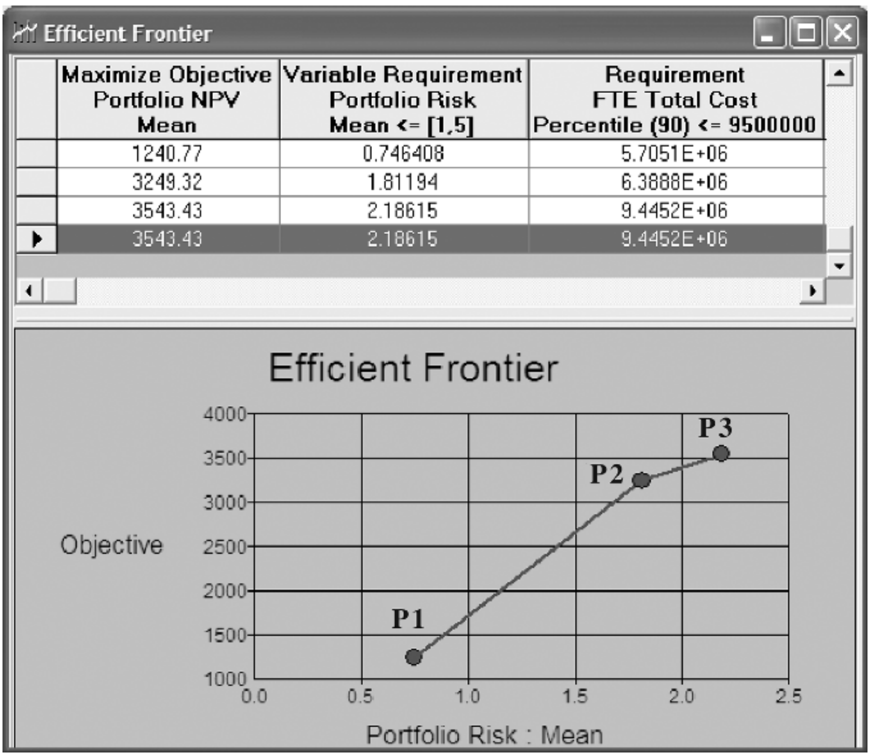


FIGURE 12.7 Efficient Frontier

TABLE 12.1 Summary of Results from Portfolio Optimization

Portfolio Characteristics	Portfolio P2	Portfolio P3
Expected eNPV (Mean)	\$3,208	\$3,532
Expected risk (Mean)	182%	218%
Budget used	\$2,776	\$3,206
90 percentile FTE total cost	\$6,368,000	\$9,423,000
90% confidence interval for eNPV	\$2,964 to \$3,443	\$3,304 to \$3,764
90% confidence range for eNPV	\$479	\$460
90% confidence interval for risk	167% to 197%	203 to 234%
90% confidence range for risk	30%	31%
Projects selected	D, E, F, H, J, M	D, E, F, H, I, J, K, M, T

Many observations can be made from Table 12.1's summary of results. While both optimal portfolios are constrained at under a \$3,500 budget and \$9,500,000 full-time equivalence (FTE) total cost 90 percent of the time, P3 uses the budget more effectively as it generates a higher eNPV for the entire portfolio but has a higher level of risk (wider range for eNPV and risk, higher volatility risk coefficient, requires more projects making it riskier, and higher total cost to implement). For the budget used and the eNPV obtained, the lowest risk level required is 218 percent, which is about 20 percent greater risk in proportion than P2. In contrast, P2 costs less and has lower risk but comes at the cost of a slightly less eNPV and IRR level. At this point the decision maker has to decide which risk–return profile to undertake. All other combinations of projects in a portfolio are by definition suboptimal to these two, given the same constraints, and should not be entertained. Hence, from a possible portfolio combination of say, 20 projects, a total of 20! or 2×10^{18} possible combinations of portfolios can exist, and through stochastic optimization with simulation and real options, we have now isolated the decision down to these two best portfolios. Finally, we can again employ a high-level portfolio real option on the decision. That is, because both portfolios require the implementation of projects D, E, F, H, J, M, do these first! Then, leave the option open to execute the remaining projects (I, K, T) if management decides to pursue P3 later.

IMPACT TO BOTTOM LINE

One of the key questions that will come up is what impact a specific project will make to the company's bottom line. This can be clearly presented through the use of revenue and cost projections in a DCF model. Depending on whether management wants to see operating income before taxes or net in-

Discounted Cash Flow								
Line Items	2000	2001	2002	2003	2004	2005	2006	2007
Revenue	300	310	360	380	432	500	550	650
Cost of Revenue	150	150	150	130	160	150	150	60
Gross Profit	150	160	210	250	272	350	400	590
Operating Expenses	200	160	155	129	137	90	100	160
Depreciation Expense	60	60	40	60	30	40	30	60
Interest Expense	5	7	10	5	8	5	5	5
Income Before Taxes	-115	-67	5	56	97	215	265	365
Taxes	-40.25	-23.45	1.75	19.6	33.95	75.25	92.75	127.75
Income After Taxes	-74.75	-43.55	3.25	36.4	63.05	139.75	172.25	237.25
Non-Cash Expenses	10	11	12	13	14	15	16	17
Cash Flow	-84.75	-54.55	-8.75	23.4	49.05	124.75	156.25	220.25

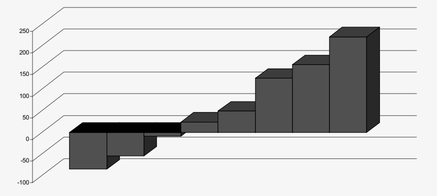
Description

This report displays your discounted cash flow (DCF) model. The DCF shows a summary of all key income statement line items used in generating free cash flows for the specified forecast years.

Key Assumptions

The key assumptions used to generate free cash flows include:

Discount rate	=	20%
Tax rate	=	35%
Depreciation profile	=	Straight Line



Results Summary

Sum of present value cash flows = $145M (no implementation cost)

Net present value (NPV)=$100M (with implementation cost)
Internal rate of return (IRR)=75.50% (with implementation cost)

FIGURE 12.8 Impact to Bottom Line

come after taxes or free cash flows after taxes, the DCF model will do a fairly decent job. Figure 12.8 illustrates a sample DCF model summary. However, one thing that must be made clear to management is that there is a difference between strategic option value and explicit value.

A discounted cash flow will show the explicit value of a project, assuming that the forecasted revenues and cost structures are correct, and the impact to bottom line will be the cash flow stream calculated in the model. However, strategic optionality value may or may not exist, depending on whether the option is executed. Assuming that a strategic option is left to expire without execution, there is actually zero value derived from the strategic flexibility inherent in an option.

CRITICAL SUCCESS FACTORS AND SENSITIVITY ANALYSIS

A sensitivity analysis similar to the one seen in Figure 12.9 is vital for management to understand what drives their business decisions. That is, what are some of the key critical success drivers of the projects? The sensitivity analysis could be done in several ways. The most prevalent method is simply choosing the resulting variable of interest, for example, net present value, and identifying all its precedent variables. Precedent variables include revenues, costs, taxes, and so forth that are required to derive the final net present

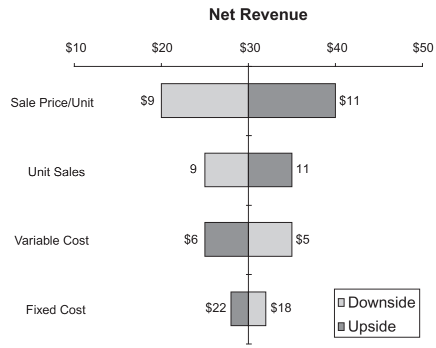
Description

This report contains a sensitivity analysis of your discounted cash flow model. The tornado chart displays each variable and the range between the variables' minimum and maximum forecast values, with the variable with the greatest range at the top.

Tornado charts are useful for measuring the sensitivity of variables that determine the free cash flows and allow you to do a quick pre-screening of the variables which drive the analysis. This analysis provides added insights into the critical success drivers in the project or strategy. In addition, it provides a list of candidates for performing Monte Carlo simulation.

Results

Based on the 10% change of the variables that drive the DCF model, the most sensitive line items in decreasing order for determining free cash flows are:



	NPV			Input		
Variable	*Downside*	*Upside*	*Range*	*Downside*	*Upside*	*Base Case*
Revenues	183.00	334.78	151.78	1.80	2.20	2.00
Tax Rate	183.29	334.46	151.17	37.80	46.20	42.00
Operating Expenses	183.29	334.46	151.17	1,350.00	1,650.00	1,500.00
Discount Factor	227.25	292.93	65.67	11.00	9.00	10.00
Depreciation	225.53	288.03	62.50	9.00	11.00	10.00

FIGURE 12.9 Critical Success Factors

value result. While holding all precedent variables *ceteris paribus,* or constant and unchanging, select one precedent variable, change its value by some predefined range, and gauge what happens to the net present value. The results can be tabulated and plotted into a Tornado diagram as seen in Figure 12.9, starting from the highest and most sensitive precedent variable and going to the lowest and least sensitive variable. Tornado and sensitivity analysis can be performed easily using the Risk Simulator software. Armed with the knowledge of which variables are the most sensitive, an analyst can decide which ones have the most variability or risk, which are then selected as prime candidates for Monte Carlo simulation.

RISK ANALYSIS AND SIMULATION ON NPV

Having performed a barrage of analyses, how confident are you of your results, and how sure are you the assumptions and data entered were correct? Because most business cases involve risks and uncertainty, there is no doubt that a margin of error exists. As we have shown in previous chapters the errors of using point estimates, as illustrated in the Flaw of Averages example, it is essential that when reporting results to management, to also present a picture of the risks involved. Typically, risk-analysis results will take the form of a Monte Carlo simulation forecast output, as shown in Figure 12.10.

Description

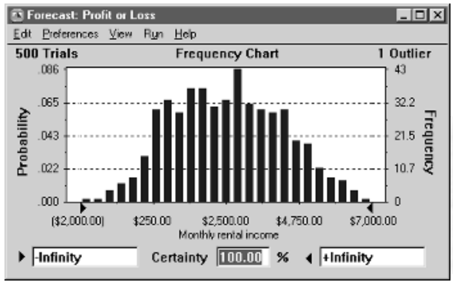
This report shows the levels of risk in your discounted cash flow analysis. The probability distribution shows each value as a fraction of the total.

Results

The chart shows all possible outcomes based on Monte Carlo simulation of the input variables in the DCF. The frequency chart shows the probability at the 90% confidence level occurring between -$250 and $6850.

Percentiles (description):

10% = $20	20% = $50	30% = $250	40% = $650
50% = $2300	60% = $2950	70% = $4200	80% = $5820
90% = $6520	100% = $7000		



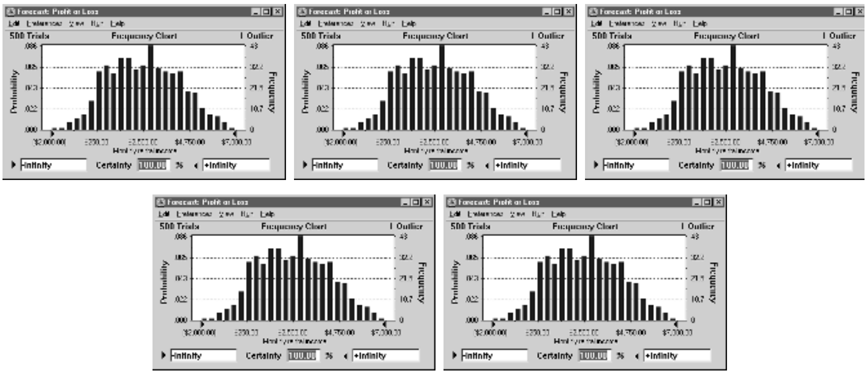
Summary

FIGURE 12.10 Project-Based Risk Analysis

BREAK-EVEN ANALYSIS AND PAYBACK PERIODS

One of the most infamous questions relating to project evaluation is the concept of payback period or break-even analysis. That is, when will the project pay for itself? As any good financial analyst knows, payback period analysis is fraught with problems. It does not account for different time horizons of projects, and the payback periods are usually based on an undiscounted cash flow basis, usually leading to wrong decisions. For example, suppose we have two projects, *A* and *B,* each costing $100. Project *A* will yield cash flows of $50 for only two years. Project *B* will yield cash flows of $49 for 10 years. Clearly Project *A* has a payback period of two years, while Project *B* has a payback period of 2.04 years. Strictly speaking, Project *A* should be undertaken; but although Project *B* yields a slightly lower cash flow, its economic life is 10 years. Net present value would have picked this up, but not a simple break-even payback analysis. Even with this said and done, management still wants to know what the payback period of a particular project is, even if it is used as a gross approximation of the value of a project.

There are ways to improve on a break-even analysis, as shown in Figure 12.11. Instead of relying on single-point estimates—for example, instead of saying that a particular project will take 4.0 years to pay back its costs—we can add slightly more sophistication. Using discounted cash flow streams in present value terms, we can say that a project has a 5 percent probability of breaking even the first year, a 15 percent probability of breaking even the second year, a 45 percent probability of breaking even the third year, a 92 percent probability of breaking even the fourth year, and a 99 percent probability of breaking even all subsequent years. So, we are stating not only when



Description

This report details the probability of the project breaking even at different time periods. This analysis uses the assumption of $45M in implementation cost in present value dollars, discounted at 5.5%. The free cash flows used to calculate the payback probabilities are derived from the NPV models and discounted using 20.0%

For year 1, there is a 5.0% probability of breaking even.
For year 2, there is a 15.5% probability of breaking even.
For year 3, there is a 45.5% probability of breaking even.
For year 4, there is a 92.5% probability of breaking even.
For year 5, there is a 99.5% probability of breaking even.

FIGURE 12.11 Simulated Discounted Payback Analysis

the break-even point will occur but also how likely it will occur on a present value basis.

DISCOUNT RATE ANALYSIS

One of the key assumptions in discounted cash flow analysis, which forms the basis for real options analysis, is the calculation of the discount rate. See Figure 12.12.

Due to the significance of the discount rate in the overall analysis, management should feel comfortable with the assumptions used. The methods of calculating discount rates abound; therefore, they should be used with due care and diligence. Even using and explaining the most widely used discount rate analysis, such as the CAPM (capital asset-pricing model) or WACC (weighted average cost of capital), it still requires significant care and caution in that the estimates are truly based on the underlying variable, which in real options are usually nontradable assets. One should not use tradable and highly liquid firm-level financial asset prices as a proxy for risk-adjustment at the project or strategy level. For instance, the beta of traded stocks for a particular firm may not be the best proxy for the beta-risk used in calculating the project-level specific risk. Rather, comparables should be used if there are insufficient historical data of the underlying variables' behavior. That is,

Description

This report shows the discount rate used in the analysis and its corresponding assumptions and results. The discount rate method used is **weighted average cost of capital.** This calculated rate is used for discounting future cash flows into today's present value.

Assumptions

Risk free rate = 8%

Return on the market = 6%

Equity risk premium = 7%

Beta = 1.1

Size premium = 5%

Minority premium = 10%

Corporate tax rate = 35%

Outstanding debt = 10

Return on debt = 8%

Shares outstanding = 1,458,990

Current stock price = $4.68

Results

CAPM = 14.6%

Percent debt = 30%

Percent equity = 15%

Equity capitalization = 14%

Total capitalization = 12%

WACC = 11.8%

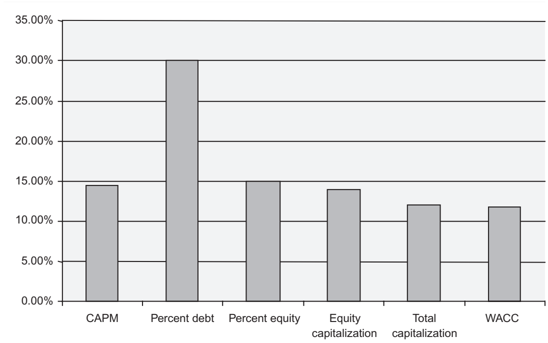


FIGURE 12.12 Discount Rate Analysis

stripped-down firms with similar functions, markets, and risks should be selected with care, and their financials should be sanitized to avoid including any anomalous nonrecurring events or any financial window-dressing phenomenon. If historical data abound, the analyst can then determine the firm's risk structure and calculate the risk-adjusted discount rate appropriately.

Appendix 2B's discussion on discount rates should suffice as a guide on how this is done. Nevertheless, management should be convinced of and comfortable with the results of such discount rate analyses. In most cases, the problem of finding the correct single discount rate for each project is a gruesome task, but given the capability of performing Monte Carlo simulation, the analyst can simulate the discount rate with certain management assumptions, and, with a 90 percent confidence that the proper discount rate is between two particular numbers, the resulting forecast will be more palatable to management. This is acceptable because discount rates measure risk while simulation captures the uncertainty of the levels of risk.

REAL OPTIONS ANALYSIS ASSUMPTIONS

The next topic is real options analysis. What are the assumptions, and where do they come from? These issues should be discussed clearly and concisely. One approach is to show management something similar to Figure 12.13. It doesn't really matter if the approach you used to solve the real options was

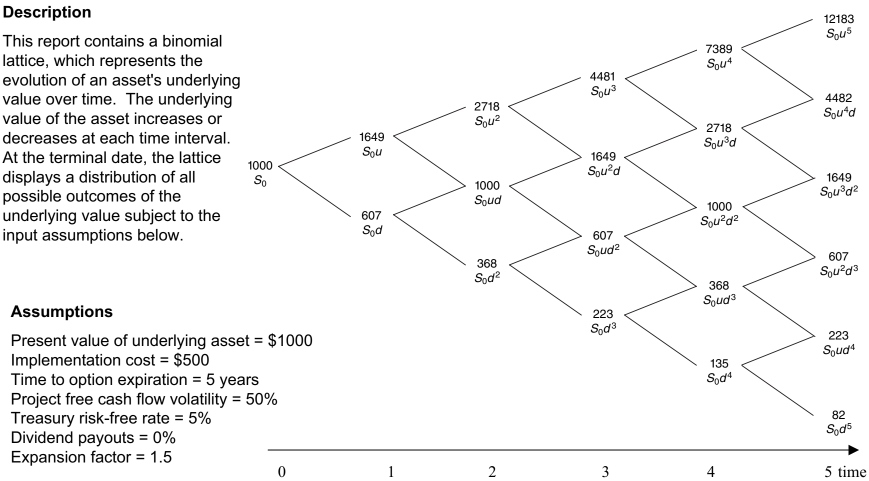


FIGURE 12.13 Real Options Assumptions

a closed-form model or stochastic differential equation. You should be able to present your results but at the same time be expositionally concise and precise. That is, you know that by using a binomial lattice with many time-steps at the limit, where the time between nodes approaches zero, the value calculated approaches the value of the closed-form equations. However, it is most certainly easier to explain and show management a simple binomial lattice than it is to explain the intricacies of a stochastic partial-differential equation. At least for presentation purposes, the binomial lattice is highly favored. The analyst can caveat the results of the lattice—say, of five steps—that it is only an approximation value, that the higher the number of time-steps, the more accurate the results become.

A good explanation of the assumptions surrounding the real options analysis should include where these assumptions come from—for example, the fact that the present value of the underlying variable comes from the sum of the present values of free cash flows in the discounted cash flow model, or that the volatility estimates come from the volatility of the simulated free cash flow's lognormal returns.

REAL OPTIONS ANALYSIS

This is the crux of the analysis and is worthy of detailed explanation, including how the valuation process works as well as the decisions that can be derived from the real options analytics. One method the author has found highly successful in disseminating results of a real options analysis is through the use of collaborating methods. For example, Figure 12.14 shows the valuation lattice for an expansion option. The value calculated is $638 million through the lattice and should be collaborated with that of a closed-form solution, if one exists. If we can show that both figures are in the ballpark, the analysis tends to hold more water. Although the analyst should understand that at the limit the binomial lattice approach is identical to the closed-form solution for generic types of options, the approaches tend to be different when you have different real options interacting with each other or when you are creating customized options analysis. However, a good sanity check of the binomial models using closed-form approximations is always warranted in such circumstances. No matter which models are used, the binomial lattice is a more powerful graphical representation of decisions than an otherwise complex mathematical model. The binomial lattice can then be converted into a series of decision nodes. These nodes correspond to optimal decisions that should be made under specific timing and underlying variable conditions. For example, in the expansion option, the decisions include when to expand, when to keep the option open for future use, or when to let the option expire without exercising it. For instance, in Figure 12.14, expanding the project prior to time period 3 is suboptimal, and it is wise to consider executing this option starting from time period 3. Also, a strategy tree should be shown (like those used to frame the options in Chapter 11) together with the lattices.

Description

This report displays the valuation of the option. The approach used is the risk-neutral probability applied backward from the terminal period to the initial period.

Results

The value of the option is calculated as $638. The option lattice has been converted into a decision tree.

Open = keeping the option open and not executing the option is the optimal decision at a particular node

Expand = exercising the option at this node is the optimal decision

Stop = the option should be left to expire without exercising it.

The bold path demarcates optimal option execution versus keeping the option open.

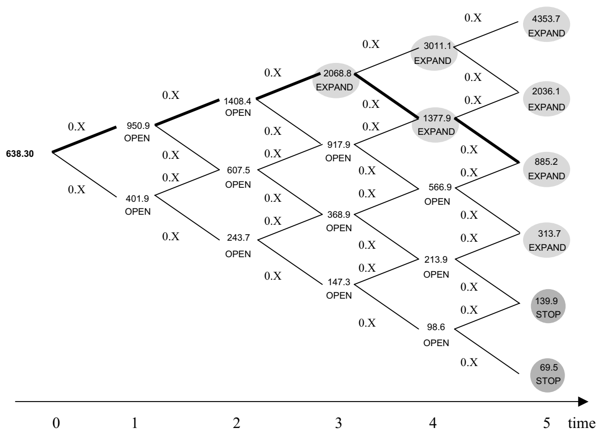


FIGURE 12.14 Real Options Analysis

REAL OPTIONS RISK ANALYSIS

Similar to the risk analysis for discounted cash flow, we can perform a risk analysis on real options. Because some of the input variables into the real options models come directly from the discounted cash flow, and if Monte Carlo simulation is performed in the discounted cash flow analysis, the simulated events flow through to the real options models. In addition, any of the other input variables that do not flow through from the discounted cash flow models can be simulated in the real options analysis. Figure 12.15 shows the distribution of results from the eNPV analysis.

THE NEXT STEPS

After completing this book, what next? How do you implement real options in practice at your corporation or on your clients' projects? What additional materials exist to assist you in your quest?

Personally, the author has found the following approaches to be rather effective in disseminating the idea and practice of real options and risk analysis within an organization:

- *It has to be a three-pronged effort.* Analysts need to be trained with the methodology and accoutered with the relevant software and tools to help implement these real options and risk analytics techniques. Middle management needs to understand the fundamentals in order to be effective middle-men between the analysts and senior management. Finally and most importantly, you need buy-in and sponsorship from senior management, otherwise the methodologies will never be implemented.

Description

This report shows the levels of risk in your option value analysis. The probability distribution shows each value as a fraction of the total.

Results

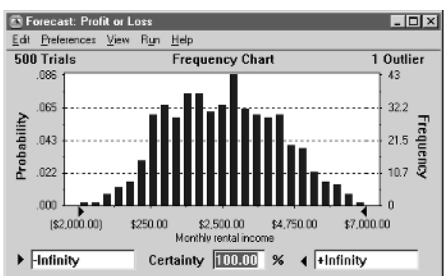
The chart shows all possible outcomes based on Monte Carlo simulation of the input variables in the real options models. The frequency chart shows the probability at the 90% confidence level occurring between $230 and $6850.

Percentiles (description):

10% = $250 20% = $350 30% = $750 40% = $1050

50% = $2550 60% = $3150 70% = $4900 80% = $6220

90% = $6520 100% = $7000



Summary

FIGURE 12.15 Real Options Risk Analysis

- ***Case studies have to be implemented.*** You need to show that real options analysis is practical and applicable in daily corporate decisions, and that real options analysis is not only applicable in the finance department, or the strategy division, or the R&D division, and so forth, but permeates all facets of the organization. Creating quick executive summary cases—several short two-page case studies within different functions and divisions of the organization—even with high-level estimates, will show management and other staff members the applicability of real options and risk analysis within your organization. This is because by using the same project names, terminologies, and lingo used in your organization, these cases become directly applicable—this psychological barrier needs to be breached. The caveat that needs to be properly addressed is that the figures used are only for illustrative purposes and more detailed analyses are required for more exact valuations. Finally, a traditional analysis needs to be presented side by side in comparison with the more advanced real options and risk analytics in order to showcase the value-added propositions of these new methodologies.
- ***Staff and management need to be educated.*** Both management and analysts need to understand the fundamental concepts of real options and risk analysis and that real options analysis is not a scary academic exercise but can be very real and practical. For instance, the author teaches public and corporate on-site courses on Real Options for Managers (a one-day high-level strategy case study discussions and corporate applications with executives), Real Options for Analysts (a two-day in-depth, hands-on software, options framing, and case study training for those who need to apply and model real options), and Risk Analysis (a two-day overall big picture of risk analysis including Monte Carlo simulation, forecasting, optimization, and the fundamentals of real options). These courses are also available on DVD by the author. Consider implementing such programs within your organization so that risk analysis and real options methodologies become routine. Knowledge provides the same terminology and the basis for more open discussions and eventual implementation.
- ***Show that the implementation cost is low and the learning curve is not steep.*** This approach is important because methodologies that take a substantial amount of time, money, and other resources will be quickly dismissed as too costly to implement. Show that books like this one exists, and software tools like the Real Options Super Lattice Solver and Risk Simulator exist, training courses exist, and competent consultants exist to help implement these strategies within your organization. It really then behooves the firm to spend only a few thousand dollars to obtain such tools and train its employees to properly value multimillion or multibillion dollar projects. The returns on investment are enormous. Even more vital is what will it cost the organization if wrong decisions

are being made because of not looking at the right valuation and decision analysis approach?

It is the author's hope that his books, software tools, training DVD videos, and training courses have helped bring the corporate world a step closer to applying such important and breakthrough analyses. It might still take a few years before such techniques become commonplace in corporate decision making, but those who take the first step and become early adopters by pushing the envelope on these new analytics might very well find themselves ahead of the crowd in terms of competitive advantage and create competitive intelligence that really will drive profit and shareholder wealth maximization. It is indeed the responsibility of senior management to perform all the due diligence it can in terms of evaluating these new techniques, so that it can make the best and most informed strategic decisions in guiding the company forward on the optimal path while navigating through the treacherous waters of uncertainty.

SUMMARY

Presenting and explaining the results of a real options analysis are vital because real options analysis has always been viewed at a distance with reverence, and its methodologies are assumed to be black box. To do a good job in explaining the results, the analyst has to make this black box transparent. A good approach is to start by comparing real options and traditional analysis, understanding that real options are built on the precepts of discounted cash flow models, where the value of an option can be seen as benefits less costs. This is similar to a discounted cash flow model, with the exception that a real options analysis assumes stochastic or unknown levels of benefits and costs. In addition, the real options process has to be transparent, indicating a step-by-step process acknowledging when the traditional analysis ends and where the new analytics begin, complete with assumptions, input data, and their results. Risk analysis results should also be presented, because single-point estimates are highly unreliable, especially when it comes to real options analysis where management is skeptical of the inputs and results; hence, providing a probability range of outcomes will make the analysis more robust and results more trustworthy.

CHAPTER 12 QUESTIONS

1. Instituting a cultural change in a company is fairly difficult. However, real options analysis has a very good chance of being adopted at major corporations. What are some of the fundamental characteristics required in

a new paradigm shift in the way decisions are made before real options analysis is accepted in a corporate setting?

2. Why is explaining to management the relevant process and steps taken in a real options analysis crucial?
3. What does critical success factor mean?
4. Why is risk analysis a recommended step in a real options analysis?
5. Why is the use of payback period flawed?

APPENDIX 12A
Summary of Articles

This section lists a summary of articles published both in academic journals and popular press. The article summary has been carefully screened, and only selected articles deemed relevant to the topics discussed are summarized. The article listings are sorted by date of publication, starting from the most current. This listing is by no means complete and exhaustive.

1. Gregory Taggart. November 2001. "Wait and Seek." Taggart talks about the value of waiting and says that "real options provide a powerful way of thinking and I can't think of any analytical framework that has been of more use to me in the past 15 years that I've been in this business."
2. Gunnar Kallberg and Peter Laurin. November 2001. "Real Options in R&D Capital Budgeting—A Case Study at Pharmacia & Upjohn." Their model shows that a user-friendly spreadsheet including an options approach could be a valuable and descriptive tool to add to a traditional NPV technique. The conclusion of the thesis is that Pharmacia & Upjohn should consider implementing the real options approach in order to value the flexibility and opportunities inherent in their future projects.
3. Jerry Flatto. November 2001. "Using Real Options in Project Evaluation." This article discusses the concept of real options, which can be used to expand your existing analysis methods. It also discusses why many of the existing analysis techniques underestimate the value of a project and how real options can capture some of the benefits that slip through the cracks under existing analysis methods.
4. John M. Charnes, David Kellogg, and Riza Demirer. October 2001. "Valuation of a Biotechnology Firm: An Application of Real Options Methodologies." This paper computes the value of a biotechnology firm, Agouron Pharmaceuticals, Inc., as the sum of the values of its current projects. Each project's value is found using the decision tree and binomial-lattice methods.
5. Robert Barker. *Business Week Online*. October 2001. "A new book on investing is being endorsed by a veritable murderer's row of finance slug-

gers, from strategist Peter Bernstein and TIAA-CREF's resident rocket scientist, Martin Leibowitz, to that DiMaggio of money managers, Legg Mason's Bill Miller. The book promotes real options as a great way to gauge hard-to-value stocks."

6. Peter Buxbaum. *BudgetLink*. October 2001. "There is a movement afoot to incorporate into business strategic thinking analytic intelligence techniques that take into account the fast-moving and uncertain world in which we live. These methodologies are increasingly being incorporated into systems that help managers plan pricing and revenue strategies, as well as supply-chain requirements. One of these emerging techniques is called real options, a concept developed in the halls of academe as an analogy to the financial option."
7. Timothy A. Luehrman, PricewaterhouseCoopers LLP. Society of Petroleum Engineers. October 2001. "Investments in oil and gas are subjected to increasingly advanced financial analyses. Real option valuation is prominent among the new financial analytical tools being applied prospectively to projects in the industry. This paper begins with the observation that many real option analyses are formally, technically correct and yet clearly lack substantial influence on decisions and ongoing project management."
8. Steven R. Rutherford, SPE, Anadarko Petroleum Corporation. Society of Petroleum Engineers. October 2001. This paper describes a real options evaluation of a real-world farm-out opportunity in case-history format.
9. S. H. Begg and R. B. Bratvold, Landmark Graphics Corporation; and J. M. Campbell, International Risk Management. Society of Petroleum Engineers. October 2001. "This paper addresses the need for a holistic, integrated approach to assessing the impacts of uncertainty on oil and gas investment decision making. Further applications are to the optimization of development plans, real options and the generation of consistent, risked cash flows for input to portfolio analysis."
10. A. Galli, SPE, ENSMP; T. Jung, Gaz de France; M. Armstrong, ENSMP; and O. Lhote, Gaz de France. Society of Petroleum Engineers. October 2001. "This case study is on a satellite platform close to a large gas and condensate oil field in the North Sea."
11. H. T. Hooper III and S. R. Rutherford, SPE, Anadarko Petroleum Corporation. Society of Petroleum Engineers. October 2001. "Three approaches to economic evaluations that have been widely discussed in the literature are decision trees, Monte Carlo simulation, and real options. Authors have shown that the incorporation of decision tree logic into Monte Carlo simulation offers an added degree of insight into the evaluation, and generally provides a more realistic valuation of an asset by incorporating some degree of management decision making. While

probabilistic economics (either decision trees or Monte Carlo simulations) and real options differ significantly in the type and amount of input data, as well as the format and applicability of output, they both have capability to capture some value of active decision making by management. This paper attempts to bridge the gap between the two approaches, at least conceptually, for the practicing engineer."

12. Soussan Faiz, SPE, Texaco Inc. Society of Petroleum Engineers. October 2001. "Various business technologies will distinguish future industry winners. The real options paradigm is emerging as the state-of-the-art method for asset valuation. The concept of an 'efficient frontier' is making headway for portfolio optimization within the energy sector."
13. Lynn B. Davidson. Society of Petroleum Engineers. September 2001. "This paper examines practical barriers to effectively using risk-based decision tools and provides guidelines to overcome the barriers. The second section explores real options: their appeal, problems, and best use. The third section of the paper focuses on challenges related to portfolio optimization. The paper ends with a summary of recommendations to improve decision quality."
14. Deloitte Consulting. Yahoo!. September 2001. "Utilities and other energy companies need better methods of deciding how heavily to bet on particular services and facilities, methods that foster flexibility in the face of an uncertain business environment. It also includes interviews with Doug Lattner, global director of the Deloitte Consulting energy practice."
15. Christopher L. Culp. *The RMA Journal.* September 2001. "This article explores common types of real options in the context of banking. Knowing the various types of options can help managers identify often-hidden opportunities and risks."
16. Kevin Sullivan, William Griswold, Yuanfang Cai, and Ben Hallen. ACM SIGSOFT Symposium and Joint International Conference on Software Engineering. September 2001. "We evaluate the potential of a new theory—developed to account for the influence of modularity on the evolution of the computer industry—to inform software design. The theory uses design structure matrices to model designs and real options techniques to value them."
17. David Newton. *Financial Times.* June 2001. Newton gives the reader a clear understanding of how real options are applicable to decisions in the R&D industry and recognizes that knowledge has a value offset by the time it takes to acquire.
18. Ash Vesudevan. *CommerceNet.* March 2001. "The greatest rewards go to those companies that can create new business models in the context of changing technological and demographic trends. Often times [sic], risk reduction becomes a competitive imperative in response to uncertainty. An

options approach, however, invokes a new perspective—profiting from uncertainty. It gives you a chance to be at the edge of the future. Doing that requires the right combination of breadth and depth. Innovation today is a competitive imperative."

19. Zeke Ashton and Bill Man. Motleyfool.com. February 2001. "Looking at some real-world companies that use Real Options."

20. Zeke Ashton. Motleyfool.com. February 2001. "Investors can use the concept of real options to explain part of the difference in market value and the intrinsic value as calculated using traditional methods. Real options represent what is possible beyond the current business operations. Investors can ignore real options, try to find real option value for free, or consciously seek out companies that have abundant real option value."

21. Shi-Jie Deng, UC Berkeley; Blake Johnson, Stanford; and Aram Sogomonian, Pacificorp. *Decision Support Systems.* January 2001. "Valuing electricity derivatives using replicating portfolios. These valuation results are used to construct real options-based valuation formulae for generation and transmission assets."

22. Hemantha S. B. Herath, University of Northern British Columbia; and Chan S. Park, Auburn University. *The Engineering Economist.* January 2001. Herath and Park show that the option value is equivalent to the Expected Value of Perfect Information (EVPI). Models include environmental preservation that involves a decision to develop a track of land and manufacturing of a new toy.

23. Chana R. Schoenberger. *Forbes Global.* December 2000. Schoenberger writes about using real options theory to value entertainment and cable stocks.

24. Rita Gunther McGrath and Ian MacMillan. *Business and Management Practices.* July 2000. McGrath and MacMillan present a methodology called STAR (strategic technology assessment review) for assessing uncertain projects that approximates option value through scoring a series of statements.

25. Diana Angelis, Naval Postgraduate School. *Business and Management Practices.* July 2000. Angelis uses Black-Scholes to capture the value of a research and development project. Her model is based on determining volatility from the underlying distributions of costs and revenue rather than net cash flows. She uses an example from Merck Pharmaceutical.

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Case Studies and Problems in Real Options

The following case studies require the use of Super Lattice Solver software.* These cases are found throughout the book and are summarized here for your convenience.

CASE STUDY: OPTION TO ABANDON

Suppose a pharmaceutical company is developing a particular drug. However, due to the uncertain nature of the drug's development progress, market demand, success in human and animal testing, and FDA approval, management has decided that it will create a strategic abandonment option. That is, at any time period within the next five years of development, management can review the progress of the R&D effort and decide whether to terminate the drug development program. After five years, the firm would have either succeeded or completely failed in its drug development initiative, and there exists no option value after that time period. If the program is terminated, the firm can potentially sell off its intellectual property rights of the drug in question to another pharmaceutical firm with which it has a contractual agreement. This contract with the other firm is exercisable at any time within this time period, at the whim of the firm owning the patents.

Using a traditional discounted cash flow model, you find the present value of the expected future cash flows discounted at an appropriate market risk-adjusted discount rate to be $150 million. Using Monte Carlo simulation, you find the implied volatility of the logarithmic returns on future cash flows to be 30 percent. The risk-free rate on a riskless asset for the same time frame is 5 percent, and you understand from the intellectual property officer of the firm that the value of the drug's patent is $100 million contractually, if sold within the next five years. For simplicity, you assume that this $100

*A full version of the software is required. Answers to the cases are available via Wiley Higher Education for downloading by registered faculty members. Wiley Higher Education may be accessed through the John Wiley & Sons, Inc., Web site (www.Wiley.com).

million salvage value is fixed for the next five years. You attempt to calculate how much this abandonment option is worth and how much this drug development effort on the whole is worth to the firm. By virtue of having this safety net of being able to abandon drug development, the value of the project is worth more than its net present value. You decide to use a closed-form approximation of an American put option because the option to abandon drug development can be exercised at any time up to the expiration date. You also decide to confirm the value of the closed-form analysis with a binomial lattice calculation. With these assumptions, do the following exercises, answering the questions that are posed:

1. Solve the abandonment option problem analytically and confirm the results using the software.
2. Select the right choice for each of the following:
   a. Increases in maturity (increase/decrease) an abandonment option value.
   b. Increases in volatility (increase/decrease) an abandonment option value.
   c. Increases in asset value (increase/decrease) an abandonment option value.
   d. Increases in risk-free rate (increase/decrease) an abandonment option value.
   e. Increases in dividend (increase/decrease) an abandonment option value.
   f. Increases in salvage value (increase/decrease) an abandonment option value.
3. Verify the results using the software's closed-form *American Put Option Approximation.*
4. Using the *Custom Lattice,* build and solve the abandonment option.
5. Use the *Black-Scholes* model to benchmark the results.
6. Apply 100 steps using the software's binomial lattice.
   a. How different are the results as compared to the 5-step lattice?
   b. How close are the closed-form results compared to the 100-step lattice?
7. Apply a 3 percent continuous dividend yield to the 100-step lattice.
   a. What happens to the results?
   b. Does a dividend yield increase or decrease the value of an abandonment option?
8. Assume that the salvage value increases at a 10 percent annual rate. Show how this can be modeled using the software.

CASE STUDY: OPTION TO EXPAND

Suppose a growth firm has a static valuation of future profitability using a discounted cash flow model (in other words, the present value of the expected future cash flows discounted at an appropriate market risk-adjusted discount rate) is found to be $400 million. Using Monte Carlo simulation, you calculate the implied volatility of the logarithmic returns on the projected future cash flows to be 35 percent. The risk-free rate on a riskless asset for the next five years is found to be yielding 7 percent. Suppose that the firm has the option to expand and double its operations by acquiring its competitor for a sum of $250 million at any time over the next five years. What is the total value of this firm assuming you account for this expansion option?

You decide to use a closed-form approximation of an American call option because the option to expand the firm's operations can be exercised at any time up to the expiration date. You also decide to confirm the value of the closed-form analysis with a binomial lattice calculation. Do the following exercises, answering the questions that are posed:

1. Solve the expansion option problem analytically, using the software.
2. Rerun the expansion option problem using the software for 100 steps, 300 steps, and 500 steps. What are your observations?
3. Show how you would use the *American Call Approximation* to estimate and benchmark the results from an expansion option. How comparable are the results?
4. Show the different levels of expansion factors but still yielding the same expanded asset value of $800. Explain your observations in terms of why the expansion value changes, and why the *Black-Scholes* and *American Option Approximation* models are insufficient to capture the fluctuation in value.
   a. Use an expansion factor of 2.00 and an asset value of $400.00 (yielding an expanded asset value of $800).
   b. Use an expansion factor of 1.25 and an asset value of $640.00 (yielding an expanded asset value of $800).
   c. Use an expansion factor of 1.50 and an asset value of $533.34 (yielding an expanded asset value of $800).
   d. Use an expansion factor of 1.75 and an asset value of $457.14 (yielding an expanded asset value of $800).
5. Add a dividend yield, and see what happens. Explain your findings.
   a. What happens when the dividend yield equals or exceeds the risk-free rate?
   b. What happens to the accuracy of closed-form solutions like the *Black-Scholes* and *American Call Approximation* models?
6. What happens to the decision to expand if a dividend yield exists?

CASE STUDY: OPTION TO CONTRACT

You work for a large aeronautical manufacturing firm that is unsure of the technological efficacy and market demand of its new fleet of long-range supersonic jets. The firm decides to hedge itself through the use of strategic options, specifically an option to contract 50 percent of its manufacturing facilities at any time within the next five years.

Suppose the firm has a current operating structure whose static valuation of future profitability using a discounted cash flow model (in other words, the present value of the expected future cash flows discounted at an appropriate market risk-adjusted discount rate) is found to be $1 billion. Using Monte Carlo simulation, you calculate the implied volatility of the logarithmic returns on the projected future cash flows to be 50 percent. The risk-free rate on a riskless asset for the next five years is found to be yielding 5 percent. Suppose the firm has the option to contract 50 percent of its current operations at any time over the next five years, thereby creating an additional $400 million in savings after this contraction. This is done through a legal contractual agreement with one of its vendors, who has agreed to take up the excess capacity and space of the firm, and at the same time, the firm can scale back its existing workforce to obtain this level of savings.

A closed-form approximation of an American option can be used, because the option to contract the firm's operations can be exercised at any time up to the expiration date and can be confirmed with a binomial lattice calculation. Do the following exercises, answering the questions that are posed:

1. Solve the contraction option problem analytically, using the software.
2. Modify the continuous dividend payout rate until the option breaks even. What observations can you make at this break-even point?
3. Use the *American Closed-Form Put Approximation* to benchmark the contraction option. What are the input parameters?
4. How can you use the *American Option to Abandon* model as a benchmark to estimate the contraction option? If it is used, are the resulting option values comparable?
5. Change the contraction factor to 0.7, and answer Question 4. Why are the answers different?

CASE STUDY: OPTION TO CHOOSE

Suppose a large manufacturing firm decides to hedge itself through the use of strategic options. Specifically, it has the option to choose among three strategies: expanding its current manufacturing operations, contracting its manufacturing operations, or completely abandoning its business unit at any time within the next five years. Suppose the firm has a current operating structure whose static valuation of future profitability using a discounted cash flow model (in other words, the present value of the future cash flows discounted at an appropriate market risk-adjusted discount rate) is found to be $100 million.

Using Monte Carlo simulation, you calculate the implied volatility of the logarithmic returns on the projected future cash flows to be 15 percent. The risk-free rate on a riskless asset for the next five years is found to be yielding 5 percent annualized returns. Suppose the firm has the option to contract 10 percent of its current operations at any time over the next five years, thereby creating an additional $25 million in savings after this contraction. The expansion option will increase the firm's operations by 30 percent, with a $20 million implementation cost. Finally, by abandoning its operations, the firm can sell its intellectual property for $100 million. Do the following exercises, answering the questions posed:

1. Solve the chooser option problem analytically, using the software.
2. Recalculate the option value accounting only for an expansion option.
3. Recalculate the option value accounting only for a contraction option.
4. Recalculate the option value accounting only for an abandonment option.
5. Compare the results of the sum of these three individual options in Questions 2 to 4 with the results obtained in Question 1 using the chooser option.
   a. Why are the results different?
   b. Which value is correct?
6. Prove that if there are many interacting options, if there is a single dominant strategy, the value of the project's option value approaches this dominant strategy's value. That is, perform the following steps, then compare and explain the results.
   a. Reduce the expansion cost to $1.
   b. Increase the contraction savings to $100.
   c. Increase the salvage value to $150.
   d. What inferences can you make based on these results?

CASE STUDY: COMPOUND OPTIONS

In a compound option analysis, the value of the option depends on the value of another option. For instance, a pharmaceutical company currently going through a particular FDA drug approval process has to go through human trials. The success of the FDA approval depends heavily on the success of human testing, both occurring at the same time. Suppose that the former costs $900 million and the latter, $500 million. Further suppose that both phases occur simultaneously and take five years to complete. Using Monte Carlo simulation, you calculate the implied volatility of the logarithmic returns on the projected future cash flows to be 25 percent. The risk-free rate on a riskless asset for the next five years is found to be yielding 7.7 percent. The drug development effort's static valuation of future profitability using a discounted cash flow model (in other words, the present value of the future cash flows discounted at an appropriate market risk-adjusted discount rate) is found to be $1 billion. Do the following exercises, answering the questions that are posed:

1. Solve the simultaneous compound option analytically, using the software. Use 5 and 100 steps for comparison.
2. Swap the implementation costs such that the first cost is $500 and the second cost is $900. Is the resulting option value similar or different? Why?
3. What happens when part of the cost of the first option is allocated to the second option? For example, make the first cost $450 and the second cost $950. Does the result change? Explain.
4. Show how an *American Call Option Approximation* can be used to benchmark the results from a simultaneous compound option.
5. Show how a *Sequential Compound Option* can also be used to calculate or at least approximate the simultaneous compound option result. Use the software's SLS and MSLS modules.

CASE STUDY: COMPOUND OPTIONS II

Compound options can also be analyzed using closed-form models rather than binomial lattices. In theory, the results obtained from binomial lattices have to approach closed-form models. As additional practice, do the following exercises, answering the questions pertaining to compound options:

1. Compute the *American Simultaneous Compound Option* in the software, obtain the option value of an asset worth $1,000, 50 percent volatility, five years to maturity, and an assumed 5 percent risk-free rate. Further, assume that the costs of the first and second options are both $500. Show the value obtained using a 5-step lattice and a 100-step super lattice analysis.
2. Now, suppose the compound option occurs in sequence and not simultaneously. That is, assume that the underlying time is now four years and the option time is two years. All other input parameters are identical. Solve the *American Sequential Compound Option* value (use 5 steps and 100 steps in the binomial lattice).

CASE STUDY: SEQUENTIAL COMPOUND OPTION

A sequential compound option exists when a project has multiple phases and latter phases depend on the success of previous phases. Suppose a project has two phases, the first of which has a one-year expiration that costs $500 million. The second phase's expiration is three years and costs $700 million. Using Monte Carlo simulation, the implied volatility of the logarithmic returns on the projected future cash flows is calculated to be 20 percent. The risk-free rate on a riskless asset for the next three years is found to be yielding 7.7 percent. The static valuation of future profitability using a discounted cash flow model—in other words, the present value of the future cash flows discounted at an appropriate market risk-adjusted discount rate—is found to be $1,000 million. Do the following exercises, answering the questions posed:

1. Solve the sequential compound option analytically, using the software. Which module do you use?
2. Change the sequence of the costs. That is, set the first phase's cost to $700 and the second phase's cost to $500. Compare your results. Explain what happens.

CASE STUDY: CHANGING STRIKES

A modification to the option types we have thus far been discussing is the idea of changing strikes—implementation costs for projects may change over time. Putting off a project for a particular period may mean a higher cost. Keep in mind that changing strikes can be applied to any previous option types as well; in other words, one can mix and match different option types. Suppose implementation of a project in the first year costs $80 million but increases to $90 million in the second year due to expected increases in the cost of raw materials and input costs. Using Monte Carlo simulation, you calculate the implied volatility of the logarithmic returns on the projected future cash flows to be 50 percent. The risk-free rate on a riskless asset for the next two years is found to be yielding 7.0 percent. The static valuation of future profitability using a discounted cash flow model (in other words, the present value of the future cash flows discounted at an appropriate market risk-adjusted discount rate) is found to be $100 million. Do the following exercises, answering the questions posed:

1. Solve the changing strikes option analytically, using the software. However, change the maturity to five years instead for the software. Use the binomial lattice of five steps.

2. Rerun the analysis after changing the first year's costs to $90 million and the second year's costs to $80 million. Explain the results. Are they intuitive?

CASE STUDY: CHANGING VOLATILITY

Instead of changing strike costs over time, in certain cases volatility on cash flow returns may differ over time. Assume a two-year option in which volatility is 20 percent in the first year and 30 percent in the second year. In this circumstance, the up and down factors are different over the two time periods. Thus, the binomial lattice will no longer be recombining. Assume an asset value of $100, implementation costs of $110, and a risk-free rate of 10 percent. (Note that changing volatility options can also be solved analytically using nonrecombining trees—see the section on nonrecombining lattices).

1. Solve the problem analytically, using the software. Which module do you use?
2. Change the first volatility to 30 percent and the second to 20 percent. What happens?

CASE STUDY: OPTION TO CONTRACT AND ABANDON

1. Solve the following Contraction and Abandonment option: Asset value of $100, five-year economic life, 5 percent annualized risk-free rate of return, 25 percent annualized volatility, 25 percent contraction with a $25 savings, and a $70 abandonment salvage value.
2. Show and explain what happens when the salvage value of abandonment far exceeds any chances of a contraction. For example, set the salvage value at $200.
3. In contrast, set the salvage value back to $70, and increase the contraction savings to $100. What happens to the value of the project?
4. Solve just the contraction option in isolation. That is, set the contraction savings to $25 and explain what happens. Change the savings to $100 and explain the change in results. What can you infer from dominant option strategies?
5. Solve just the abandonment option in isolation. That is, set the salvage value to $70, and explain what happens. Change the salvage value to $200, and explain the change in results. What can you infer from dominant option strategies?

6. Redo all the preceding questions using the Lattice Maker module and explain your observations pertaining to the decision lattices.

CASE STUDY: BASIC BLACK-SCHOLES WITH DIVIDENDS

The Black-Scholes equation is applicable for analyzing European-type options—that is, options that can be executed only at maturity and not before. The original Black-Scholes model cannot solve an option problem when there are dividend payments. However, extensions of the Black-Scholes model, termed the Generalized Black-Scholes model, can accommodate a continuous dividend payout for a European Option.

Do the following exercises and answer the questions posed, assuming that a European call option's asset value and strike cost are $100, subject to 25 percent volatility. The maturity on this option is five years, and the corresponding risk-free rate on a similar asset maturity is 5 percent.

1. Using the software, calculate the European call option.
2. Compare your results using 5, 10, 50, 100, 300, 500, and 1,000 steps in the super lattice routine. Explain what happens when the number of steps gets higher.
3. Now assume that a continuous dividend payout yielding 3 percent exists. What happens to the value of the option?
4. Show that the value of an American option is identical to the European option when no dividends are paid. That is, it is never optimal to execute an American call option early when no dividend payouts exist.
5. Show that as a 3 percent dividend yield exists, the value of the American call option exceeds the value of a European option. Why is this so?

CASE STUDY: BARRIER OPTIONS

Barrier options are combinations of call and put options such that they become in-the-money or out-of-the-money when the asset value breaches an artificial barrier.

Standard single upper barrier options can be call-up-and-in, call-up-and-out, put-up-and-in, and put-up-and-out. Standard single lower barrier options can be call-down-and-in, call-down-and-out, put-down-and-in, and put-down-and-out. Double barrier options are combinations of standard single upper and lower barriers.

1. Using the double barrier option, change each input parameter, and explain the effects on the up-and-in and down-and-in call option, up-and-in and down-and-in put option, up-and-out and down-and-out call option, and up-and-out and down-and-out put option. Explain your observations when the barrier levels change or when volatility increases.
2. Replicate the analysis using a standard lower barrier option.
3. Replicate the analysis using a standard upper barrier option.

Answers to Chapter Questions

CHAPTER 1

1. What are some of the characteristics of a project or a firm that is best suited for a real options analysis? *The project has to be faced with uncertainty, a financial model can be built, management must have flexibility or options to make midcourse corrections when uncertainty becomes resolved over time, and management must be credible enough to execute the profit-maximizing behavior at the appropriate time.*
2. Define the following:
   a. Compound option. *The value of an option depends on the value of another option executed either concurrently or in sequence.*
   b. Barrier option. *The value of an option depends on the asset's breaching or not breaching an artificial barrier.*
   c. Expansion option. *The value of an option where a project has the strategic ability but not the obligation to expand its existing operations.*
3. If management is not credible in acting appropriately through profit-maximizing behavior, are strategic real options still worth anything? *No. All the strategic options in the world are worthless if they will all be left to expire without execution because management does not act appropriately.*

CHAPTER 2

1. What are the three traditional approaches to valuation? *The market approach, the income approach, and the cost approach.*
2. Why should benefits and costs be discounted at two separate discount rates? *Benefits and costs have different sets of risks. Benefits are usually driven by a project's or firm's revenues, which are subject to market risks and uncertainty; hence, benefits should be discounted at the market risk-adjusted rate of return. In retrospect, costs are usually faced with private*

risk, which means that the market will not compensate the project for this risk; hence, costs should be discounted at the risk-free rate.

3. Is the following statement true? Why or why not? "The value of a firm is simply the sum of all its individual projects." *False. The value of a firm is greater than the sum of its parts. This is due to network effects, diversification, synergy, and the ability to leverage on existing projects to provide growth options for the future.*
4. What are some of the assumptions required in order for the CAPM to work? *Investors are risk-averse individuals who maximize their expected utility of their end-of-period wealth; investors are price-takers and have homogeneous beliefs and expectations about asset returns; there exists a risk-free asset and investors may borrow or lend unlimited amounts at the risk-free rate; the quantities of assets are fixed and all assets are marketable and perfectly divisible; asset markets are frictionless and information is costless and available to all investors; and there are no market imperfections like taxes, regulations, or restrictions on short sales.*
5. The answer is available for downloading by registered faculty members on the John Wiley & Sons, Inc., Web site (www.Wiley.com).

CHAPTER 3

1. Can an option take on a negative value? *No, the value of an option is always positive or zero, by definition. However, the value of an option may be surpassed by the premium required to create the strategic option to begin with. Hence, the net value may be negative, but the option itself is never negative.*
2. Why are real options sometimes viewed as strategic maps of convoluted pathways? *In traditional analyses, the discounted cash flow assumes that all decisions are made at the outset, with no recourse for midcourse corrections. In retrospect, real options analysis assumes that future outcomes are uncertain and that management has the strategic flexibility to make midcourse corrections whenever it deems appropriate, when some of these uncertainties become known. That is, management sees projects as having different potential outcomes, akin to a strategic roadmap, which it can navigate.*
3. Why are real options seen as risk-reduction and value-enhancement strategies? *Because real options imply that certain projects should not be executed if conditions are poor, or that instituting certain strategic options increases the value of the project. Thus, real options mitigate downside risks. This is the risk reduction effect, while the revenue enhancement effect comes from the ability of real options to leverage cer-*

tain exogenous business conditions, such as expanding when conditions are appropriate, to capture the upside potential of a project.

4. Why are the real options names usually self-explanatory and not based on names of mathematical models? *Using basic names is important—when it comes to explaining the process and results to management, it makes it easier for management to understand, and therefore increases the chances of acceptance of the methodology and results. The use of mathematical or formulaic names is irrelevant and serves no special purpose here.*
5. What is a Tornado diagram as presented in Figure 3.5's example? *A tornado diagram is named for its shape—it lists the variables that drive an analysis, where the most sensitive variables are listed first, in descending order of magnitude. For example, in a net present value analysis, a tornado diagram will list the inputs that the net present value is most sensitive to. Armed with this information, the analyst can then decide which key variables are highly uncertain in the future and which are deterministic. The uncertain key variables that drive the net present value and hence the decision are called critical success drivers. These critical success drivers are prime candidates for Monte Carlo simulation.*

CHAPTER 4

1. What is Monte Carlo simulation? *Monte Carlo simulation is a generic type of parametric simulation, that is, a simulation where the particular variable to be simulated is assumed to follow certain distributional parameters, hence the term parametric. The variable under simulation is replaced with randomly generated numbers from the specified distribution and its associated parameters thousands of times. This is akin to creating thousands of scenario analyses. The result of a simulation is usually a distribution of forecasts of the variable of interest, complete with probabilities of outcomes.*
2. What is portfolio optimization? *Portfolio optimization considers the interconnected behavior and diversification effects of multiple projects grouped into a portfolio. The goal of portfolio optimization is usually to maximize a certain variable (returns, profits, profit-to-risk index) or to minimize a certain variable (cost, risks, and so forth) in the context of a rolled-up portfolio, while considering the project's interrelationships. These goals are achieved subject to certain requirements or constraints (budget, timing resources, and so forth). The results are usually a set of criteria on how resources should be optimally allocated across multiple projects to meet these goals.*

3. Why is update analysis required in a real options analysis framework? *Because real options analysis is a dynamic decision analysis, where there is value in uncertainty, updates are required because when uncertainty becomes resolved through the passage of time, decisions need to be made. Update analysis reflects the decisions made and what future actions may be appropriate going forward.*
4. What is problem framing? *Problem framing is viewing the project under analysis within a real options paradigm—in other words, identifying where strategic flexibility value exists and how management can exert its flexibility in decision making and create value.*
5. Why are reports important? *Reports are important because they provide a concise and coherent view of the analytical process as well as the results obtained through this process.*

CHAPTER 5

1. What do you believe are the three most important differences between financial options and real options? *The length of maturity in real options far surpasses that of a financial option. The underlying assets in financial options are highly liquid and tradable, and historical data are available. In real options, assets are usually nontradable and illiquid. The value of a financial option is relatively small compared to significant values in real options.*
2. In the Flaw of Averages example, a nonparametric simulation approach is used. What does nonparametric simulation mean? *Nonparametric means no parameters. That is, nonparametric simulation does not make any distributional and parameter assumptions. Instead, the simulation uses historical data.*
3. In simulating a sample stock price path, a stochastic process called Geometric Brownian Motion is used. What does a stochastic process mean? *A stochastic process is the opposite of a deterministic process. That is, the outcome of a stochastic process cannot be predicted in advance; instead, it has an uncertainty variable that changes at every simulation trial. A stochastic process can be easily valued through Monte Carlo simulation.*
4. What are some of the restrictive assumptions used in the Black-Scholes equation? *The stocks underlying the call or put options provide no dividends during the life of the option; there are no transaction costs involved with the sale or purchase of either the stock or the option; the short-term risk-free interest rate is known and is constant during the life of the option; the security buyers may borrow any fraction of the purchase price at the short-term risk-free rate; short-term selling is permit-*

ted without penalty, and sellers immediately receive the full cash proceeds at today's price for securities sold short; call or put options can be exercised only on their expiration date; security trading takes place in continuous time, and stock prices move in continuous time.

5. The answer is available for downloading by registered faculty members on the John Wiley & Sons, Inc., Web site (www.Wiley.com).

CHAPTER 6

1. Why does solving a real options problem using the binomial lattices approach the results generated through closed-form models? *The underlying mathematical structures of both approaches are identical. That is, closed-form models are derived through stochastic differential calculus to obtain the results of a continuous simulation process. In retrospect, the binomial lattices approximate this continuous process through the creation of a discrete simulation. Hence, when the number of steps in a binomial lattice approaches infinity, such that the time between steps in a lattice approaches zero, the discrete lattice simulation becomes a continuous simulation; thus, the results are identical.*
2. Is real options analysis a special case of discounted cash flow analysis, or is discounted cash flow analysis a special case of real options analysis? *Discounted cash flow analysis is a special case of real options analysis. This is because when all uncertainty is resolved (volatility equals zero), or at the point of expiration with no time left for the option, the value of the project is exactly the discounted cash flow result, because the real options value is zero at that point.*
3. Explain what a risk-neutral probability means. *Risk-neutralization means taking the risk away from something. Risk-neutral probability means to risk-adjust the asset values at each node by taking away the risk through an adjustment of the probabilities that lead to these asset values in the first place. It is used in valuing options with binomial lattices.*
4. What is the difference between a recombining lattice and a nonrecombining lattice? *In recombining lattices—a binomial lattice, for example—the middle branches of the lattice converge to the same result; in a nonrecombining lattice, they do not. Sometimes nonrecombining lattices are required, especially when there are two or more stochastic underlying variables, or when volatility of the single underlying variable changes over time.*
5. The answer is available for downloading by registered faculty members on the John Wiley & Sons, Inc., Web site (www.Wiley.com).

CHAPTER 7

Answers are available for downloading by registered faculty members on the John Wiley & Sons, Inc., Web site (www.Wiley.com).

CHAPTER 8

1. Decision trees are considered inappropriate when used to solve real options problems. Why is this so? *Decision trees are great for setting up a real options problem as well as presenting the results. However, standalone decision trees are inadequate in solving real options problems. This is because subjective probabilities have to be assigned to each branch on a decision tree, and in a complex tree, incorrect probabilities will be compounded over different periods and the errors will grow the further out in time. In addition, because decision trees have different strategies attached to each decision node, the values assigned to each node have to be discounted using a different market risk-adjusted discount rate. Establishing the correct risk-based discount rate at each node is fairly difficult to do, and errors tend to compound over time.*
2. What are some of the assumptions required for risk-neutral probabilities to work? *Risk-neutral probabilities can be applied to binomial lattices and not to decision trees because of some of their underlying assumptions. Recall that in order for a risk-neutral probability to work, the underlying asset evolution lattice needs to be created using discrete Brownian Motion simulations. For example, a binomial lattice node has two bifurcations, one above and one below its current level, a property called the Martingale process. This spreads out to multiple time periods. However, in a decision tree analysis, the values on each strategy node in the future do not necessarily have to be above as well as below the origin node or have the same magnitude. Thus, using risk-neutralized probabilities in discounting a decision tree back to its origination point will yield grossly incorrect results.*
3. What is stochastic optimization? *Stochastic optimization is similar to a simple optimization analysis with the exception that its inputs are stochastic and changing. For instance, in Modern Portfolio Theory, the optimal portfolio allocation of resources is obtained through the maximization of returns and the minimization of risks, resulting in an efficient frontier, which can be solved mathematically. However, in a stochastic optimization problem, the inputs that drive returns and risks are stochastic and changing at every instance. Hence, stochastic optimization that utilizes Monte Carlo simulation is required to obtain the optimal values, rather than being solved mathematically.*

4. The answer is available for downloading by registered faculty members on the John Wiley & Sons, Inc., Web site (www.Wiley.com).

CHAPTER 12

1. Instituting a cultural change in a company is fairly difficult. However, real options analysis has a very good chance of being adopted at major corporations. What are some of the fundamental characteristics required in a new paradigm shift in the way decisions are made before real options analysis is accepted in a corporate setting? *The new shift in paradigm must be applicable to solving particular problems, flexible enough to be applicable across multiple types of problems, compatible with the old approach, and able to provide significant value-added insights and competitive advantage.*
2. Why is explaining to management the relevant process and steps taken in a real options analysis crucial? *Management may find it difficult to accept the results stemming from a series of black-box analyses. Instead, if a series of logical and transparent steps are instituted and explained, management buy-in may be simpler.*
3. What does critical success factor mean? *Critical success factors are the key variables that are highly uncertain and variable, and that also drive the value of the project, such that the success or failure of the project, measured financially, is subject to these factors.*
4. Why is risk analysis a recommended step in a real options analysis? *Risk analysis using Monte Carlo simulation provides a probability range of outcomes that will make the analysis more robust and results more trustworthy. Simulation also accounts for the uncertainty in the input variables that affect the analysis results.*
5. Why is the use of payback period flawed? *Payback period or break-even analysis ignores the time value of money and ignores the highly valuable stream of future cash flows beyond the break-even point. Making capital investment decisions based solely on payback period will yield incorrect and oftentimes disastrous decisions.*

Notes

CHAPTER 1. A NEW PARADIGM?

1. For a more detailed listing of articles, see Appendix 12A.

APPENDIX 1B. SCHLUMBERGER ON REAL OPTIONS IN OIL AND GAS

1. Unlike sandstones—which are made from mineral grains—carbonate rocks (like chalk) are made up of much finer particles (the calcified remains of plankton and other tiny sea creatures). As such, they have a much smaller porosity but may still contain quantities of extractable hydrocarbons that are found in fractures (from microns to meters in length) that are common to such rocks.
2. R. C. Selley, *Elements of Petroleum Geology—Second Edition*, Academic Press, 1998.
3. Pressure is needed to produce from the fluids from deep down in the ground. Sometimes the reservoir has enough pressure of its own to allow us to produce without any assistance. Other times we need to assist the reservoir in some way (most commonly by injecting water). Once we start producing oil (and/or gas), reservoir pressure goes down—as with a toy water pistol, which when fully primed can produce a high-pressure stream of water; but once the pressure is lower, the water jet becomes weak and eventually dies out. The same occurs in a reservoir, so it is important to maintain the pressure necessary to maintain flow.
4. Often (especially in the latter stages of the life of a reservoir) oil flows in conjunction with water and sometimes solid particles (like sand and possibly scale deposits). These flows need to be treated in a separator before being sent off along a pipeline (or vessel) to be refined.
5. FPSO: Floating Production and Storage Operation. These can have several forms—storage can be made on the rig and then offloaded by a buoy to a dedicated tanker that commutes back and forth from the refinery.

6. Oil (and gas) is considered to "sweep" through a reservoir. We may observe areas of a field which, for some reason, have not been swept, thereby residual deposits remain to be exploited.
7. J. Paddock, D. Siegel, and J. Smith, "Option Valuation of Claims on Physical Assets: The Case of Offshore Petroleum Leases," *Quarterly Journal of Economics* 103, no. 3 (1988), 479–508.
8. T. A. Luehrman, "Extending the Influence of Real Options: Problems and Opportunities," paper SPE 71407, presented at the 2001 Annual Technical Conference and Exhibition, New Orleans, Louisiana, September 30 to October 3, 2001.

APPENDIX 1C. INTELLECTUAL PROPERTY ECONOMICS ON REAL OPTIONS IN PATENT AND INTANGIBLE VALUATION

1. Source: Brookings Institute.
2. See Business Week Online, "Royalties: A Royal Pain for Net Radio," by Stephen Wildstrom, McGraw-Hill Publishing, March 29, 2002.

APPENDIX 1E. SPRINT ON REAL OPTIONS IN TELECOMMUNICATIONS

1. Federal Communications Commission, Trends in Telephone Service, August 2001, pg. 16-3.
2. Federal Communications Commission, Trends in Telephone Service, August 2001, pg. 12-3.
3. AT&T 10-K.

CHAPTER 2. TRADITIONAL VALUATION APPROACHES

1. The NPV is simply the sum of the present values of future cash flows less the implementation cost. The IRR is the implicit discount rate that forces the NPV to be zero. Both calculations can be easily performed in Excel using its "NPV(. . .)" and "IRR(. . .)" functions.
2. See Appendix 2B for a more detailed discussion on discount rate models.
3. A multiple regression or principal component analysis can be performed but probably with only limited success for physical assets as opposed to financial assets because there are usually very little historical data available for such analyses.

4. Chapter 9 details the steps and requirements for a Monte Carlo simulation.
5. Appendix 2A provides details on calculating free cash flows from financial statements.

APPENDIX 2B. DISCOUNT RATE VERSUS RISK-FREE RATE

1. Use the after-tax cost of debt because interest paid on debt is tax deductible. We need to include this tax shield. Therefore, *Cost of Debt = Interest Paid – Taxes Saved*. Similarly, we have *Cost of Debt* $= K_d - TK_d = K_d\,(1 - T)$.
2. The cost of preferred stock is $K_{ps} = D_{ps} \div P_{net}$, where D is the dividend paid (assumed to be a perpetuity) and P is the net or clean price paid on the preferred stock after taking into account any accrued interest and carrying costs.
3. There are generally three accepted methods to calculating the cost of equity: (a) The CAPM Approach uses $K_s = K_{rf} + \beta_i(K_m - K_{rf})$, where β is the beta-risk coefficient of the company's equity, K_m is the equity market portfolio rate of return, and K_{rf} is the corresponding maturity's risk-free Treasury rate. (b) The Discounted Cash Flow (Gordon Growth Model) assumes $K_s = [D_1 \div P_0(1 - F)] + g$, where g = Retention Rate × Return on Equity and F is the floatation cost. (c) The Risk Premium over Bond Yield approach assumes that K_s = *Bond Yield* + *Risk Premium*, corresponding to the appropriate risk structure.
4. Suppose you have an asset that costs \$100 and increases to \$110 in the first period but reverts to \$100 the second period. The return in period one is 10%, and the return in period two is −9.09%. Hence, the arithmetic average of both periods' returns is 0.455%, but it is illogical because you ended up with what you started off with. The geometric average is calculated as

$$\sqrt[2]{\frac{110}{100} \times \frac{100}{110}} - 1 = 0\%$$

which seems more logical.
5. Book value is generally used because it captures the value of the security when it was issued. However, critics have argued that the market value more closely reflects the current situation the firm faces when operating in its current condition. Furthermore, market values tend to be forward-looking, and book values tend to be backward-looking. Because the valuation analysis looks at forecast values, we can argue for the use of market

value weightings. The problem with that logic is magnified when there is significant volatility in the equity and debt market due to speculation.

6. The assumptions for the CAPM include the following: investors are risk-averse individuals who maximize their expected utility of their end-of-period wealth; investors are price-takers and have homogeneous beliefs and expectations about asset returns; there exists a risk-free asset, and investors may borrow or lend unlimited amounts at the risk-free rate; the quantities of assets are fixed and all assets are marketable and perfectly divisible; asset markets are frictionless, and information is costless and available to all investors; and there are no market imperfections like taxes, regulations, or restrictions on short sales.
7. The CAPM requires that in equilibrium the market portfolio be efficient. It must lie on the upper half of the minimum variance opportunity set where the marginal rate of substitution equals the marginal rate of transformation ($MRS = MRT$). The efficiency can be established based on homogeneous expectation assumptions. Given this, they will all perceive the same minimum variance opportunity set. The market portfolio must hence be efficient because the market is simply the sum of all holdings and all individual holdings are efficient. Given market efficiency, the market portfolio M where all assets are held according to their market value weights by simple algebraic manipulation, that is, equating the slope of the capital market line with the slope of the opportunity set, we can derive the following expression: $E(R_i) = R_f + [E(R_m) - R_f]\ (\sigma_{i,m}/\sigma^2{}_m)$. This CAPM model can also be derived using the $MRT = MRS$ convention, where a linear programming method is used to solve for minimum variance opportunity set and the maximum expected return efficiency set.
8. For instance, multicollinearity, autocorrelation, random walk (nonstationarity), seasonality, and heteroskedasticity pose a problem in macroeconomic time series. The model should be developed carefully.

CHAPTER 3. REAL OPTIONS ANALYSIS

1. These figures are for illustrative purposes. We will work through similar problems as well as more complicated real option models in later chapters.
2. This is obtained using the Gordon constant growth model for collapsing all future cash flows into a single figure. See Appendix 2A on financial statements analysis for details.

CHAPTER 4. THE REAL OPTIONS PROCESS

1. See Appendix 9B on Monte Carlo simulation for the technical details on how specific distributions are chosen and what some of the simulation conditions are.

2. Chapter 7 provides the technical step-by-step approach to applying real options analysis for multiple types of options. Chapter 8 previews more of the mathematical intricacies in options modeling.
3. Appendix 9D explains the approach to portfolio optimization.
4. Chapter 12 shows a step-by-step series of reports and how to present them to senior management, providing a novel way to explain and break down a difficult series of black-box analysis into clear, concise, and transparent procedures.

CHAPTER 5. REAL OPTIONS, FINANCIAL OPTIONS, MONTE CARLO SIMULATION, AND OPTIMIZATION

1. In an abandonment option, there is usually a maximum to the salvage value; thus, the payoff function may actually look like a put but with a limit cap on the upside.
2. In this example, the median is a better measure of central tendency.
3. See Appendix 8A for details on Geometric Brownian Motion.

CHAPTER 6. BEHIND THE SCENES

1. Appendix 7B shows the derivation of the Black-Scholes model, while Appendix 8C shows some of the closed-form solutions of the Generalized Black-Scholes model and other exotic options.
2. This is simply an illustration of the size and computational requirements for an exact binomial approximation where data from all the simulated trials are saved.
3. The simulated actual values are based on a Geometric Brownian Motion with a volatility of 20 percent calculated as the standard deviation of the simulated natural logarithms of historical returns.
4. See Appendix 8A for details on Brownian Motions.
5. In certain rare cases, these equations need to be modified, cases when multinomial lattices are used or when there are complex stochastic processes that need to be incorporated, including jump-diffusions or mean reversion.
6. In reality, this drift rate in a Martingale process is the risk-free rate, which is also used in discounting the binomial lattice values back in time.
7. This assumes a continuous discounting approach. The continuous discounting approach ($e^{-rf(\delta t)}$) is used throughout the book rather than a discrete discounting approach ($(1 + rf)^{-\delta t}$) because both approaches will provide identical results when a high number of time-steps is used (usually above 10 steps). In addition, because the enclosed software

allows the user to calculate a high number of time-steps quickly, using the continuous discounting approach will facilitate the convergence of the results at a higher rate.

CHAPTER 7. REAL OPTIONS MODEL

1. A similar approach is to use the Roll-Geske-Whaley (RGW) approximation. Note that these approximation models cannot be readily or easily solved within an Excel environment but instead require some programming scripts or the use of software. The Real Options Super Lattice software CD-ROM has these American approximation models as well as the ability to solve up to thousands of time-steps in the binomial approach.
2. Note that these approximation models cannot be readily or easily solved within an Excel environment but instead require some programming scripts or the use of software. Be aware that closed-form American option approximation models can only provide benchmark values for an expansion option.
3. There is an end-of-chapter problem analyzing the expansion option when the competition grows at a different rate and faces different risk structures. The problem can be easily tackled using binomial lattices.
4. The model is shown in Appendix 8C. Note that these approximation models cannot be readily or easily solved within an Excel environment but instead require some programming scripts or the use of software.

APPENDIX 7A. VOLATILITY ESTIMATES

1. Go to http://finance.yahoo.com and enter a stock symbol (e.g., MSFT). Click on Quotes: Historical Prices and select Weekly and select the period of interest. You can then download the data to a spreadsheet for analysis.

APPENDIX 7F. REALITY CHECKS

1. The modified internal rate of return (MIRR) takes the sum of the future values of all cash flows and discounts it to equal the present value of implementation costs. The discount rate that equates these two values is the MIRR.
2. The Wilcoxon *signed-rank* statistic calculation is represented by

$$W(X, \theta) = \sum_{i=1}^{n} S(X_i - \theta)\psi(|X_i - \theta|)$$

where S is the sign function and ψ is the rank function. From here, the p-value is calculated using

$$P\left(\left|\frac{W(\theta)-E(\theta)}{\sqrt{\frac{n(n+1)(2n+1)}{6}}}\right|<b^*\right)\cong 1-\alpha$$

APPENDIX 7G. APPLYING MONTE CARLO SIMULATION TO SOLVE REAL OPTIONS

1. This is due to the mathematical properties of American options, which require the knowledge of what the optimal stopping times and optimal execution barriers are. Using simulation to obtain American-type options is fairly difficult and is beyond the scope of this book.
2. The Excel spreadsheet is located in the *Examples* folder under the name *Simulating Options Analysis*. Note that the example spreadsheet requires that Risk Simulator software be installed to run properly. To obtain similar results shown, simply open the spreadsheet and hit the *RUN* icon. Finally, note that because Monte Carlo simulation is by definition a random selection of values from predefined distributions, the results may not match exactly those seen in the examples.

CHAPTER 8. ADVANCED OPTIONS PROBLEMS

1. The derivation of this optimal value is beyond the scope of this book, because it applies stochastic calculus analytics. Dixit and Pindyck's *Investment under Uncertainty* (1994) provides a good guide to some of the analytics of stochastic derivations.

APPENDIX 9C. FORECASTING

1. The following texts provide detailed explanations of time-series forecasting and regression analysis, arranged from more advanced to more basic applications:

   Time Series Analysis, by James D. Hamilton, Princeton University Press, 1994.

   Forecasting with Dynamic Regression Models, by Alan Pankratz, Wiley Series in Probability and Mathematical Statistics, 1991.

   Econometrics, edited by John Eatwell, Peter Newman, and Murray Milgate, W.W. Norton, 1990.

Handbook of Financial Analysis, Forecasting and Modeling, by Jae Shim and Joel Siegel, Prentice Hall, 1988.

CHAPTER 10. REAL OPTIONS VALUATION APPLICATION CASES

1. http://www.treas.gov/offices/domestic-finance/debt-management/interest-rate/yield-hist.html
2. See the section on Expansion Option for more examples on how this start-up's technology can be used as a platform to further develop newer technologies that can be worth a lot more than just the abandonment option.

CHAPTER 11. REAL OPTIONS CASE STUDIES

1. *Wall Street Journal*, April 21, 2004.
2. Financial Accounting Standards Board Web site: www.fasb.org.
3. See Johnathan Mun's *Valuing Employee Stock Options Under 2004 FAS 123* (Wiley, 2004) for details on the case study.
4. The GBM accounts for dividends on European options but the basic BSM does not.
5. American options are exercisable at any time up to and including the expiration date. European options are exercisable only at termination or maturity expiration date. Most ESOs are a mixture of both—European option during the vesting period (the option cannot be exercised prior to vesting), reverting to an American option after the vesting period.
6. These could be cliff vesting (the options are all void if the employee leaves or is terminated before this cliff vesting period) or graded monthly/quarterly/annually vesting (a certain proportion of the options vest after a specified period of employment service to the firm).
7. The BSM described in this case study refers to the original model developed by Fisher Black, Myron Scholes, and Robert Merton. Although significant advances have been made such that the BSM can be modified to take into consideration some of the exotic issues discussed in this case study, it is mathematically very complex and is highly impractical for use.
8. This multiple is the ratio of the stock price when the option is exercised to the contractual strike price, and is tabulated based on historical information. Post- and near-termination exercise behaviors are excluded.
9. A tornado chart lists all the inputs that drive the model, starting from the input variable that has the most effect on the results. The chart is obtained by perturbing each input at some consistent range (e.g., ±10 percent from the base case) one at a time, and comparing its results to the base case.

10. Different input levels yield different tornado charts but in most cases, volatility is not the only dominant variable. Forfeiture, vesting, and suboptimal exercise behavior multiples all tend to either dominate over or be as dominant as volatility.
11. A spider chart looks like a spider with a central body and its many legs protruding. The positively sloped lines indicate a positive relationship (e.g., the higher the stock price, the higher the option value as seen in Figure 11.18), while a negatively sloped line indicates a negative relationship. Further, spider charts can be used to visualize linear and nonlinear relationships.
12. People tend to exhibit suboptimal exercise behavior due to many reasons—the need for liquidity, risk aversion, personal preferences, expectations, and so forth.
13. Assumptions used: stock and strike price of $25, 10-year maturity, 5 percent risk-free rate, 50 percent volatility, 0 percent dividends, suboptimal exercise behavior multiple range of 1 to 20, vesting period of 1 to 10 years, and tested with 100 to 5,000 binomial lattice steps.
14. Assumptions used: stock and strike price range of $5 to $100, 10-year maturity, 5 percent risk-free rate, 50 percent volatility, 0 percent dividends, suboptimal exercise behavior multiple range of 1 to 20, 4-year vesting, and tested with 100 to 5,000 binomial lattice steps.
15. Assumptions used: stock and strike price of $25, 10-year maturity, 5 percent risk-free rate, 10 percent to 100 percent volatility range, 0 percent dividends, suboptimal exercise behavior multiple range of 1 to 20, 1-year vesting, and tested with 100 to 5,000 binomial lattice steps.
16. Assumptions used: stock and strike price of $25, 10-year maturity, 5 percent risk-free rate, 50 percent volatility, 0 percent dividends, suboptimal behavior 1.01, vesting period of 1 to 10 years, forfeiture range 0 percent to 50 percent, and tested with 100 to 5,000 binomial lattice steps.
17. The results only illustrate a typical case and should not be generalized across all possible cases.
18. Of the 6,553 stocks analyzed, 2,924 of them pays dividends; 2,140 of them yielding at or below 5 percent; 2,282 at or below 6 percent; 2,503 at or below 7 percent; and 2,830 at or below 10 percent.
19. Stock price and strike price are set at $100, 5-year maturity, 5 percent risk-free rate, 75 percent volatility, and 1,000 steps in the customized lattice. Other exotic variable inputs are listed in Table 11.4.
20. Stock price and strike price are set at $100, 5-year maturity, 5 percent risk-free rate, 75 percent volatility, 1,000 steps in the customized lattice, 1.8 behavior multiple, 10 percent forfeiture rate, and 1-year vesting.
21. Stock and strike price of $100, 75 percent volatility, 5 percent risk-free rate, 10-year maturity, no dividends, 1-year vesting, 10 percent forfeiture rate, and 1,000 lattice steps.

22. Stock and strike price range of \$30 to \$100, 45 percent volatility, 5 percent risk-free rate, 10-year maturity, dividend range 0 percent to 10 percent, vesting of 1 year to 4 years, 5 percent to 14 percent forfeiture rate, suboptimal exercise behavior multiple range of 1.8 to 3.0, and 1,000 lattice steps.
23. Assumptions used: stock and strike price of \$100, 10-year maturity, 1-year vesting, 35 percent volatility, 0 percent dividends, 5 percent risk-free rate, suboptimal exercise behavior multiple range of 1.2 to 3.0, forfeiture range of 0 percent to 40 percent, and 1,000-step customized lattice.
24. An alternative method is to calculate the relevant carrying cost adjustment by artificially inserting an inflated dividend yield to convert the ESO into a "soft option," thereby discounting the value of the ESO. This method is more difficult to apply and is susceptible to more subjectivity than using a put option.
25. Cedric Jolidon finds the mean values of marketability discounts to be between 20 percent and 35 percent in his article, "The Application of the Marketability Discount in the Valuation of Swiss Companies" (Swiss Private Equity Corporate Finance Association). A typical marketability range of 10 percent to 40 percent was found in several discount court cases. In the *CPA Journal* (February 2001), M. Greene and D. Schnapp found that a typical range was somewhere between 30 percent and 35 percent. Another article in the *Business Valuation Review* finds that 35 percent is the typical value (Jay Abrams, "Discount for Lack of Marketability"). In the *Fair Value* newsletter, Michael Paschall finds that 30 percent to 50 percent is the typical marketability discount used in the market.
26. Risk Simulator software was used to simulate the input variables.
27. Any level of precision and confidence can be chosen. Here, the 99.9 percent statistical confidence with a \$0.01 error precision (\$0.01 fluctuation around the average option value) is fairly restrictive. Of course, the level of precision attained is contingent on the inputs and their distributional parameters being accurate.
28. This assumes that the inputs are valid and accurate.
29. The spot rate curve used in the analysis was averaged around the past four weeks of the valuation date to obtain a better market consensus of the economic expectations.
30. This is only a sample GARCH model used to illustrate the analysis.
31. The R-squared (R^2), or coefficient of determination, is an error measurement that looks at the percent variation of the dependent variable that can be explained by the variation in the independent variable for a regression analysis, and ranges from 0 to 1.0. The higher the R^2 value, the better the model fits and explains the data. In this case, an R^2 of 0.0105 (Figure 11.27) means a bad fit and the model is not statistically significant. Thus its results could not be relied on.

32. Examples of goodness-of-fit statistics include the t-statistic and the F-statistic. The former is used to test if *each* of the estimated slope and intercepts is statistically significant, that is, if it is statistically significantly different from zero (therefore making sure that the intercept and slope estimates are statistically valid). The latter applies the same concepts but simultaneously tests the entire regression equation including the intercept and slopes. The calculated F-statistic of 1.8650 and a corresponding p-value of 0.1147 (Figure 11.27) indicate collectively that the model is statistically insignificant and the results cannot be relied on.
33. Carpenter, J. 1998. "The Exercise and Valuation of Executive Stock Options." *Journal of Financial Economics*, vol. 48, no. 2 (May).
34. Huddart, S., and Lang, M. 1996. "Employee Stock Option Exercises: An Empirical Analysis." *Journal of Accounting and Economics*, vol. 21, no. 1 (February).
35. Using an inverted Brownian Motion stochastic process, the 99.99 percent cutoff point was determined for the stock price within the specified time period given the volatility measure.
36. The higher the suboptimal exercise behavior multiple is set, the higher the option value—a conservative estimate of the multiple means that it is set higher so as not to undervalue the option.
37. A 1,000-step customized binomial lattice is generally used unless otherwise noted. Sometimes increments from 1,000 to 5,000 steps may be used to check for convergence. However, due to the nonrecombining nature of changing volatility options, a lower number of steps may have to be employed.
38. This proprietary algorithm was developed by Dr. Johnathan Mun based on his analytical work with FASB in 2003–2004; his books: *Valuing Employee Stock Options Under the 2004 FAS 123 Requirements* (Wiley, 2004), *Real Options Analysis: Tools and Techniques* (Wiley, 2002), *Real Options Analysis Course* (Wiley, 2003), *Applied Risk Analysis: Moving Beyond Uncertainty* (Wiley, 2003); his software, Real Options Analysis Toolkit (versions 1.0 and 2.0); his academic research; and his previous valuation consulting experience at KPMG Consulting.
39. A nonrecombining binomial lattice bifurcates (splits into two) every step it takes, so starting from one value, it branches out to two values on the first step (2^1), two becomes four in the second step (2^2), and four becomes eight in the third step (2^3), and so forth, until the 1,000th step (2^{1000} or over 10^{301} values to calculate, and the world's fastest supercomputer won't be able to calculate the result within our lifetimes).
40. The Law of Large Numbers stipulates that the central tendency (mean) of a distribution of averages is an unbiased estimator of the true population average. The results from 4,200 steps show a mean value

comparable to the median of the distribution of averages, and, hence, 4,200 steps is chosen as the input into the binomial lattice.

41. This has the same effect as multiplying the number of grants by (1 – Forfeiture) because total valuation is *Price* × *Quantity* × (1 – *Forfeiture*), so it does not matter whether the forfeiture adjustment is made on the option price or the quantity of option grants, as long as it is applied only once.
42. This is the extreme case where we assume 100 percent of the employee stock options will be executed once they become fully vested, to minimize the BSM results.

CHAPTER 12. RESULTS INTERPRETATION AND PRESENTATION

1. This is also sometimes referred to as the inverse of the coefficient of variation, and its concept is related to the profitability index, Tobin's *q*-ratio, the Sharpe ratio, and Jensen's alpha measure.

About the CD-ROM

INTRODUCTION

This appendix provides you with information on the contents of the CD that accompanies this book. For the latest and greatest information, please refer to the ReadMe file located at the root of the CD.

SYSTEM REQUIREMENTS

- BM PC or compatible computer with Pentium III or higher processor.
- 128 MB RAM (256 MB recommended) and 25 MB hard-disk space.
- CD-ROM drive, SVGA monitor with 256 Color.
- Excel, XP, 2003, or later.
- Windows 2000, ME, XP, or higher.
- An Internet connection is required to install the software.

USING THE CD WITH WINDOWS

There is an automated setup program available in the CD-ROM for the Real Options Valuation's Super Lattice Solver software and the Risk Simulator software. You must first be connected to the Internet to install these software. To run the setup program, do the following:

1. Insert the enclosed CD into the CD-ROM drive of your computer.
2. The setup program should come up automatically. If it does not, open Windows Explorer double click the CDAutorun.exe file to launch the interface.
3. Click on Install Real Options SLS, and click Run. Follow the onscreen instructions.
4. When prompted, enter the following user name and license key for a 30-day trial of the SLS software:

Name: **30 Day License** Key: **513C-27D2-DC6B-9666**

5. Then go back and click on Install Risk Simulator, and click Run. Follow the onscreen instructions. Do not click Finish until the secondary installation is complete at the end.

To obtain a permanent license or an extended academic trial (a special offer for professors and students), please contact Admin@RealOptionsValuation.com for details.

WHAT'S ON THE CD

The following sections provide a summary of the software and other materials you'll find on the CD.

CONTENT

The enclosed CD-ROM contains a 30-day trial version of Real Options Valuation's Super Lattice Solver software and the Risk Simulator software.

Refer to Chapter 9 for details on using the Real Options Valuation's Super Lattice Solver software and Risk Simulator software.

This CD contains an installer for the Real Options Super Lattice Solver software and Risk Simulator software. To install these software, insert the CD-ROM and the installer will automatically appear. If not, browse this CD and double click on CDAutorun.exe. You must first be connected to the Internet before installation can occur, as the installer will be downloading the latest setup files for these software from the Real Options Valuation, Inc. website (www.realoptionsvaluation.com).

This CD also contains Answers and Solutions to some of the end-of-chapter questions.

Finally, Risk Simulator requires Microsoft .NET Framework installed to function. Most new computers come with .NET Framework preinstalled. For older machines, you may have to install this manually. The DOT NET Framework folder on the CD-ROM has a file called dotnetfx.exe.

Install this file if you do not have .NET Framework preinstalled. If you do not know if you have .NET Framework on your computer, you can install this file just to make sure.

Trial, demo, or evaluation versions are usually limited either by time or functionality (such as being unable to save projects). Some trial versions are very sensitive to system date changes. If you alter your computer's date, the programs will "time out" and no longer be functional.

CUSTOMER CARE

If you have trouble with the CD-ROM, please call the Wiley Product Technical Support phone number at (800) 762-2974. Outside the United States, call 1(317) 572-3994. You can also contact Wiley Product Technical Support at http://techsupport.wiley.com. John Wiley & Sons will provide technical support only for installation and other general quality control items. For technical support on the applications themselves, consult the program's vendor or author.

To place additional orders or to request information about other Wiley products, please call (877) 762-2974.

If you have questions about obtaining the fully functional software, please contact Dr. Johnathan Mun at JohnathanMun@cs.com.

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For more information about the CD-ROM, see the **About the CD-ROM** section on page 645.

CUSTOMER NOTE
IF THIS BOOK IS ACCOMPANIED BY SOFTWARE, PLEASE READ THE FOLLOWING BEFORE OPENING THE PACKAGE.

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